Computer algebra independent integration tests

`1_Algebraic_functions/1.2_Trinomial_products/1.2.2Quartic/1.2.2.5P(x)(a+bx^2+cx^4)^p`

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Contents

1 Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from Albert Rich Rubi web site.

1.1 Listing of CAS systems tested

The following systems were tested at this time.

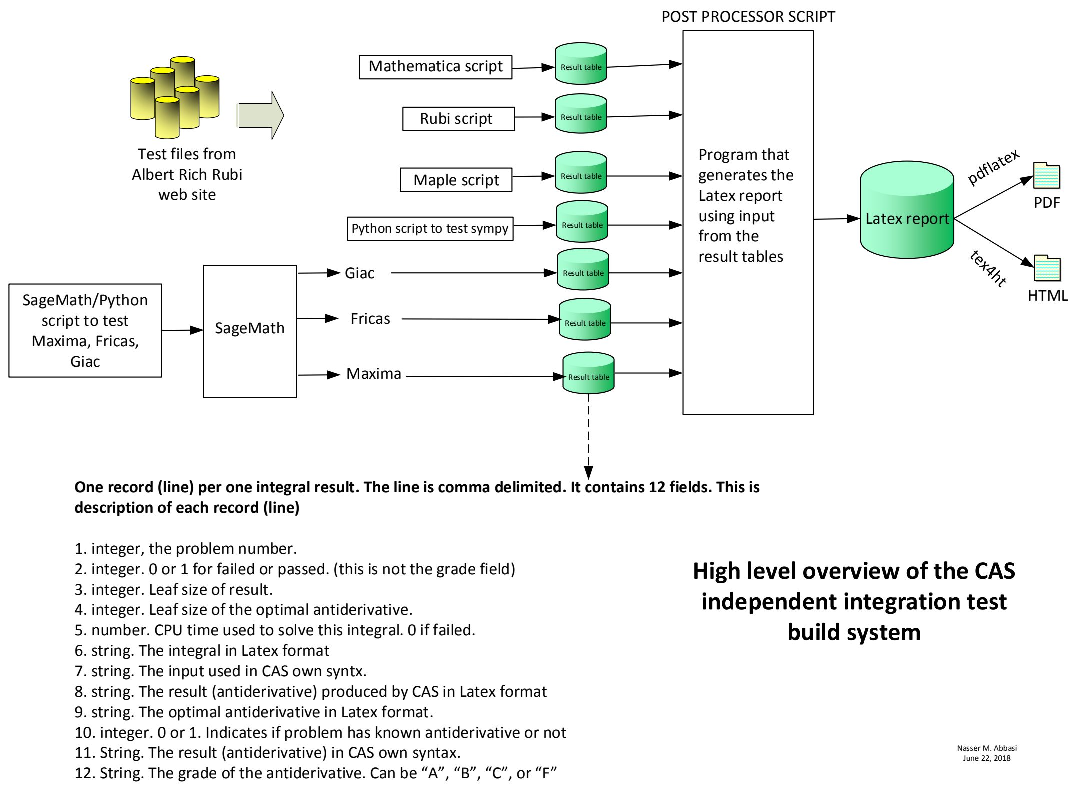
1. Mathematica 11.3 (64 bit).
2. Rubi 4.15.2 in Mathematica 11.3.
3. Rubi in Sympy (Version 1.3) under Python 3.7.0 using Anaconda distribution.
4. Maple 2018.1 (64 bit).
5. Maxima 5.41 Using Lisp ECL 16.1.2.
6. Fricas 1.3.4.
7. Sympy 1.3 under Python 3.7.0 using Anaconda distribution.
8. Giac/Xcas 1.4.9.

Maxima, Fricas and Giac/Xcas were called from inside SageMath version 8.3. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python. Rubi in Sympy was also called directly using sympy 1.3 in python.

1.2 Design of the test system

The following diagram gives a high level view of the current test build system.



1.3 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.4 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.5 Important notes about some of the results

Important note about Maxima results Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS and Giac/XCAS results There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

Important note about finding leaf size of antiderivative For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

```
#see https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-express

def tree(expr):
    if expr.operator() is None:
       return expr
    else:
       return [expr.operator()]+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

When these cas systems implement a buildin function to find the leaf size of expressions, it will be used instead, and these tests run again.

1.6 Grading of results

The table below summarizes the grading of each CAS system.

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100. (111)	% 0. (0)
Rubi in Sympy	% 47.75 (53)	% 52.25 (58)
Mathematica	% 100. (111)	% 0. (0)
Maple	% 100. (111)	% 0. (0)
Maxima	% 74.77 (83)	% 25.23 (28)
Fricas	% 74.77 (83)	% 25.23 (28)
Sympy	% 42.34 (47)	% 57.66 (64)
Giac	% 81.98 (91)	% 18.02 (20)

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

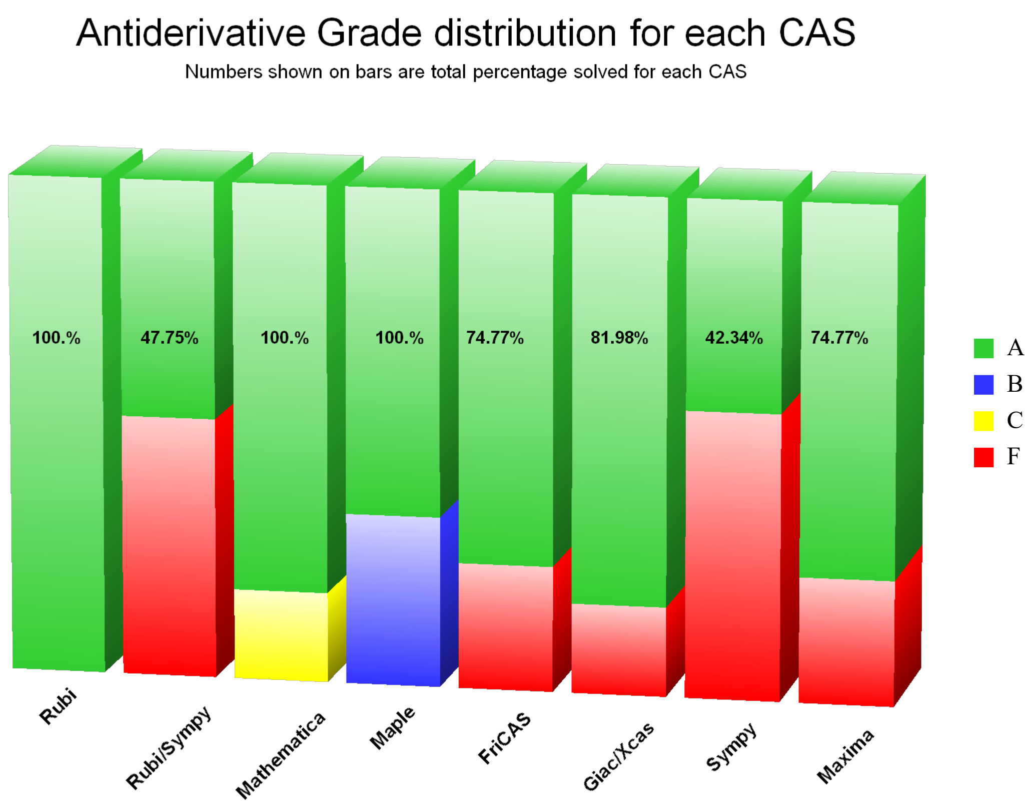
Grading is currently implemented only for for Mathematica, Rubi and Maple results. For all other CAS systems (Maxima, Fricas, Sympy, Giac, Rubi in sympy), the grading function is not yet implemented.

For these systems, a grade of A is assigned if the integrate command completes successfully and a grade of F otherwise.

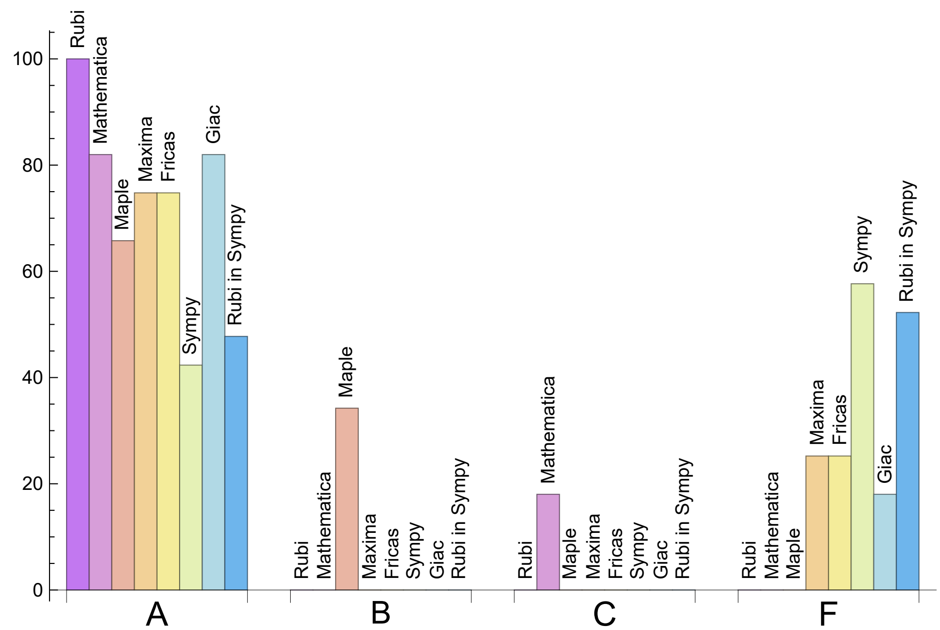
Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	100.	0.	0.	0.
Rubi in Sympy	47.75	0.	0.	52.25
Mathematica	81.98	0.	18.02	0.
Maple	65.77	34.23	0.	0.
Maxima	74.77	0.	0.	25.23
Fricas	74.77	0.	0.	25.23
Sympy	42.34	0.	0.	57.66
Giac	81.98	0.	0.	18.02

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



1.7 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	1.75	217.75	1.	140.	1.
Rubi in Sympy	58.06	149.3	0.87	126.	0.87
Mathematica	1.34	275.04	1.1	146.	1.03
Maple	0.04	2152.19	3.99	186.	1.54
Maxima	0.72	136.18	1.18	119.	1.17
Fricas	2.01	202.67	1.63	113.	1.52
Sympy	17.42	711.74	6.89	165.	1.21
Giac	1.49	141.51	1.32	130.	1.29

1.8 list of integrals that has no closed form antiderivative

{}

1.9 list of integrals not solved by each system

Not solved by Rubi {}

Not solved by Rubi in Sympy {1, 2, 3, 4, 5, 6, 7, 8, 9, 13, 14, 19, 24, 25, 37, 38, 39, 40, 41, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 65, 66, 68, 69, 70, 71, 72, 75, 76, 77, 78, 81, 82, 83, 84, 86, 87, 88, 89, 90, 96, 98, 99, 100, 101, 102, 107}

Not solved by Mathematica {}

Not solved by Maple {}

Not solved by Maxima {20, 21, 22, 23, 24, 25, 36, 37, 38, 39, 40, 41, 52, 53, 54, 55, 56, 57, 58, 59, 64, 65, 66, 103, 104, 105, 106, 107}

Not solved by Fricas {20, 21, 22, 23, 24, 25, 36, 37, 38, 39, 40, 41, 52, 53, 54, 55, 56, 57, 58, 59, 64, 65, 66, 103, 104, 105, 106, 107}

Not solved by Sympy {12, 13, 14, 17, 18, 19, 20, 21, 22, 23, 24, 25, 28, 29, 30, 33, 34, 35, 36, 37, 38, 39, 40, 41, 44, 45, 46, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 64, 65, 66, 83, 84, 87, 88, 89, 90, 93, 94, 95, 96, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111}

Not solved by Giac {25, 36, 37, 38, 39, 40, 41, 54, 55, 57, 58, 59, 64, 65, 66, 103, 104, 105, 106, 107}

1.10 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Rubi in Sympy {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {103, 104, 107}

Mathematica {15, 16, 17, 31, 32, 33, 34, 35, 47, 48, 49, 50, 51}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Rubi in Sympy Verification phase not implemented yet.

2 detailed summary tables of results

2.1 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	50	41	54	1	46	58	0
normalized size	1	1.	1.	0.82	1.08	0.02	0.92	1.16	0.
time (sec)	N/A	0.087	0.005	0.001	0.696	0.255	0.09	0.278	0.

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	69	58	77	1	65	86	0
normalized size	1	1.	1.	0.84	1.12	0.01	0.94	1.25	0.
time (sec)	N/A	0.091	0.037	0.001	0.697	0.241	0.103	0.286	0.

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	88	75	100	1	83	115	0
normalized size	1	1.	1.	0.85	1.14	0.01	0.94	1.31	0.
time (sec)	N/A	0.145	0.037	0.001	0.702	0.241	0.12	0.287	0.

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	105	90	120	1	102	143	0
normalized size	1	1.	1.	0.86	1.14	0.01	0.97	1.36	0.
time (sec)	N/A	0.203	0.065	0.001	0.701	0.24	0.144	0.291	0.

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	122	105	140	1	121	171	0
normalized size	1	1.	1.	0.86	1.15	0.01	0.99	1.4	0.
time (sec)	N/A	0.243	0.077	0.001	0.701	0.242	0.156	0.29	0.

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	97	95	127	1	116	143	0
normalized size	1	1.	0.87	0.85	1.13	0.01	1.04	1.28	0.
time (sec)	N/A	0.262	0.089	0.001	0.701	0.236	0.154	0.295	0.

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	154	139	186	1	165	212	0
normalized size	1	1.	1.	0.9	1.21	0.01	1.07	1.38	0.
time (sec)	N/A	0.275	0.097	0.001	0.702	0.239	0.174	0.294	0.

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	196	183	246	1	209	281	0
normalized size	1	1.	1.	0.93	1.26	0.01	1.07	1.43	0.
time (sec)	N/A	0.388	0.118	0.001	0.699	0.24	0.206	0.286	0.

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	234	234	219	294	1	258	350	0
normalized size	1	1.	1.	0.94	1.26	0.	1.1	1.5	0.
time (sec)	N/A	0.582	0.286	0.001	0.706	0.241	0.231	0.288	0.

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	50	58	58	58	515	69	34
normalized size	1	1.	1.11	1.29	1.29	1.29	11.44	1.53	0.76
time (sec)	N/A	0.066	0.032	0.013	0.702	0.272	6.81	0.302	22.007

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	58	86	69	69	2195	80	42
normalized size	1	1.	1.14	1.69	1.35	1.35	43.04	1.57	0.82
time (sec)	N/A	0.117	0.05	0.011	0.702	0.324	88.818	0.29	23.52

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	68	114	82	82	0	93	51
normalized size	1	1.	1.19	2.	1.44	1.44	0.	1.63	0.89
time (sec)	N/A	0.175	0.059	0.013	0.706	0.484	0.	0.292	33.667

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	81	145	97	97	0	108	0
normalized size	1	1.	1.27	2.27	1.52	1.52	0.	1.69	0.
time (sec)	N/A	0.315	0.088	0.014	0.705	1.518	0.	0.298	0.

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	98	179	119	119	0	130	0
normalized size	1	1.	1.29	2.36	1.57	1.57	0.	1.71	0.
time (sec)	N/A	0.364	0.135	0.014	0.708	6.913	0.	0.302	0.

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	A	A	A	A	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	98	92	88	93	923	90	95
normalized size	1	1.	1.07	1.	0.96	1.01	10.03	0.98	1.03
time (sec)	N/A	0.159	0.325	0.008	0.783	0.267	6.457	0.281	21.704

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	A	A	A	A	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	121	148	101	107	3589	104	105
normalized size	1	1.	1.16	1.42	0.97	1.03	34.51	1.	1.01
time (sec)	N/A	0.189	0.271	0.007	0.79	0.29	81.866	0.274	33.262

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	A	A	F(-1)	A	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	150	204	112	117	0	115	122
normalized size	1	1.	1.18	1.61	0.88	0.92	0.	0.91	0.96
time (sec)	N/A	0.239	1.422	0.007	0.785	0.45	0.	0.275	43.082

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	165	241	124	135	0	127	133
normalized size	1	1.	1.21	1.77	0.91	0.99	0.	0.93	0.98
time (sec)	N/A	0.303	1.193	0.006	0.779	1.397	0.	0.274	66.066

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	187	303	143	157	0	146	0
normalized size	1	1.	1.24	2.01	0.95	1.04	0.	0.97	0.
time (sec)	N/A	0.353	1.444	0.011	0.795	5.727	0.	0.274	0.

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	F(-2)	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	194	231	0	0	0	1	177
normalized size	1	1.	1.03	1.22	0.	0.	0.	0.01	0.94
time (sec)	N/A	0.459	0.706	0.039	0.	0.	0.	0.648	42.969

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	F(-2)	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	234	616	0	0	0	1	221
normalized size	1	1.	1.11	2.92	0.	0.	0.	0.	1.05
time (sec)	N/A	0.551	0.433	0.03	0.	0.	0.	1.053	53.876

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	F(-2)	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	245	245	280	866	0	0	0	1	250
normalized size	1	1.	1.14	3.53	0.	0.	0.	0.	1.02
time (sec)	N/A	0.516	0.573	0.031	0.	0.	0.	1.288	69.66

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	F(-1)	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	290	290	383	1132	0	0	0	1	286
normalized size	1	1.	1.32	3.9	0.	0.	0.	0.	0.99
time (sec)	N/A	1.519	1.078	0.039	0.	0.	0.	1.697	143.637

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	F(-1)	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	321	321	441	1435	0	0	0	1	0
normalized size	1	1.	1.37	4.47	0.	0.	0.	0.	0.
time (sec)	N/A	1.341	1.566	0.041	0.	0.	0.	2.034	0.

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	F(-1)	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	545	545	816	3835	0	0	0	0	0
normalized size	1	1.	1.5	7.04	0.	0.	0.	0.	0.
time (sec)	N/A	10.772	3.525	0.067	0.	0.	0.	0.	0.

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	90	122	112	228	604	126	71
normalized size	1	1.	0.96	1.3	1.19	2.43	6.43	1.34	0.76
time (sec)	N/A	0.121	0.121	0.026	0.705	0.325	8.946	0.29	31.853

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	112	182	143	293	2689	155	88
normalized size	1	1.	0.97	1.58	1.24	2.55	23.38	1.35	0.77
time (sec)	N/A	0.286	0.219	0.026	0.705	0.33	109.052	0.273	40.634

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	134	242	171	354	0	184	105
normalized size	1	1.	0.97	1.75	1.24	2.57	0.	1.33	0.76
time (sec)	N/A	0.352	0.101	0.027	0.704	0.561	0.	0.274	45.987

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	159	302	196	410	0	213	122
normalized size	1	1.	1.06	2.01	1.31	2.73	0.	1.42	0.81
time (sec)	N/A	0.431	0.14	0.028	0.699	1.766	0.	0.278	58.549

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	185	362	220	467	0	242	139
normalized size	1	1.	1.14	2.23	1.36	2.88	0.	1.49	0.86
time (sec)	N/A	0.47	0.188	0.028	0.71	8.518	0.	0.276	69.378

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	A	A	A	A	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	146	146	130	219	952	135	126
normalized size	1	1.	1.04	1.04	0.93	1.56	6.8	0.96	0.9
time (sec)	N/A	0.218	0.986	0.016	0.778	0.273	8.573	0.274	39.682

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	A	A	A	A	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	186	214	162	297	4107	173	144
normalized size	1	1.	1.13	1.3	0.98	1.8	24.89	1.05	0.87
time (sec)	N/A	0.297	0.828	0.016	0.781	0.312	91.825	0.281	52.599

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	A	A	F(-1)	A	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	200	260	182	333	0	192	151
normalized size	1	1.	1.12	1.45	1.02	1.86	0.	1.07	0.84
time (sec)	N/A	0.348	0.98	0.016	0.78	0.476	0.	0.275	59.469

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	A	A	F(-1)	A	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	234	328	193	355	0	209	167
normalized size	1	1.	1.25	1.75	1.03	1.9	0.	1.12	0.89
time (sec)	N/A	0.413	1.279	0.017	0.782	1.377	0.	0.275	71.168

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	A	A	F(-1)	A	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	243	374	209	387	0	228	175
normalized size	1	1.	1.25	1.93	1.08	1.99	0.	1.18	0.9
time (sec)	N/A	0.403	1.424	0.018	0.779	6.361	0.	0.276	82.39

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	F(-2)	F(-1)	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	330	330	341	1241	0	0	0	0	296
normalized size	1	1.	1.03	3.76	0.	0.	0.	0.	0.9
time (sec)	N/A	1.56	1.608	0.143	0.	0.	0.	0.	110.156

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	F(-2)	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	368	368	398	2851	0	0	0	0	0
normalized size	1	1.	1.08	7.75	0.	0.	0.	0.	0.
time (sec)	N/A	1.925	2.533	0.163	0.	0.	0.	0.	0.

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	F(-1)	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	386	386	421	3544	0	0	0	0	0
normalized size	1	1.	1.09	9.18	0.	0.	0.	0.	0.
time (sec)	N/A	1.387	2.967	0.161	0.	0.	0.	0.	0.

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	F(-1)	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	439	439	489	7598	0	0	0	0	0
normalized size	1	1.	1.11	17.31	0.	0.	0.	0.	0.
time (sec)	N/A	4.413	4.529	0.153	0.	0.	0.	0.	0.

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	F(-1)	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	468	468	524	8189	0	0	0	0	0
normalized size	1	1.	1.12	17.5	0.	0.	0.	0.	0.
time (sec)	N/A	3.876	5.208	0.093	0.	0.	0.	0.	0.

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	F(-1)	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	770	770	1109	16517	0	0	0	0	0
normalized size	1	1.	1.44	21.45	0.	0.	0.	0.	0.
time (sec)	N/A	25.763	8.767	0.151	0.	0.	0.	0.	0.

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	128	186	163	414	668	166	110
normalized size	1	1.	0.9	1.3	1.14	2.9	4.67	1.16	0.77
time (sec)	N/A	0.175	0.178	0.029	0.711	0.303	9.177	0.28	48.481

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	161	278	209	525	2822	212	134
normalized size	1	1.	0.92	1.59	1.19	3.	16.13	1.21	0.77
time (sec)	N/A	0.452	0.265	0.029	0.705	0.349	118.807	0.272	62.935

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	193	370	254	635	0	257	158
normalized size	1	1.	0.95	1.81	1.25	3.11	0.	1.26	0.77
time (sec)	N/A	0.514	0.165	0.031	0.71	0.628	0.	0.266	75.312

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	224	231	462	289	734	0	302	182
normalized size	1	1.	1.03	2.06	1.29	3.28	0.	1.35	0.81
time (sec)	N/A	0.626	0.236	0.032	0.707	2.014	0.	0.267	88.878

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	239	261	554	321	832	0	347	196
normalized size	1	1.	1.09	2.32	1.34	3.48	0.	1.45	0.82
time (sec)	N/A	0.661	0.305	0.034	0.71	8.885	0.	0.273	104.153

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	A	A	A	A	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	186	180	185	387	1103	177	168
normalized size	1	1.	1.01	0.97	1.	2.09	5.96	0.96	0.91
time (sec)	N/A	0.258	2.187	0.023	0.775	0.278	9.103	0.266	57.28

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	A	A	A	A	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	235	264	234	531	4498	231	194
normalized size	1	1.	1.05	1.18	1.05	2.38	20.17	1.04	0.87
time (sec)	N/A	0.468	1.147	0.022	0.784	0.317	106.84	0.266	75.624

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	A	A	F(-1)	A	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	243	243	259	322	270	599	0	267	207
normalized size	1	1.	1.07	1.33	1.11	2.47	0.	1.1	0.85
time (sec)	N/A	0.524	1.315	0.023	0.785	0.504	0.	0.273	83.973

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	A	A	F(-1)	A	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	263	263	303	396	293	666	0	308	233
normalized size	1	1.	1.15	1.51	1.11	2.53	0.	1.17	0.89
time (sec)	N/A	0.614	1.834	0.023	0.786	1.518	0.	0.268	102.35

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	A	A	F(-1)	A	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	269	269	325	454	309	714	0	344	240
normalized size	1	1.	1.21	1.69	1.15	2.65	0.	1.28	0.89
time (sec)	N/A	0.629	2.225	0.025	0.779	7.228	0.	0.268	115.85

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	F(-2)	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	474	474	488	3733	0	0	0	1	0
normalized size	1	1.	1.03	7.88	0.	0.	0.	0.	0.
time (sec)	N/A	4.503	4.371	0.326	0.	0.	0.	34.836	0.

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	F(-1)	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	621	621	683	10809	0	0	0	1	0
normalized size	1	1.	1.1	17.41	0.	0.	0.	0.	0.
time (sec)	N/A	10.425	6.614	0.398	0.	0.	0.	50.972	0.

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	F(-1)	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	646	646	733	13757	0	0	0	0	0
normalized size	1	1.	1.13	21.3	0.	0.	0.	0.	0.
time (sec)	N/A	8.792	6.756	0.393	0.	0.	0.	0.	0.

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	F(-1)	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	679	679	845	19742	0	0	0	0	0
normalized size	1	1.	1.24	29.08	0.	0.	0.	0.	0.
time (sec)	N/A	10.095	7.075	0.314	0.	0.	0.	0.	0.

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	F(-1)	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	728	728	980	21161	0	0	0	1	0
normalized size	1	1.	1.35	29.07	0.	0.	0.	0.	0.
time (sec)	N/A	8.19	7.398	0.143	0.	0.	0.	19.711	0.

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	F(-1)	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1150	1144	1590	36326	0	0	0	0	0
normalized size	1	0.99	1.38	31.59	0.	0.	0.	0.	0.
time (sec)	N/A	24.669	9.317	0.344	0.	0.	0.	0.	0.

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	F(-1)	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	645	645	908	14103	0	0	0	0	0
normalized size	1	1.	1.41	21.87	0.	0.	0.	0.	0.
time (sec)	N/A	8.507	7.886	0.186	0.	0.	0.	0.	0.

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	F(-1)	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1177	1179	1649	35336	0	0	0	0	0
normalized size	1	1.	1.4	30.02	0.	0.	0.	0.	0.
time (sec)	N/A	21.295	8.936	0.361	0.	0.	0.	0.	0.

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	416	416	416	829	564	1	503	645	0
normalized size	1	1.	1.	1.99	1.36	0.	1.21	1.55	0.
time (sec)	N/A	1.221	0.208	0.002	0.71	0.248	0.36	0.304	0.

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	259	259	259	354	339	1	309	398	0
normalized size	1	1.	1.	1.37	1.31	0.	1.19	1.54	0.
time (sec)	N/A	0.676	0.145	0.001	0.705	0.249	0.259	0.284	0.

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	154	161	186	1	165	212	0
normalized size	1	1.	1.	1.05	1.21	0.01	1.07	1.38	0.
time (sec)	N/A	0.334	0.095	0.002	0.704	0.246	0.172	0.287	0.

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	17	22	22	15	23	0
normalized size	1	1.	1.	0.85	1.1	1.1	0.75	1.15	0.
time (sec)	N/A	0.029	0.003	0.002	0.702	0.262	0.145	0.318	0.

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	F(-2)	F(-1)	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	234	616	0	0	0	0	221
normalized size	1	1.	1.11	2.92	0.	0.	0.	0.	1.05
time (sec)	N/A	0.652	0.466	0.004	0.	0.	0.	0.	72.682

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	F(-2)	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	368	368	398	2851	0	0	0	0	0
normalized size	1	1.	1.08	7.75	0.	0.	0.	0.	0.
time (sec)	N/A	1.9	2.557	0.009	0.	0.	0.	0.	0.

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	F(-1)	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	621	621	683	10809	0	0	0	0	0
normalized size	1	1.	1.1	17.41	0.	0.	0.	0.	0.
time (sec)	N/A	10.327	6.606	0.015	0.	0.	0.	0.	0.

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	4	4	4	5	5	5	3	7	3
normalized size	1	1.	1.	1.25	1.25	1.25	0.75	1.75	0.75
time (sec)	N/A	0.012	0.001	0.002	0.701	0.266	0.089	0.3	4.917

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	16	18	19	19	12	23	0
normalized size	1	1.	1.14	1.29	1.36	1.36	0.86	1.64	0.
time (sec)	N/A	0.04	0.007	0.003	0.708	0.264	1.026	0.28	0.

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	30	35	36	36	26	41	0
normalized size	1	1.	0.97	1.13	1.16	1.16	0.84	1.32	0.
time (sec)	N/A	0.072	0.02	0.004	0.701	0.266	1.109	0.283	0.

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	45	58	58	58	41	66	0
normalized size	1	1.	0.88	1.14	1.14	1.14	0.8	1.29	0.
time (sec)	N/A	0.128	0.043	0.004	0.693	0.268	1.216	0.289	0.

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	68	87	84	84	63	100	0
normalized size	1	1.	1.	1.28	1.24	1.24	0.93	1.47	0.
time (sec)	N/A	0.2	0.041	0.005	0.7	0.247	1.322	0.282	0.

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	92	122	113	113	88	142	0
normalized size	1	1.	1.	1.33	1.23	1.23	0.96	1.54	0.
time (sec)	N/A	0.268	0.064	0.004	0.703	0.282	1.463	0.285	0.

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	15	15	8	18	8
normalized size	1	1.	1.	1.09	1.36	1.36	0.73	1.64	0.73
time (sec)	N/A	0.016	0.005	0.008	0.695	0.249	0.181	0.283	4.274

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	23	29	30	30	29	35	17
normalized size	1	1.	1.05	1.32	1.36	1.36	1.32	1.59	0.77
time (sec)	N/A	0.05	0.013	0.008	0.703	0.257	0.726	0.283	8.674

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	30	45	39	39	44	45	0
normalized size	1	1.	1.03	1.55	1.34	1.34	1.52	1.55	0.
time (sec)	N/A	0.082	0.023	0.009	0.703	0.254	2.243	0.283	0.

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	44	69	61	61	66	66	0
normalized size	1	1.	0.94	1.47	1.3	1.3	1.4	1.4	0.
time (sec)	N/A	0.122	0.041	0.009	0.704	0.256	3.353	0.286	0.

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	67	98	84	84	94	93	0
normalized size	1	1.	1.02	1.48	1.27	1.27	1.42	1.41	0.
time (sec)	N/A	0.173	0.055	0.01	0.705	0.259	5.442	0.285	0.

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	91	134	113	113	122	131	0
normalized size	1	1.	1.01	1.49	1.26	1.26	1.36	1.46	0.
time (sec)	N/A	0.214	0.076	0.01	0.71	0.274	8.601	0.286	0.

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	20	26	26	19	30	22
normalized size	1	1.	1.	0.69	0.9	0.9	0.66	1.03	0.76
time (sec)	N/A	0.039	0.01	0.01	0.7	0.262	0.277	0.288	26.83

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	39	44	43	43	304	51	36
normalized size	1	1.	0.93	1.05	1.02	1.02	7.24	1.21	0.86
time (sec)	N/A	0.098	0.028	0.009	0.699	0.298	3.682	0.284	23.56

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	44	65	50	50	716	58	0
normalized size	1	1.	0.94	1.38	1.06	1.06	15.23	1.23	0.
time (sec)	N/A	0.125	0.04	0.01	0.701	0.261	15.602	0.286	0.

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	55	89	63	63	1389	72	0
normalized size	1	1.	0.96	1.56	1.11	1.11	24.37	1.26	0.
time (sec)	N/A	0.147	0.052	0.011	0.7	0.287	71.271	0.295	0.

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	71	120	84	84	0	92	0
normalized size	1	1.	0.96	1.62	1.14	1.14	0.	1.24	0.
time (sec)	N/A	0.237	0.075	0.011	0.703	0.286	0.	0.283	0.

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	91	156	111	111	0	122	0
normalized size	1	1.	0.95	1.62	1.16	1.16	0.	1.27	0.
time (sec)	N/A	0.275	0.104	0.011	0.704	0.33	0.	0.284	0.

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	42	33	43	61	34	49	34
normalized size	1	1.	0.91	0.72	0.93	1.33	0.74	1.07	0.74
time (sec)	N/A	0.089	0.037	0.015	0.701	0.256	0.689	0.287	23.054

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	66	74	77	126	1188	89	0
normalized size	1	1.	0.93	1.04	1.08	1.77	16.73	1.25	0.
time (sec)	N/A	0.329	0.071	0.016	0.702	0.271	16.666	0.284	0.

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	77	110	92	157	0	104	0
normalized size	1	1.	0.94	1.34	1.12	1.91	0.	1.27	0.
time (sec)	N/A	0.363	0.102	0.017	0.701	0.357	0.	0.285	0.

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	90	146	109	190	0	122	0
normalized size	1	1.	0.95	1.54	1.15	2.	0.	1.28	0.
time (sec)	N/A	0.42	0.09	0.016	0.702	0.875	0.	0.292	0.

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	102	182	124	221	0	136	0
normalized size	1	1.	0.96	1.72	1.17	2.08	0.	1.28	0.
time (sec)	N/A	0.486	0.114	0.017	0.695	4.173	0.	0.284	0.

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	118	221	146	270	0	158	0
normalized size	1	1.	0.97	1.81	1.2	2.21	0.	1.3	0.
time (sec)	N/A	0.603	0.139	0.018	0.716	25.419	0.	0.289	0.

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	48	40	57	97	44	62	44
normalized size	1	1.	0.86	0.71	1.02	1.73	0.79	1.11	0.79
time (sec)	N/A	0.132	0.046	0.018	0.7	0.259	0.784	0.283	31.265

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	80	90	101	207	1255	115	78
normalized size	1	1.	0.9	1.01	1.13	2.33	14.1	1.29	0.88
time (sec)	N/A	0.531	0.089	0.02	0.698	0.294	17.003	0.286	48.11

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	97	134	123	258	0	136	102
normalized size	1	1.	0.92	1.28	1.17	2.46	0.	1.3	0.97
time (sec)	N/A	0.639	0.144	0.02	0.697	0.355	0.	0.286	63.156

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	114	178	144	309	0	158	122
normalized size	1	1.	0.97	1.52	1.23	2.64	0.	1.35	1.04
time (sec)	N/A	0.486	0.114	0.021	0.706	0.899	0.	0.282	105.42

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	136	222	166	360	0	180	148
normalized size	1	1.	1.04	1.69	1.27	2.75	0.	1.37	1.13
time (sec)	N/A	0.56	0.123	0.021	0.699	4.625	0.	0.284	137.068

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	153	266	188	412	0	201	0
normalized size	1	1.	1.04	1.81	1.28	2.8	0.	1.37	0.
time (sec)	N/A	0.637	0.195	0.022	0.709	27.121	0.	0.287	0.

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	60	47	70	139	53	76	48
normalized size	1	1.	0.88	0.69	1.03	2.04	0.78	1.12	0.71
time (sec)	N/A	0.109	0.055	0.021	0.707	0.255	0.846	0.288	21.859

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	97	106	119	285	1032	132	0
normalized size	1	1.	0.92	1.01	1.13	2.71	9.83	1.26	0.
time (sec)	N/A	0.387	0.165	0.022	0.7	0.277	16.452	0.287	0.

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	121	158	146	360	0	159	0
normalized size	1	1.	0.99	1.3	1.2	2.95	0.	1.3	0.
time (sec)	N/A	0.439	0.095	0.022	0.705	0.367	0.	0.286	0.

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	144	210	170	433	0	184	0
normalized size	1	1.	1.02	1.49	1.21	3.07	0.	1.3	0.
time (sec)	N/A	0.508	0.15	0.024	0.715	0.913	0.	0.285	0.

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	169	262	196	508	0	209	0
normalized size	1	1.	1.07	1.66	1.24	3.22	0.	1.32	0.
time (sec)	N/A	0.606	0.204	0.024	0.71	4.587	0.	0.287	0.

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	195	314	220	581	0	234	0
normalized size	1	1.	1.1	1.77	1.24	3.28	0.	1.32	0.
time (sec)	N/A	0.725	0.226	0.025	0.707	26.779	0.	0.286	0.

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F	F	A
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	717	717	2588	3038	0	0	0	0	709
normalized size	1	1.	3.61	4.24	0.	0.	0.	0.	0.99
time (sec)	N/A	1.394	5.372	0.013	0.	0.	0.	0.	132.275

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F	F	A
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	505	505	1534	1585	0	0	0	0	474
normalized size	1	1.	3.04	3.14	0.	0.	0.	0.	0.94
time (sec)	N/A	0.772	6.341	0.01	0.	0.	0.	0.	85.169

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	359	359	526	453	0	0	0	0	325
normalized size	1	1.	1.47	1.26	0.	0.	0.	0.	0.91
time (sec)	N/A	0.473	2.602	0.008	0.	0.	0.	0.	61.977

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	447	447	513	1005	0	0	0	0	435
normalized size	1	1.	1.15	2.25	0.	0.	0.	0.	0.97
time (sec)	N/A	0.756	2.771	0.009	0.	0.	0.	0.	124.925

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F(-1)	F(-2)	F(-1)
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	680	680	598	1395	0	0	0	0	0
normalized size	1	1.	0.88	2.05	0.	0.	0.	0.	0.
time (sec)	N/A	1.235	4.795	0.082	0.	0.	0.	0.	0.

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	18	23	23	0	95	17
normalized size	1	1.	1.	0.95	1.21	1.21	0.	5.	0.89
time (sec)	N/A	0.015	0.045	0.007	0.768	0.26	0.	0.329	13.007

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	51	52	69	111	0	228	48
normalized size	1	1.	0.89	0.91	1.21	1.95	0.	4.	0.84
time (sec)	N/A	0.108	0.073	0.007	0.762	0.26	0.	0.316	17.885

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	48	53	66	108	0	221	46
normalized size	1	1.	0.84	0.93	1.16	1.89	0.	3.88	0.81
time (sec)	N/A	0.132	0.075	0.008	0.771	0.282	0.	0.314	18.318

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	61	63	127	124	0	262	54
normalized size	1	1.	0.88	0.91	1.84	1.8	0.	3.8	0.78
time (sec)	N/A	0.201	0.092	0.008	0.781	0.267	0.	0.314	22.647

2.2 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [47] had the largest ratio of [0.6875]

Table 1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	2	1	1.	18	0.056
2	A	2	1	1.	23	0.043
3	A	2	1	1.	28	0.036
4	A	2	1	1.	33	0.03
5	A	2	1	1.	38	0.026
6	A	2	1	1.	20	0.05
7	A	2	1	1.	25	0.04
8	A	2	1	1.	30	0.033
9	A	2	1	1.	35	0.029
10	A	10	7	1.	18	0.389
11	A	9	7	1.	23	0.304

Continued on next page

Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
12	A	8	6	1.	28	0.214
13	A	10	7	1.	33	0.212
14	A	12	8	1.	38	0.21
15	A	15	8	1.	16	0.5
16	A	14	8	1.	21	0.381
17	A	15	7	1.	26	0.269
18	A	17	8	1.	31	0.258
19	A	19	9	1.	36	0.25
20	A	9	7	1.	20	0.35
21	A	8	7	1.	25	0.28
22	A	9	8	1.	30	0.267
23	A	11	9	1.	35	0.257
24	A	13	10	1.	40	0.25
25	A	13	10	1.	55	0.182
26	A	12	9	1.	18	0.5
27	A	11	9	1.	23	0.391
28	A	10	8	1.	28	0.286
29	A	10	8	1.	33	0.242
30	A	11	9	1.	38	0.237
31	A	17	10	1.	16	0.625
32	A	16	10	1.	21	0.476
33	A	15	9	1.	26	0.346
34	A	15	9	1.	31	0.29
35	A	16	10	1.	36	0.278
36	A	11	9	1.	20	0.45
37	A	10	9	1.	25	0.36
38	A	9	8	1.	30	0.267
39	A	9	8	1.	35	0.229
40	A	10	9	1.	40	0.225
41	A	13	11	1.	55	0.2
42	A	14	10	1.	18	0.556
43	A	13	9	1.	23	0.391
44	A	12	9	1.	28	0.321
45	A	12	10	1.	33	0.303

Continued on next page

Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
46	A	13	11	1.	38	0.29
47	A	19	11	1.	16	0.688
48	A	18	10	1.	21	0.476
49	A	17	10	1.	26	0.385
50	A	17	11	1.	31	0.355
51	A	18	12	1.	36	0.333
52	A	13	10	1.	20	0.5
53	A	12	9	1.	25	0.36
54	A	11	9	1.	30	0.3
55	A	11	10	1.	35	0.286
56	A	12	11	1.	40	0.275
57	A	11	9	0.99	55	0.164
58	A	11	10	1.	50	0.2
59	A	13	10	1.	50	0.2
60	A	2	1	1.	63	0.016
61	A	2	1	1.	63	0.016
62	A	2	1	1.	61	0.016
63	A	2	1	1.	63	0.016
64	A	9	8	1.	63	0.127
65	A	11	10	1.	63	0.159
66	A	13	10	1.	63	0.159
67	A	2	2	1.	26	0.077
68	A	3	2	1.	31	0.065
69	A	3	2	1.	36	0.056
70	A	3	2	1.	41	0.049
71	A	3	2	1.	46	0.043
72	A	3	2	1.	51	0.039
73	A	4	3	1.	21	0.143
74	A	4	3	1.	26	0.115
75	A	6	4	1.	31	0.129
76	A	6	4	1.	36	0.111
77	A	6	4	1.	41	0.098
78	A	6	4	1.	46	0.087
79	A	3	2	1.	16	0.125

Continued on next page

Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
80	A	3	2	1.	21	0.095
81	A	3	2	1.	26	0.077
82	A	3	2	1.	31	0.065
83	A	3	2	1.	36	0.056
84	A	3	2	1.	41	0.049
85	A	3	2	1.	26	0.077
86	A	3	2	1.	31	0.065
87	A	3	2	1.	36	0.056
88	A	3	2	1.	41	0.049
89	A	3	2	1.	46	0.043
90	A	3	2	1.	51	0.039
91	A	9	5	1.	21	0.238
92	A	9	5	1.	26	0.192
93	A	9	5	1.	31	0.161
94	A	3	2	1.	36	0.056
95	A	3	2	1.	41	0.049
96	A	3	2	1.	46	0.043
97	A	3	2	1.	16	0.125
98	A	3	2	1.	21	0.095
99	A	3	2	1.	26	0.077
100	A	3	2	1.	31	0.065
101	A	3	2	1.	36	0.056
102	A	3	2	1.	41	0.049
103	A	12	10	1.	32	0.312
104	A	10	10	1.	32	0.312
105	A	8	8	1.	32	0.25
106	A	7	7	1.	32	0.219
107	A	9	8	1.	32	0.25
108	A	1	1	1.	28	0.036
109	A	5	5	1.	31	0.161
110	A	5	5	1.	33	0.152
111	A	4	4	1.	36	0.111

3 Listing of integrals

3.1 $\int (d+ex)\left(a+bx^2+cx^4\right)\,dx$

Optimal. Leaf size=50

$$adx+\frac{1}{2}aex^2+\frac{1}{3}bdx^3+\frac{1}{4}bex^4+\frac{1}{5}cdx^5+\frac{1}{6}cex^6$$

```
[Out] a*d*x + (a*e*x^2)/2 + (b*d*x^3)/3 + (b*e*x^4)/4 + (c*d*x^5)/5 + (
c*e*x^6)/6
```

Rubi [A] time = 0.0874187, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056

$$adx+\frac{1}{2}aex^2+\frac{1}{3}bdx^3+\frac{1}{4}bex^4+\frac{1}{5}cdx^5+\frac{1}{6}cex^6$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x)*(a + b*x^2 + c*x^4),x]
```

```
[Out] a*d*x + (a*e*x^2)/2 + (b*d*x^3)/3 + (b*e*x^4)/4 + (c*d*x^5)/5 + (
c*e*x^6)/6
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$ae\int x\,dx+\frac{bdx^3}{3}+\frac{bex^4}{4}+\frac{cdx^5}{5}+\frac{cex^6}{6}+d\int a\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x+d)*(c*x**4+b*x**2+a),x)
```

```
[Out] a*e*Integral(x, x) + b*d*x**3/3 + b*e*x**4/4 + c*d*x**5/5 + c*e*x
**6/6 + d*Integral(a, x)
```

Mathematica [A] time = 0.00528132, size = 50, normalized size = 1.

$$adx+\frac{1}{2}aex^2+\frac{1}{3}bdx^3+\frac{1}{4}bex^4+\frac{1}{5}cdx^5+\frac{1}{6}cex^6$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x)*(a + b*x^2 + c*x^4),x]
```

```
[Out] a*d*x + (a*e*x^2)/2 + (b*d*x^3)/3 + (b*e*x^4)/4 + (c*d*x^5)/5 + (
c*e*x^6)/6
```

Maple [A] time = 0.001, size = 41, normalized size = 0.8

$$adx + \frac{aex^2}{2} + \frac{bdx^3}{3} + \frac{bex^4}{4} + \frac{cdx^5}{5} + \frac{cex^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x+d)*(c*x^4+b*x^2+a),x)
```

```
[Out] a*d*x+1/2*a*e*x^2+1/3*b*d*x^3+1/4*b*e*x^4+1/5*c*d*x^5+1/6*c*e*x^6
```

Maxima [A] time = 0.695756, size = 54, normalized size = 1.08

$$\frac{1}{6}\,cex^6 + \frac{1}{5}\,cdx^5 + \frac{1}{4}\,bex^4 + \frac{1}{3}\,bdx^3 + \frac{1}{2}\,aex^2 + adx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)*(e*x + d),x, algorithm="maxima")
```

```
[Out] 1/6*c*e*x^6 + 1/5*c*d*x^5 + 1/4*b*e*x^4 + 1/3*b*d*x^3 + 1/2*a*e*x
^2 + a*d*x
```

Fricas [A] time = 0.25548, size = 1, normalized size = 0.02

$$\frac{1}{6}x^6ec + \frac{1}{5}x^5dc + \frac{1}{4}x^4eb + \frac{1}{3}x^3db + \frac{1}{2}x^2ea + xda$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)*(e*x + d),x, algorithm="fricas")
```

```
[Out] 1/6*x^6*e*c + 1/5*x^5*d*c + 1/4*x^4*e*b + 1/3*x^3*d*b + 1/2*x^2*e
*a + x*d*a
```

Sympy [A] time = 0.089687, size = 46, normalized size = 0.92

$$adx + \frac{aex^2}{2} + \frac{bdx^3}{3} + \frac{bex^4}{4} + \frac{cdx^5}{5} + \frac{cex^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x+d)*(c*x**4+b*x**2+a),x)
```

```
[Out] a*d*x + a*e*x**2/2 + b*d*x**3/3 + b*e*x**4/4 + c*d*x**5/5 + c*e*x
**6/6
```

GIAC/XCAS [A] time = 0.277799, size = 58, normalized size = 1.16

$$\frac{1}{6}\, cx^6 e + \frac{1}{5}\, cdx^5 + \frac{1}{4}\, bx^4 e + \frac{1}{3}\, bdx^3 + \frac{1}{2}\, ax^2 e + adx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)*(e*x + d),x, algorithm="giac")
```

```
[Out] 1/6*c*x^6*e + 1/5*c*d*x^5 + 1/4*b*x^4*e + 1/3*b*d*x^3 + 1/2*a*x^2
*e + a*d*x
```

3.2 $\int \left(d+ex+fx^2\right)\left(a+bx^2+cx^4\right)\,dx$

Optimal. Leaf size=69

$$\frac{1}{3}x^3(af+bd)+adx+\frac{1}{2}aex^2+\frac{1}{5}x^5(bf+cd)+\frac{1}{4}bex^4+\frac{1}{6}cex^6+\frac{1}{7}cfx^7$$

```
[Out] a*d*x + (a*e*x^2)/2 + ((b*d + a*f)*x^3)/3 + (b*e*x^4)/4 + ((c*d +
 b*f)*x^5)/5 + (c*e*x^6)/6 + (c*f*x^7)/7
```

Rubi [A] time = 0.0913705, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043

$$\frac{1}{3}x^3(af+bd)+adx+\frac{1}{2}aex^2+\frac{1}{5}x^5(bf+cd)+\frac{1}{4}bex^4+\frac{1}{6}cex^6+\frac{1}{7}cfx^7$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x + f*x^2)*(a + b*x^2 + c*x^4),x]
```

```
[Out] a*d*x + (a*e*x^2)/2 + ((b*d + a*f)*x^3)/3 + (b*e*x^4)/4 + ((c*d +
 b*f)*x^5)/5 + (c*e*x^6)/6 + (c*f*x^7)/7
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$ae\int x\,dx+\frac{bex^4}{4}+\frac{cex^6}{6}+\frac{cfx^7}{7}+d\int a\,dx+x^5\left(\frac{bf}{5}+\frac{cd}{5}\right)+x^3\left(\frac{af}{3}+\frac{bd}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**2+e*x+d)*(c*x**4+b*x**2+a),x)
```

```
[Out] a*e*Integral(x, x) + b*e*x**4/4 + c*e*x**6/6 + c*f*x**7/7 + d*Int
egral(a, x) + x**5*(b*f/5 + c*d/5) + x**3*(a*f/3 + b*d/3)
```

Mathematica [A] time = 0.0374614, size = 69, normalized size = 1.

$$\frac{1}{3}x^3(af+bd)+adx+\frac{1}{2}aex^2+\frac{1}{5}x^5(bf+cd)+\frac{1}{4}bex^4+\frac{1}{6}cex^6+\frac{1}{7}cfx^7$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x + f*x^2)*(a + b*x^2 + c*x^4),x]
```

```
[Out] a*d*x + (a*e*x^2)/2 + ((b*d + a*f)*x^3)/3 + (b*e*x^4)/4 + ((c*d +
 b*f)*x^5)/5 + (c*e*x^6)/6 + (c*f*x^7)/7
```

Maple [A] time = 0.001, size = 58, normalized size = 0.8

$$adx + \frac{aex^2}{2} + \frac{(fa + bd)\, x^3}{3} + \frac{bex^4}{4} + \frac{(bf + cd)\, x^5}{5} + \frac{cex^6}{6} + \frac{cfx^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x^2+e*x+d)*(c*x^4+b*x^2+a),x)
```

```
[Out] a*d*x+1/2*a*e*x^2+1/3*(a*f+b*d)*x^3+1/4*b*e*x^4+1/5*(b*f+c*d)*x^5
+1/6*c*e*x^6+1/7*c*f*x^7
```

Maxima [A] time = 0.697054, size = 77, normalized size = 1.12

$$\frac{1}{7}\, cfx^7 + \frac{1}{6}\, cex^6 + \frac{1}{4}\, bex^4 + \frac{1}{5}\, (cd + bf)x^5 + \frac{1}{2}\, aex^2 + \frac{1}{3}\, (bd + af)x^3 + adx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2 + a)*(f*x^2 + e*x + d),x, algorithm="maxima")
```

```
[Out] 1/7*c*f*x^7 + 1/6*c*e*x^6 + 1/4*b*e*x^4 + 1/5*(c*d + b*f)*x^5 + 1
/2*a*e*x^2 + 1/3*(b*d + a*f)*x^3 + a*d*x
```

Fricas [A] time = 0.241331, size = 1, normalized size = 0.01

$$\frac{1}{7}x^7 fc + \frac{1}{6}x^6 ec + \frac{1}{5}x^5 dc + \frac{1}{5}x^5 fb + \frac{1}{4}x^4 eb + \frac{1}{3}x^3 db + \frac{1}{3}x^3 fa + \frac{1}{2}x^2 ea + xda$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2 + a)*(f*x^2 + e*x + d),x, algorithm="fricas")
```

```
[Out] 1/7*x^7*f*c + 1/6*x^6*e*c + 1/5*x^5*d*c + 1/5*x^5*f*b + 1/4*x^4*e
*b + 1/3*x^3*d*b + 1/3*x^3*f*a + 1/2*x^2*e*a + x*d*a
```

Sympy [A] time = 0.102619, size = 65, normalized size = 0.94

$$adx + \frac{aex^2}{2} + \frac{bex^4}{4} + \frac{cex^6}{6} + \frac{cfx^7}{7} + x^5\left(\frac{bf}{5} + \frac{cd}{5}\right) + x^3\left(\frac{af}{3} + \frac{bd}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x**2+e*x+d)*(c*x**4+b*x**2+a),x)
```

```
[Out] a*d*x + a*e*x**2/2 + b*e*x**4/4 + c*e*x**6/6 + c*f*x**7/7 + x**5*
(b*f/5 + c*d/5) + x**3*(a*f/3 + b*d/3)
```

GIAC/XCAS [A] time = 0.285878, size = 86, normalized size = 1.25

$$\frac{1}{7}cfx^7 + \frac{1}{6}cx^6e + \frac{1}{5}cdx^5 + \frac{1}{5}bfx^5 + \frac{1}{4}bx^4e + \frac{1}{3}bdx^3 + \frac{1}{3}afx^3 + \frac{1}{2}ax^2e + adx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)*(f*x^2 + e*x + d),x, algorithm="giac")
```

```
[Out] 1/7*c*f*x^7 + 1/6*c*x^6*e + 1/5*c*d*x^5 + 1/5*b*f*x^5 + 1/4*b*x^4
*e + 1/3*b*d*x^3 + 1/3*a*f*x^3 + 1/2*a*x^2*e + a*d*x
```

3.3 $\int \left(d+ex+fx^2+gx^3\right) \left(a+bx^2+cx^4\right) \, dx$

Optimal. Leaf size=88

$$\frac{1}{3}x^3(af+bd)+\frac{1}{4}x^4(ag+be)+adx+\frac{1}{2}aex^2+\frac{1}{5}x^5(bf+cd)+\frac{1}{6}x^6(bg+ce)+\frac{1}{7}cfx^7+\frac{1}{8}cgx^8$$

```
[Out] a*d*x + (a*e*x^2)/2 + ((b*d + a*f)*x^3)/3 + ((b*e + a*g)*x^4)/4 +
 ((c*d + b*f)*x^5)/5 + ((c*e + b*g)*x^6)/6 + (c*f*x^7)/7 + (c*g*x
^8)/8
```

Rubi [A] time = 0.144957, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.036$

$$\frac{1}{3}x^3(af+bd)+\frac{1}{4}x^4(ag+be)+adx+\frac{1}{2}aex^2+\frac{1}{5}x^5(bf+cd)+\frac{1}{6}x^6(bg+ce)+\frac{1}{7}cfx^7+\frac{1}{8}cgx^8$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x + f*x^2 + g*x^3)*(a + b*x^2 + c*x^4),x]
```

```
[Out] a*d*x + (a*e*x^2)/2 + ((b*d + a*f)*x^3)/3 + ((b*e + a*g)*x^4)/4 +
 ((c*d + b*f)*x^5)/5 + ((c*e + b*g)*x^6)/6 + (c*f*x^7)/7 + (c*g*x
^8)/8
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$ae\int x\,dx+\frac{cfx^7}{7}+\frac{cgx^8}{8}+d\int a\,dx+x^6\left(\frac{bg}{6}+\frac{ce}{6}\right)+x^5\left(\frac{bf}{5}+\frac{cd}{5}\right)+x^4\left(\frac{ag}{4}+\frac{be}{4}\right)+x^3\left(\frac{af}{3}+\frac{bd}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((g*x**3+f*x**2+e*x+d)*(c*x**4+b*x**2+a),x)
```

```
[Out] a*e*Integral(x, x) + c*f*x**7/7 + c*g*x**8/8 + d*Integral(a, x) +
 x**6*(b*g/6 + c*e/6) + x**5*(b*f/5 + c*d/5) + x**4*(a*g/4 + b*e/
4) + x**3*(a*f/3 + b*d/3)
```

Mathematica [A] time = 0.0365296, size = 88, normalized size = 1.

$$\frac{1}{3}x^3(af+bd)+\frac{1}{4}x^4(ag+be)+adx+\frac{1}{2}aex^2+\frac{1}{5}x^5(bf+cd)+\frac{1}{6}x^6(bg+ce)+\frac{1}{7}cfx^7+\frac{1}{8}cgx^8$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x + f*x^2 + g*x^3)*(a + b*x^2 + c*x^4),x]
```

```
[Out] a*d*x + (a*e*x^2)/2 + ((b*d + a*f)*x^3)/3 + ((b*e + a*g)*x^4)/4 +
 ((c*d + b*f)*x^5)/5 + ((c*e + b*g)*x^6)/6 + (c*f*x^7)/7 + (c*g*x
^8)/8
```

Maple [A] time = 0.001, size = 75, normalized size = 0.9

$$adx + \frac{aex^2}{2} + \frac{(fa+bd)\,x^3}{3} + \frac{(ag+be)\,x^4}{4} + \frac{(bf+cd)\,x^5}{5} + \frac{(bg+ce)\,x^6}{6} + \frac{cfx^7}{7} + \frac{cgx^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((g*x^3+f*x^2+e*x+d)*(c*x^4+b*x^2+a),x)
```

```
[Out] a*d*x+1/2*a*e*x^2+1/3*(a*f+b*d)*x^3+1/4*(a*g+b*e)*x^4+1/5*(b*f+c*
d)*x^5+1/6*(b*g+c*e)*x^6+1/7*c*f*x^7+1/8*c*g*x^8
```

Maxima [A] time = 0.702221, size = 100, normalized size = 1.14

$$\frac{1}{8}\,cgx^8 + \frac{1}{7}\,cfx^7 + \frac{1}{6}\,(ce+bg)x^6 + \frac{1}{5}\,(cd+bf)x^5 + \frac{1}{4}\,(be+ag)x^4 + \frac{1}{2}\,aex^2 + \frac{1}{3}\,(bd+af)x^3 + adx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)*(g*x^3 + f*x^2 + e*x + d),x, algorithm="maxima")
```

```
[Out] 1/8*c*g*x^8 + 1/7*c*f*x^7 + 1/6*(c*e + b*g)*x^6 + 1/5*(c*d + b*f)
*x^5 + 1/4*(b*e + a*g)*x^4 + 1/2*a*e*x^2 + 1/3*(b*d + a*f)*x^3 +
a*d*x
```

Fricas [A] time = 0.241215, size = 1, normalized size = 0.01

$$\frac{1}{8}x^8gc + \frac{1}{7}x^7fc + \frac{1}{6}x^6ec + \frac{1}{6}x^6gb + \frac{1}{5}x^5dc + \frac{1}{5}x^5fb + \frac{1}{4}x^4eb + \frac{1}{4}x^4ga + \frac{1}{3}x^3db + \frac{1}{3}x^3fa + \frac{1}{2}x^2ea + xda$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2 + a)*(g*x^3 + f*x^2 + e*x + d),x, algorithm="fricas")
```

```
[Out] 1/8*x^8*g*c + 1/7*x^7*f*c + 1/6*x^6*e*c + 1/6*x^6*g*b + 1/5*x^5*d
*c + 1/5*x^5*f*b + 1/4*x^4*e*b + 1/4*x^4*g*a + 1/3*x^3*d*b + 1/3*
x^3*f*a + 1/2*x^2*e*a + x*d*a
```

Sympy [A] time = 0.120193, size = 83, normalized size = 0.94

$$adx + \frac{aex^2}{2} + \frac{cfx^7}{7} + \frac{cgx^8}{8} + x^6\left(\frac{bg}{6} + \frac{ce}{6}\right) + x^5\left(\frac{bf}{5} + \frac{cd}{5}\right) + x^4\left(\frac{ag}{4} + \frac{be}{4}\right) + x^3\left(\frac{af}{3} + \frac{bd}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**3+f*x**2+e*x+d)*(c*x**4+b*x**2+a),x)
```

```
[Out] a*d*x + a*e*x**2/2 + c*f*x**7/7 + c*g*x**8/8 + x**6*(b*g/6 + c*e/
6) + x**5*(b*f/5 + c*d/5) + x**4*(a*g/4 + b*e/4) + x**3*(a*f/3 +
b*d/3)
```

GIAC/XCAS [A] time = 0.287478, size = 115, normalized size = 1.31

$$\frac{1}{8}cgx^8 + \frac{1}{7}cfx^7 + \frac{1}{6}bgx^6 + \frac{1}{6}cx^6e + \frac{1}{5}cdx^5 + \frac{1}{5}bfx^5 + \frac{1}{4}agx^4 + \frac{1}{4}bx^4e + \frac{1}{3}bdx^3 + \frac{1}{3}afx^3 + \frac{1}{2}ax^2e + adx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2 + a)*(g*x^3 + f*x^2 + e*x + d),x, algorithm="giac")
```

```
[Out] 1/8*c*g*x^8 + 1/7*c*f*x^7 + 1/6*b*g*x^6 + 1/6*c*x^6*e + 1/5*c*d*x
^5 + 1/5*b*f*x^5 + 1/4*a*g*x^4 + 1/4*b*x^4*e + 1/3*b*d*x^3 + 1/3*
a*f*x^3 + 1/2*a*x^2*e + a*d*x
```

3.4 $\int (a + bx^2 + cx^4) (d + ex + fx^2 + gx^3 + hx^4) \, dx$

Optimal. Leaf size=105

$$\frac{1}{5}x^5(ah + bf + cd) + \frac{1}{3}x^3(af + bd) + \frac{1}{4}x^4(ag + be) + adx + \frac{1}{2}aex^2 + \frac{1}{6}x^6(bg + ce) + \frac{1}{7}x^7(bh + cf) + \frac{1}{8}cgx^8 + \frac{1}{9}chx^9$$

```
[Out] a*d*x + (a*e*x^2)/2 + ((b*d + a*f)*x^3)/3 + ((b*e + a*g)*x^4)/4 +
 ((c*d + b*f + a*h)*x^5)/5 + ((c*e + b*g)*x^6)/6 + ((c*f + b*h)*x
^7)/7 + (c*g*x^8)/8 + (c*h*x^9)/9
```

Rubi [A] time = 0.203448, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.03

$$\frac{1}{5}x^5(ah + bf + cd) + \frac{1}{3}x^3(af + bd) + \frac{1}{4}x^4(ag + be) + adx + \frac{1}{2}aex^2 + \frac{1}{6}x^6(bg + ce) + \frac{1}{7}x^7(bh + cf) + \frac{1}{8}cgx^8 + \frac{1}{9}chx^9$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2 + c*x^4)*(d + e*x + f*x^2 + g*x^3 + h*x^4),x]
```

```
[Out] a*d*x + (a*e*x^2)/2 + ((b*d + a*f)*x^3)/3 + ((b*e + a*g)*x^4)/4 +
 ((c*d + b*f + a*h)*x^5)/5 + ((c*e + b*g)*x^6)/6 + ((c*f + b*h)*x
^7)/7 + (c*g*x^8)/8 + (c*h*x^9)/9
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$ae\int x\,dx + \frac{cgx^8}{8} + \frac{chx^9}{9} + d\int a\,dx + x^7\left(\frac{bh}{7} + \frac{cf}{7}\right) + x^6\left(\frac{bg}{6} + \frac{ce}{6}\right) + x^5\left(\frac{ah}{5} + \frac{bf}{5} + \frac{cd}{5}\right) + x^4\left(\frac{ag}{4} + \frac{be}{4}\right) + x^3\left(\frac{af}{3} + \frac{bd}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2+a)*(h*x**4+g*x**3+f*x**2+e*x+d),x)
```

```
[Out] a*e*Integral(x, x) + c*g*x**8/8 + c*h*x**9/9 + d*Integral(a, x) +
 x**7*(b*h/7 + c*f/7) + x**6*(b*g/6 + c*e/6) + x**5*(a*h/5 + b*f/
```

```
5 + c*d/5) + x**4*(a*g/4 + b*e/4) + x**3*(a*f/3 + b*d/3)
```

Mathematica [A] time = 0.0650132, size = 105, normalized size = 1.

$$\frac{1}{5}x^5(ah+bf+cd)+\frac{1}{3}x^3(af+bd)+\frac{1}{4}x^4(ag+be)+adx+\frac{1}{2}aex^2+\frac{1}{6}x^6(bg+ce)+\frac{1}{7}x^7(bh+cf)+\frac{1}{8}cgx^8+\frac{1}{9}chx^9$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2 + c*x^4)*(d + e*x + f*x^2 + g*x^3 + h*x^4),x]
```

```
[Out] a*d*x + (a*e*x^2)/2 + ((b*d + a*f)*x^3)/3 + ((b*e + a*g)*x^4)/4 +
 ((c*d + b*f + a*h)*x^5)/5 + ((c*e + b*g)*x^6)/6 + ((c*f + b*h)*x
^7)/7 + (c*g*x^8)/8 + (c*h*x^9)/9
```

Maple [A] time = 0.001, size = 90, normalized size = 0.9

$$adx+\frac{aex^2}{2}+\frac{(fa+bd)x^3}{3}+\frac{(ag+be)x^4}{4}+\frac{(ah+bf+cd)x^5}{5}+\frac{(bg+ce)x^6}{6}+\frac{(bh+cf)x^7}{7}+\frac{cgx^8}{8}+\frac{chx^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2+a)*(h*x^4+g*x^3+f*x^2+e*x+d),x)
```

```
[Out] a*d*x+1/2*a*e*x^2+1/3*(a*f+b*d)*x^3+1/4*(a*g+b*e)*x^4+1/5*(a*h+b*
f+c*d)*x^5+1/6*(b*g+c*e)*x^6+1/7*(b*h+c*f)*x^7+1/8*c*g*x^8+1/9*c*
h*x^9
```

Maxima [A] time = 0.701268, size = 120, normalized size = 1.14

$$\frac{1}{9}chx^9+\frac{1}{8}cgx^8+\frac{1}{7}(cf+bh)x^7+\frac{1}{6}(ce+bg)x^6+\frac{1}{5}(cd+bf+ah)x^5+\frac{1}{4}(be+ag)x^4+\frac{1}{2}aex^2+\frac{1}{3}(bd+af)x^3+adx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2 + a)*(h*x^4 + g*x^3 + f*x^2 + e*x + d),x, algorithm="maxi
```

```
[Out] 1/9*c*h*x^9 + 1/8*c*g*x^8 + 1/7*(c*f + b*h)*x^7 + 1/6*(c*e + b*g)
*x^6 + 1/5*(c*d + b*f + a*h)*x^5 + 1/4*(b*e + a*g)*x^4 + 1/2*a*e*
x^2 + 1/3*(b*d + a*f)*x^3 + a*d*x
```

Fricas [A] time = 0.240363, size = 1, normalized size = 0.01

$$\frac{1}{9}x^9hc + \frac{1}{8}x^8gc + \frac{1}{7}x^7fc + \frac{1}{7}x^7hb + \frac{1}{6}x^6ec + \frac{1}{6}x^6gb + \frac{1}{5}x^5dc + \frac{1}{5}x^5fb + \frac{1}{5}x^5ha + \frac{1}{4}x^4eb + \frac{1}{4}x^4ga + \frac{1}{3}x^3db + \frac{1}{3}x^3fa + \frac{1}{2}x^2ea + xda$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)*(h*x^4 + g*x^3 + f*x^2 + e*x + d),x, algorithm="fric
```

```
[Out] 1/9*x^9*h*c + 1/8*x^8*g*c + 1/7*x^7*f*c + 1/7*x^7*h*b + 1/6*x^6*e
*c + 1/6*x^6*g*b + 1/5*x^5*d*c + 1/5*x^5*f*b + 1/5*x^5*h*a + 1/4*
x^4*e*b + 1/4*x^4*g*a + 1/3*x^3*d*b + 1/3*x^3*f*a + 1/2*x^2*e*a +
 x*d*a
```

Sympy [A] time = 0.144266, size = 102, normalized size = 0.97

$$adx + \frac{aex^2}{2} + \frac{cgx^8}{8} + \frac{chx^9}{9} + x^7\left(\frac{bh}{7} + \frac{cf}{7}\right) + x^6\left(\frac{bg}{6} + \frac{ce}{6}\right) + x^5\left(\frac{ah}{5} + \frac{bf}{5} + \frac{cd}{5}\right) + x^4\left(\frac{ag}{4} + \frac{be}{4}\right) + x^3\left(\frac{af}{3} + \frac{bd}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2+a)*(h*x**4+g*x**3+f*x**2+e*x+d),x)
```

```
[Out] a*d*x + a*e*x**2/2 + c*g*x**8/8 + c*h*x**9/9 + x**7*(b*h/7 + c*f/
7) + x**6*(b*g/6 + c*e/6) + x**5*(a*h/5 + b*f/5 + c*d/5) + x**4*(
a*g/4 + b*e/4) + x**3*(a*f/3 + b*d/3)
```

GIAC/XCAS [A] time = 0.291434, size = 143, normalized size = 1.36

$$\frac{1}{9}chx^9 + \frac{1}{8}cgx^8 + \frac{1}{7}cfx^7 + \frac{1}{7}bhx^7 + \frac{1}{6}bgx^6 + \frac{1}{6}cx^6e + \frac{1}{5}cdx^5 + \frac{1}{5}bfx^5 + \frac{1}{5}ahx^5 + \frac{1}{4}agx^4 + \frac{1}{4}bx^4e + \frac{1}{3}bdx^3 + \frac{1}{3}afx^3 + \frac{1}{2}ax^2e + adx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)*(h*x^4 + g*x^3 + f*x^2 + e*x + d),x, algorithm="giac
```

```
[Out] 1/9*c*h*x^9 + 1/8*c*g*x^8 + 1/7*c*f*x^7 + 1/7*b*h*x^7 + 1/6*b*g*x
^6 + 1/6*c*x^6*e + 1/5*c*d*x^5 + 1/5*b*f*x^5 + 1/5*a*h*x^5 + 1/4*
a*g*x^4 + 1/4*b*x^4*e + 1/3*b*d*x^3 + 1/3*a*f*x^3 + 1/2*a*x^2*e +
 a*d*x
```

3.5 $\int \left(a+bx^2+cx^4\right)\left(d+ex+fx^2+gx^3+hx^4+ix^5\right)\,dx$

Optimal. Leaf size=122

$$\frac{1}{5}x^5(ah+bf+cd)+\frac{1}{6}x^6(ai+bg+ce)+\frac{1}{3}x^3(af+bd)+\frac{1}{4}x^4(ag+be)+adx+\frac{1}{2}aex^2+\frac{1}{7}x^7(bh+cf)+\frac{1}{8}x^8(bi+cg)+\frac{1}{9}chx^9+\frac{1}{10}cix^{10}$$

```
[Out] a*d*x + (a*e*x^2)/2 + ((b*d + a*f)*x^3)/3 + ((b*e + a*g)*x^4)/4 +
 ((c*d + b*f + a*h)*x^5)/5 + ((c*e + b*g + a*i)*x^6)/6 + ((c*f +
b*h)*x^7)/7 + ((c*g + b*i)*x^8)/8 + (c*h*x^9)/9 + (c*i*x^10)/10
```

Rubi [A] time = 0.243356, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.026$

$$\frac{1}{5}x^5(ah+bf+cd)+\frac{1}{6}x^6(ai+bg+ce)+\frac{1}{3}x^3(af+bd)+\frac{1}{4}x^4(ag+be)+adx+\frac{1}{2}aex^2+\frac{1}{7}x^7(bh+cf)+\frac{1}{8}x^8(bi+cg)+\frac{1}{9}chx^9+\frac{1}{10}cix^{10}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2 + c*x^4)*(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5),x]
```

```
[Out] a*d*x + (a*e*x^2)/2 + ((b*d + a*f)*x^3)/3 + ((b*e + a*g)*x^4)/4 +
 ((c*d + b*f + a*h)*x^5)/5 + ((c*e + b*g + a*i)*x^6)/6 + ((c*f +
b*h)*x^7)/7 + ((c*g + b*i)*x^8)/8 + (c*h*x^9)/9 + (c*i*x^10)/10
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$ae\int x\,dx+\frac{chx^9}{9}+\frac{cix^{10}}{10}+d\int a\,dx+x^8\left(\frac{bi}{8}+\frac{cg}{8}\right)+x^7\left(\frac{bh}{7}+\frac{cf}{7}\right)+x^6\left(\frac{ai}{6}+\frac{bg}{6}+\frac{ce}{6}\right)+x^5\left(\frac{ah}{5}+\frac{bf}{5}+\frac{cd}{5}\right)+x^4\left(\frac{ag}{4}+\frac{be}{4}\right)+x^3\left(\frac{af}{3}+\frac{bd}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2+a)*(i*x**5+h*x**4+g*x**3+f*x**2+e*x+d),x)
```

```
[Out] a*e*Integral(x, x) + c*h*x**9/9 + c*i*x**10/10 + d*Integral(a, x)
 + x**8*(b*i/8 + c*g/8) + x**7*(b*h/7 + c*f/7) + x**6*(a*i/6 + b*
```

```
g/6 + c*e/6) + x**5*(a*h/5 + b*f/5 + c*d/5) + x**4*(a*g/4 + b*e/4
) + x**3*(a*f/3 + b*d/3)
```

Mathematica [A] time = 0.0767009, size = 122, normalized size = 1.

$$\frac{1}{5}x^5(ah+bf+cd)+\frac{1}{6}x^6(ai+bg+ce)+\frac{1}{3}x^3(af+bd)+\frac{1}{4}x^4(ag+be)+adx+\frac{1}{2}aex^2+\frac{1}{7}x^7(bh+cf)+\frac{1}{8}x^8(bi+cg)+\frac{1}{9}chx^9+\frac{1}{10}cix^{10}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)*(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5),x]
```

```
[Out] a*d*x + (a*e*x^2)/2 + ((b*d + a*f)*x^3)/3 + ((b*e + a*g)*x^4)/4 +
 ((c*d + b*f + a*h)*x^5)/5 + ((c*e + b*g + a*i)*x^6)/6 + ((c*f +
b*h)*x^7)/7 + ((c*g + b*i)*x^8)/8 + (c*h*x^9)/9 + (c*i*x^10)/10
```

Maple [A] time = 0.001, size = 105, normalized size = 0.9

$$adx+\frac{aex^2}{2}+\frac{(fa+bd)\,x^3}{3}+\frac{(ag+be)\,x^4}{4}+\frac{(ah+bf+cd)\,x^5}{5}+\frac{(ai+bg+ce)\,x^6}{6}+\frac{(bh+cf)\,x^7}{7}+\frac{(bi+cg)\,x^8}{8}+\frac{chx^9}{9}+\frac{cix^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)*(i*x^5+h*x^4+g*x^3+f*x^2+e*x+d),x)
```

```
[Out] a*d*x+1/2*a*e*x^2+1/3*(a*f+b*d)*x^3+1/4*(a*g+b*e)*x^4+1/5*(a*h+b*
f+c*d)*x^5+1/6*(a*i+b*g+c*e)*x^6+1/7*(b*h+c*f)*x^7+1/8*(b*i+c*g)*
x^8+1/9*c*h*x^9+1/10*c*i*x^10
```

Maxima [A] time = 0.701168, size = 140, normalized size = 1.15

$$\frac{1}{10}\,cix^{10}+\frac{1}{9}\,chx^9+\frac{1}{8}\,(cg+bi)x^8+\frac{1}{7}\,(cf+bh)x^7+\frac{1}{6}\,(ce+bg+ai)x^6+\frac{1}{5}\,(cd+bf+ah)x^5+\frac{1}{4}\,(be+ag)x^4+\frac{1}{2}\,aex^2+\frac{1}{3}\,(bd+af)x^3+adx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^5 + h*x^4 + g*x^3 + f*x^2 + e*x + d)*(c*x^4 + b*x^2 + a),x, algorith
```

```
[Out] 1/10*c*i*x^10 + 1/9*c*h*x^9 + 1/8*(c*g + b*i)*x^8 + 1/7*(c*f + b*
h)*x^7 + 1/6*(c*e + b*g + a*i)*x^6 + 1/5*(c*d + b*f + a*h)*x^5 +
1/4*(b*e + a*g)*x^4 + 1/2*a*e*x^2 + 1/3*(b*d + a*f)*x^3 + a*d*x
```

Fricas [A] time = 0.24205, size = 1, normalized size = 0.01

$$\frac{1}{10}x^{10}ic + \frac{1}{9}x^9hc + \frac{1}{8}x^8gc + \frac{1}{8}x^8ib + \frac{1}{7}x^7fc + \frac{1}{7}x^7hb + \frac{1}{6}x^6ec + \frac{1}{6}x^6gb + \frac{1}{6}x^6ia + \frac{1}{5}x^5dc + \frac{1}{5}x^5fb + \frac{1}{5}x^5ha + \frac{1}{4}x^4eb + \frac{1}{4}x^4ga + \frac{1}{3}x^3db + \frac{1}{3}x^3fa + \frac{1}{2}x^2ea + xda$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^5 + h*x^4 + g*x^3 + f*x^2 + e*x + d)*(c*x^4 + b*x^2 + a),x, algorith
```

```
[Out] 1/10*x^10*i*c + 1/9*x^9*h*c + 1/8*x^8*g*c + 1/8*x^8*i*b + 1/7*x^7
*f*c + 1/7*x^7*h*b + 1/6*x^6*e*c + 1/6*x^6*g*b + 1/6*x^6*i*a + 1/
5*x^5*d*c + 1/5*x^5*f*b + 1/5*x^5*h*a + 1/4*x^4*e*b + 1/4*x^4*g*a
 + 1/3*x^3*d*b + 1/3*x^3*f*a + 1/2*x^2*e*a + x*d*a
```

Sympy [A] time = 0.156229, size = 121, normalized size = 0.99

$$adx + \frac{aex^2}{2} + \frac{chx^9}{9} + \frac{cix^{10}}{10} + x^8\left(\frac{bi}{8} + \frac{cg}{8}\right) + x^7\left(\frac{bh}{7} + \frac{cf}{7}\right) + x^6\left(\frac{ai}{6} + \frac{bg}{6} + \frac{ce}{6}\right) + x^5\left(\frac{ah}{5} + \frac{bf}{5} + \frac{cd}{5}\right) + x^4\left(\frac{ag}{4} + \frac{be}{4}\right) + x^3\left(\frac{af}{3} + \frac{bd}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)*(i*x**5+h*x**4+g*x**3+f*x**2+e*x+d),x)
```

```
[Out] a*d*x + a*e*x**2/2 + c*h*x**9/9 + c*i*x**10/10 + x**8*(b*i/8 + c*
g/8) + x**7*(b*h/7 + c*f/7) + x**6*(a*i/6 + b*g/6 + c*e/6) + x**5
*(a*h/5 + b*f/5 + c*d/5) + x**4*(a*g/4 + b*e/4) + x**3*(a*f/3 + b
*d/3)
```

GIAC/XCAS [A] time = 0.290297, size = 171, normalized size = 1.4

$$\frac{1}{10}\,cix^{10}+\frac{1}{9}\,chx^{9}+\frac{1}{8}\,cgx^{8}+\frac{1}{8}\,bix^{8}+\frac{1}{7}\,cfx^{7}+\frac{1}{7}\,bhx^{7}+\frac{1}{6}\,bgx^{6}+\frac{1}{6}\,aix^{6}+\frac{1}{6}\,cx^{6}e+\frac{1}{5}\,cdx^{5}+\frac{1}{5}\,bfx^{5}+\frac{1}{5}\,ahx^{5}+\frac{1}{4}\,agx^{4}+\frac{1}{4}\,bx^{4}e+\frac{1}{3}\,bdx^{3}+\frac{1}{3}\,afx^{3}+\frac{1}{2}\,ax^{2}e+adx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^5 + h*x^4 + g*x^3 + f*x^2 + e*x + d)*(c*x^4 + b*x^2 + a),x, algorith
```

```
[Out] 1/10*c*i*x^10 + 1/9*c*h*x^9 + 1/8*c*g*x^8 + 1/8*b*i*x^8 + 1/7*c*f
*x^7 + 1/7*b*h*x^7 + 1/6*b*g*x^6 + 1/6*a*i*x^6 + 1/6*c*x^6*e + 1/
5*c*d*x^5 + 1/5*b*f*x^5 + 1/5*a*h*x^5 + 1/4*a*g*x^4 + 1/4*b*x^4*e
 + 1/3*b*d*x^3 + 1/3*a*f*x^3 + 1/2*a*x^2*e + a*d*x
```

3.6 $\int (d+ex)\left(a+bx^2+cx^4\right)^2\,dx$

Optimal. Leaf size=112

$$a^2dx+\frac{1}{2}a^2ex^2+\frac{1}{5}dx^5\left(2ac+b^2\right)+\frac{1}{6}ex^6\left(2ac+b^2\right)+\frac{2}{3}abdx^3+\frac{1}{2}abex^4+\frac{2}{7}bcdx^7+\frac{1}{4}bcex^8+\frac{1}{9}c^2dx^9+\frac{1}{10}c^2ex^{10}$$

```
[Out] a^2*d*x + (a^2*e*x^2)/2 + (2*a*b*d*x^3)/3 + (a*b*e*x^4)/2 + ((b^2
 + 2*a*c)*d*x^5)/5 + ((b^2 + 2*a*c)*e*x^6)/6 + (2*b*c*d*x^7)/7 +
(b*c*e*x^8)/4 + (c^2*d*x^9)/9 + (c^2*e*x^10)/10
```

Rubi [A] time = 0.262465, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05

$$a^2dx+\frac{1}{2}a^2ex^2+\frac{1}{5}dx^5\left(2ac+b^2\right)+\frac{1}{6}ex^6\left(2ac+b^2\right)+\frac{2}{3}abdx^3+\frac{1}{2}abex^4+\frac{2}{7}bcdx^7+\frac{1}{4}bcex^8+\frac{1}{9}c^2dx^9+\frac{1}{10}c^2ex^{10}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x)*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] a^2*d*x + (a^2*e*x^2)/2 + (2*a*b*d*x^3)/3 + (a*b*e*x^4)/2 + ((b^2
 + 2*a*c)*d*x^5)/5 + ((b^2 + 2*a*c)*e*x^6)/6 + (2*b*c*d*x^7)/7 +
(b*c*e*x^8)/4 + (c^2*d*x^9)/9 + (c^2*e*x^10)/10
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^2e\int x\,dx+a^2\int d\,dx+\frac{2abdx^3}{3}+\frac{abex^4}{2}+\frac{2bcdx^7}{7}+\frac{bcex^8}{4}+\frac{c^2dx^9}{9}+\frac{c^2ex^{10}}{10}+\frac{dx^5\left(2ac+b^2\right)}{5}+\frac{ex^6\left(2ac+b^2\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x+d)*(c*x**4+b*x**2+a)**2,x)
```

```
[Out] a**2*e*Integral(x, x) + a**2*Integral(d, x) + 2*a*b*d*x**3/3 + a*
b*e*x**4/2 + 2*b*c*d*x**7/7 + b*c*e*x**8/4 + c**2*d*x**9/9 + c**2
```

```
*e*x**10/10 + d*x**5*(2*a*c + b**2)/5 + e*x**6*(2*a*c + b**2)/6
```

Mathematica [A] time = 0.0890925, size = 97, normalized size = 0.87

$$\frac{630a^2x(2d+ex)+42a\left(5bx^3(4d+3ex)+2cx^5(6d+5ex)\right)+42b^2x^5(6d+5ex)+45bcx^7(8d+7ex)+14c^2x^9(10d+9ex)}{1260}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x)*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (630*a^2*x*(2*d + e*x) + 42*b^2*x^5*(6*d + 5*e*x) + 45*b*c*x^7*(8
*d + 7*e*x) + 14*c^2*x^9*(10*d + 9*e*x) + 42*a*(5*b*x^3*(4*d + 3*
e*x) + 2*c*x^5*(6*d + 5*e*x)))/1260
```

Maple [A] time = 0.001, size = 95, normalized size = 0.9

$$a^2dx+\frac{a^2ex^2}{2}+\frac{2\,abdx^3}{3}+\frac{abex^4}{2}+\frac{\left(2\,ac+b^2\right)\,dx^5}{5}+\frac{\left(2\,ac+b^2\right)\,ex^6}{6}+\frac{2\,bcdx^7}{7}+\frac{bcex^8}{4}+\frac{c^2dx^9}{9}+\frac{c^2ex^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x+d)*(c*x^4+b*x^2+a)^2,x)
```

```
[Out] a^2*d*x+1/2*a^2*e*x^2+2/3*a*b*d*x^3+1/2*a*b*e*x^4+1/5*(2*a*c+b^2)
*d*x^5+1/6*(2*a*c+b^2)*e*x^6+2/7*b*c*d*x^7+1/4*b*c*e*x^8+1/9*c^2*
d*x^9+1/10*c^2*e*x^10
```

Maxima [A] time = 0.700677, size = 127, normalized size = 1.13

$$\frac{1}{10}\,c^2ex^{10}+\frac{1}{9}\,c^2dx^9+\frac{1}{4}\,bcex^8+\frac{2}{7}\,bcdx^7+\frac{1}{6}\,\left(b^2+2\,ac\right)ex^6+\frac{1}{2}\,abex^4+\frac{1}{5}\,\left(b^2+2\,ac\right)dx^5+\frac{2}{3}\,abdx^3+\frac{1}{2}\,a^2ex^2+a^2dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^2*(e*x + d),x, algorithm="maxima")
```

```
[Out] 1/10*c^2*e*x^10 + 1/9*c^2*d*x^9 + 1/4*b*c*e*x^8 + 2/7*b*c*d*x^7 +
 1/6*(b^2 + 2*a*c)*e*x^6 + 1/2*a*b*e*x^4 + 1/5*(b^2 + 2*a*c)*d*x^
```

```
5 + 2/3*a*b*d*x^3 + 1/2*a^2*e*x^2 + a^2*d*x
```

Fricas [A] time = 0.235922, size = 1, normalized size = 0.01

$$\frac{1}{10}x^{10}ec^2 + \frac{1}{9}x^9dc^2 + \frac{1}{4}x^8ecb + \frac{2}{7}x^7dcb + \frac{1}{6}x^6eb^2 + \frac{1}{3}x^6eca + \frac{1}{5}x^5db^2 + \frac{2}{5}x^5dca + \frac{1}{2}x^4eba + \frac{2}{3}x^3dba + \frac{1}{2}x^2ea^2 + xda^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^2*(e*x + d),x, algorithm="fricas")
```

```
[Out] 1/10*x^10*e*c^2 + 1/9*x^9*d*c^2 + 1/4*x^8*e*c*b + 2/7*x^7*d*c*b +
 1/6*x^6*e*b^2 + 1/3*x^6*e*c*a + 1/5*x^5*d*b^2 + 2/5*x^5*d*c*a +
1/2*x^4*e*b*a + 2/3*x^3*d*b*a + 1/2*x^2*e*a^2 + x*d*a^2
```

Sympy [A] time = 0.153786, size = 116, normalized size = 1.04

$$a^2dx + \frac{a^2ex^2}{2} + \frac{2abdx^3}{3} + \frac{abex^4}{2} + \frac{2bcdx^7}{7} + \frac{bcex^8}{4} + \frac{c^2dx^9}{9} + \frac{c^2ex^{10}}{10} + x^6\left(\frac{ace}{3} + \frac{b^2e}{6}\right) + x^5\left(\frac{2acd}{5} + \frac{b^2d}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x+d)*(c*x**4+b*x**2+a)**2,x)
```

```
[Out] a**2*d*x + a**2*e*x**2/2 + 2*a*b*d*x**3/3 + a*b*e*x**4/2 + 2*b*c*
d*x**7/7 + b*c*e*x**8/4 + c**2*d*x**9/9 + c**2*e*x**10/10 + x**6*
(a*c*e/3 + b**2*e/6) + x**5*(2*a*c*d/5 + b**2*d/5)
```

GIAC/XCAS [A] time = 0.294979, size = 143, normalized size = 1.28

$$\frac{1}{10}\,c^2x^{10}e + \frac{1}{9}\,c^2dx^9 + \frac{1}{4}\,bcx^8e + \frac{2}{7}\,bcdx^7 + \frac{1}{6}\,b^2x^6e + \frac{1}{3}\,acx^6e + \frac{1}{5}\,b^2dx^5 + \frac{2}{5}\,acdx^5 + \frac{1}{2}\,abx^4e + \frac{2}{3}\,abdx^3 + \frac{1}{2}\,a^2x^2e + a^2dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^2*(e*x + d),x, algorithm="giac")
```

```
[Out] 1/10*c^2*x^10*e + 1/9*c^2*d*x^9 + 1/4*b*c*x^8*e + 2/7*b*c*d*x^7 +
 1/6*b^2*x^6*e + 1/3*a*c*x^6*e + 1/5*b^2*d*x^5 + 2/5*a*c*d*x^5 +
1/2*a*b*x^4*e + 2/3*a*b*d*x^3 + 1/2*a^2*x^2*e + a^2*d*x
```

3.7 $\int \left(d+ex+fx^2\right)\left(a+bx^2+cx^4\right)^2\,dx$

Optimal. Leaf size=154

$$a^2dx+\frac{1}{2}a^2ex^2+\frac{1}{7}x^7\left(2acf+b^2f+2bcd\right)+\frac{1}{5}x^5\left(2abf+2acd+b^2d\right)+\frac{1}{6}ex^6\left(2ac+b^2\right)+\frac{1}{3}ax^3(af+2bd)+\frac{1}{2}abex^4+\frac{1}{9}cx^9(2bf+cd)+\frac{1}{4}bcex^8+\frac{1}{10}c^2ex^{10}+\frac{1}{11}c^2fx^{11}$$

```
[Out] a^2*d*x + (a^2*e*x^2)/2 + (a*(2*b*d + a*f)*x^3)/3 + (a*b*e*x^4)/2
 + ((b^2*d + 2*a*c*d + 2*a*b*f)*x^5)/5 + ((b^2 + 2*a*c)*e*x^6)/6
+ ((2*b*c*d + b^2*f + 2*a*c*f)*x^7)/7 + (b*c*e*x^8)/4 + (c*(c*d +
 2*b*f)*x^9)/9 + (c^2*e*x^10)/10 + (c^2*f*x^11)/11
```

Rubi [A] time = 0.275443, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.04$

$$a^2dx+\frac{1}{2}a^2ex^2+\frac{1}{7}x^7\left(2acf+b^2f+2bcd\right)+\frac{1}{5}x^5\left(2abf+2acd+b^2d\right)+\frac{1}{6}ex^6\left(2ac+b^2\right)+\frac{1}{3}ax^3(af+2bd)+\frac{1}{2}abex^4+\frac{1}{9}cx^9(2bf+cd)+\frac{1}{4}bcex^8+\frac{1}{10}c^2ex^{10}+\frac{1}{11}c^2fx^{11}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x + f*x^2)*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] a^2*d*x + (a^2*e*x^2)/2 + (a*(2*b*d + a*f)*x^3)/3 + (a*b*e*x^4)/2
 + ((b^2*d + 2*a*c*d + 2*a*b*f)*x^5)/5 + ((b^2 + 2*a*c)*e*x^6)/6
+ ((2*b*c*d + b^2*f + 2*a*c*f)*x^7)/7 + (b*c*e*x^8)/4 + (c*(c*d +
 2*b*f)*x^9)/9 + (c^2*e*x^10)/10 + (c^2*f*x^11)/11
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^2e\int x\,dx+a^2\int d\,dx+\frac{abex^4}{2}+\frac{ax^3\left(af+2bd\right)}{3}+\frac{bcex^8}{4}+\frac{c^2ex^{10}}{10}+\frac{c^2fx^{11}}{11}+\frac{cx^9\left(2bf+cd\right)}{9}+\frac{ex^6\left(2ac+b^2\right)}{6}+x^7\left(\frac{2acf}{7}+\frac{b^2f}{7}+\frac{2bcd}{7}\right)+x^5\left(\frac{2abf}{5}+\frac{2acd}{5}+\frac{b^2d}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**2+e*x+d)*(c*x**4+b*x**2+a)**2,x)
```

```
[Out] a**2*e*Integral(x, x) + a**2*Integral(d, x) + a*b*e*x**4/2 + a*x*
*3*(a*f + 2*b*d)/3 + b*c*e*x**8/4 + c**2*e*x**10/10 + c**2*f*x**1
1/11 + c*x**9*(2*b*f + c*d)/9 + e*x**6*(2*a*c + b**2)/6 + x**7*(2
*a*c*f/7 + b**2*f/7 + 2*b*c*d/7) + x**5*(2*a*b*f/5 + 2*a*c*d/5 +
b**2*d/5)
```

Mathematica [A] time = 0.096575, size = 154, normalized size = 1.

$$a^2dx+\frac{1}{2}a^2ex^2+\frac{1}{7}x^7\left(2acf+b^2f+2bcd\right)+\frac{1}{5}x^5\left(2abf+2acd+b^2d\right)+\frac{1}{6}ex^6\left(2ac+b^2\right)+\frac{1}{3}ax^3(af+2bd)+\frac{1}{2}abex^4+\frac{1}{9}cx^9(2bf+cd)+\frac{1}{4}bcex^8+\frac{1}{10}c^2ex^{10}+\frac{1}{11}c^2fx^{11}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x + f*x^2)*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] a^2*d*x + (a^2*e*x^2)/2 + (a*(2*b*d + a*f)*x^3)/3 + (a*b*e*x^4)/2
 + ((b^2*d + 2*a*c*d + 2*a*b*f)*x^5)/5 + ((b^2 + 2*a*c)*e*x^6)/6
+ ((2*b*c*d + b^2*f + 2*a*c*f)*x^7)/7 + (b*c*e*x^8)/4 + (c*(c*d +
 2*b*f)*x^9)/9 + (c^2*e*x^10)/10 + (c^2*f*x^11)/11
```

Maple [A] time = 0.001, size = 139, normalized size = 0.9

$$\frac{c^2fx^{11}}{11}+\frac{c^2ex^{10}}{10}+\frac{\left(2\,bcf+c^2d\right)x^9}{9}+\frac{bcex^8}{4}+\frac{\left(2\,bcd+f\left(2\,ac+b^2\right)\right)x^7}{7}+\frac{\left(2\,ac+b^2\right)ex^6}{6}+\frac{\left(d\left(2\,ac+b^2\right)+2\,abf\right)x^5}{5}+\frac{abex^4}{2}+\frac{\left(a^2f+2\,abd\right)x^3}{3}+\frac{a^2ex^2}{2}+a^2dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x^2+e*x+d)*(c*x^4+b*x^2+a)^2,x)
```

```
[Out] 1/11*c^2*f*x^11+1/10*c^2*e*x^10+1/9*(2*b*c*f+c^2*d)*x^9+1/4*b*c*e
*x^8+1/7*(2*b*c*d+f*(2*a*c+b^2))*x^7+1/6*(2*a*c+b^2)*e*x^6+1/5*(d
*(2*a*c+b^2)+2*a*b*f)*x^5+1/2*a*b*e*x^4+1/3*(a^2*f+2*a*b*d)*x^3+1
/2*a^2*e*x^2+a^2*d*x
```

Maxima [A] time = 0.701541, size = 186, normalized size = 1.21

$$\frac{1}{11}\,c^2fx^{11}+\frac{1}{10}\,c^2ex^{10}+\frac{1}{4}\,bcex^8+\frac{1}{9}\,\left(c^2d+2\,bcf\right)x^9+\frac{1}{6}\,\left(b^2+2\,ac\right)ex^6+\frac{1}{7}\,\left(2\,bcd+\left(b^2+2\,ac\right)f\right)x^7+\frac{1}{2}\,abex^4+\frac{1}{5}\,\left(2\,abf+\left(b^2+2\,ac\right)d\right)x^5+\frac{1}{2}\,a^2ex^2+a^2dx+\frac{1}{3}\,\left(2\,abd+a^2f\right)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2 + a)^2*(f*x^2 + e*x + d),x, algorithm="maxima")
```

```
[Out] 1/11*c^2*f*x^11 + 1/10*c^2*e*x^10 + 1/4*b*c*e*x^8 + 1/9*(c^2*d +
2*b*c*f)*x^9 + 1/6*(b^2 + 2*a*c)*e*x^6 + 1/7*(2*b*c*d + (b^2 + 2*
a*c)*f)*x^7 + 1/2*a*b*e*x^4 + 1/5*(2*a*b*f + (b^2 + 2*a*c)*d)*x^5
 + 1/2*a^2*e*x^2 + a^2*d*x + 1/3*(2*a*b*d + a^2*f)*x^3
```

Fricas [A] time = 0.239069, size = 1, normalized size = 0.01

$$\frac{1}{11}x^{11}fc^2+\frac{1}{10}x^{10}ec^2+\frac{1}{9}x^9dc^2+\frac{2}{9}x^9fcb+\frac{1}{4}x^8ecb+\frac{2}{7}x^7dcb+\frac{1}{7}x^7fb^2+\frac{2}{7}x^7fca+\frac{1}{6}x^6eb^2+\frac{1}{3}x^6eca+\frac{1}{5}x^5db^2+\frac{2}{5}x^5dca+\frac{2}{5}x^5fba+\frac{1}{2}x^4eba+\frac{2}{3}x^3dba+\frac{1}{3}x^3fa^2+\frac{1}{2}x^2ea^2+xda^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2 + a)^2*(f*x^2 + e*x + d),x, algorithm="fricas")
```

```
[Out] 1/11*x^11*f*c^2 + 1/10*x^10*e*c^2 + 1/9*x^9*d*c^2 + 2/9*x^9*f*c*b
 + 1/4*x^8*e*c*b + 2/7*x^7*d*c*b + 1/7*x^7*f*b^2 + 2/7*x^7*f*c*a
+ 1/6*x^6*e*b^2 + 1/3*x^6*e*c*a + 1/5*x^5*d*b^2 + 2/5*x^5*d*c*a +
 2/5*x^5*f*b*a + 1/2*x^4*e*b*a + 2/3*x^3*d*b*a + 1/3*x^3*f*a^2 +
1/2*x^2*e*a^2 + x*d*a^2
```

Sympy [A] time = 0.174018, size = 165, normalized size = 1.07

$$a^2dx+\frac{a^2ex^2}{2}+\frac{abex^4}{2}+\frac{bcex^8}{4}+\frac{c^2ex^{10}}{10}+\frac{c^2fx^{11}}{11}+x^9\left(\frac{2bcf}{9}+\frac{c^2d}{9}\right)+x^7\left(\frac{2acf}{7}+\frac{b^2f}{7}+\frac{2bcd}{7}\right)+x^6\left(\frac{ace}{3}+\frac{b^2e}{6}\right)+x^5\left(\frac{2abf}{5}+\frac{2acd}{5}+\frac{b^2d}{5}\right)+x^3\left(\frac{a^2f}{3}+\frac{2abd}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x**2+e*x+d)*(c*x**4+b*x**2+a)**2,x)
```

```
[Out] a**2*d*x + a**2*e*x**2/2 + a*b*e*x**4/2 + b*c*e*x**8/4 + c**2*e*x
**10/10 + c**2*f*x**11/11 + x**9*(2*b*c*f/9 + c**2*d/9) + x**7*(2
*a*c*f/7 + b**2*f/7 + 2*b*c*d/7) + x**6*(a*c*e/3 + b**2*e/6) + x*
*5*(2*a*b*f/5 + 2*a*c*d/5 + b**2*d/5) + x**3*(a**2*f/3 + 2*a*b*d/
3)
```

GIAC/XCAS [A] time = 0.294016, size = 212, normalized size = 1.38

$$\frac{1}{11}\,c^2fx^{11}+\frac{1}{10}\,c^2x^{10}e+\frac{1}{9}\,c^2dx^9+\frac{2}{9}\,bcfx^9+\frac{1}{4}\,bcx^8e+\frac{2}{7}\,bcdx^7+\frac{1}{7}\,b^2fx^7+\frac{2}{7}\,acfx^7+\frac{1}{6}\,b^2x^6e+\frac{1}{3}\,acx^6e+\frac{1}{5}\,b^2dx^5+\frac{2}{5}\,acdx^5+\frac{2}{5}\,abfx^5+\frac{1}{2}\,abx^4e+\frac{2}{3}\,abdx^3+\frac{1}{3}\,a^2fx^3+\frac{1}{2}\,a^2x^2e+a^2dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^2*(f*x^2 + e*x + d),x, algorithm="giac")
```

```
[Out] 1/11*c^2*f*x^11 + 1/10*c^2*x^10*e + 1/9*c^2*d*x^9 + 2/9*b*c*f*x^9
 + 1/4*b*c*x^8*e + 2/7*b*c*d*x^7 + 1/7*b^2*f*x^7 + 2/7*a*c*f*x^7
+ 1/6*b^2*x^6*e + 1/3*a*c*x^6*e + 1/5*b^2*d*x^5 + 2/5*a*c*d*x^5 +
 2/5*a*b*f*x^5 + 1/2*a*b*x^4*e + 2/3*a*b*d*x^3 + 1/3*a^2*f*x^3 +
1/2*a^2*x^2*e + a^2*d*x
```

3.8 $\int \left(d+ex+fx^2+gx^3\right)\left(a+bx^2+cx^4\right)^2\,dx$

Optimal. Leaf size=196

$$a^2dx+\frac{1}{2}a^2ex^2+\frac{1}{7}x^7\left(2acf+b^2f+2bcd\right)+\frac{1}{5}x^5\left(2abf+2acd+b^2d\right)+\frac{1}{8}x^8\left(2acg+b^2g+2bce\right)+\frac{1}{6}x^6\left(2abg+2ace+b^2e\right)+\frac{1}{3}ax^3(af+2bd)+\frac{1}{4}ax^4(ag+2be)+\frac{1}{9}cx^9(2bf+cd)+\frac{1}{10}cx^{10}(2bg+ce)+\frac{1}{11}c^2fx^{11}+\frac{1}{12}c^2gx^{12}$$

```
[Out] a^2*d*x + (a^2*e*x^2)/2 + (a*(2*b*d + a*f)*x^3)/3 + (a*(2*b*e + a
*g)*x^4)/4 + ((b^2*d + 2*a*c*d + 2*a*b*f)*x^5)/5 + ((b^2*e + 2*a*
c*e + 2*a*b*g)*x^6)/6 + ((2*b*c*d + b^2*f + 2*a*c*f)*x^7)/7 + ((2
*b*c*e + b^2*g + 2*a*c*g)*x^8)/8 + (c*(c*d + 2*b*f)*x^9)/9 + (c*(
c*e + 2*b*g)*x^10)/10 + (c^2*f*x^11)/11 + (c^2*g*x^12)/12
```

Rubi [A] time = 0.387965, antiderivative size = 196, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.033

$$a^2dx+\frac{1}{2}a^2ex^2+\frac{1}{7}x^7\left(2acf+b^2f+2bcd\right)+\frac{1}{5}x^5\left(2abf+2acd+b^2d\right)+\frac{1}{8}x^8\left(2acg+b^2g+2bce\right)+\frac{1}{6}x^6\left(2abg+2ace+b^2e\right)+\frac{1}{3}ax^3(af+2bd)+\frac{1}{4}ax^4(ag+2be)+\frac{1}{9}cx^9(2bf+cd)+\frac{1}{10}cx^{10}(2bg+ce)+\frac{1}{11}c^2fx^{11}+\frac{1}{12}c^2gx^{12}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x + f*x^2 + g*x^3)*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] a^2*d*x + (a^2*e*x^2)/2 + (a*(2*b*d + a*f)*x^3)/3 + (a*(2*b*e + a
*g)*x^4)/4 + ((b^2*d + 2*a*c*d + 2*a*b*f)*x^5)/5 + ((b^2*e + 2*a*
c*e + 2*a*b*g)*x^6)/6 + ((2*b*c*d + b^2*f + 2*a*c*f)*x^7)/7 + ((2
*b*c*e + b^2*g + 2*a*c*g)*x^8)/8 + (c*(c*d + 2*b*f)*x^9)/9 + (c*(
c*e + 2*b*g)*x^10)/10 + (c^2*f*x^11)/11 + (c^2*g*x^12)/12
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^2e\int x\,dx+a^2\int d\,dx+\frac{ax^4(ag+2be)}{4}+\frac{ax^3(af+2bd)}{3}+\frac{c^2fx^{11}}{11}+\frac{c^2gx^{12}}{12}+\frac{cx^{10}(2bg+ce)}{10}+\frac{cx^9(2bf+cd)}{9}+x^8\left(\frac{acg}{4}+\frac{b^2g}{8}+\frac{bce}{4}\right)+x^7\left(\frac{2acf}{7}+\frac{b^2f}{7}+\frac{2bcd}{7}\right)+x^6\left(\frac{abg}{3}+\frac{ace}{3}+\frac{b^2e}{6}\right)+x^5\left(\frac{2abf}{5}+\frac{2acd}{5}+\frac{b^2d}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((g*x**3+f*x**2+e*x+d)*(c*x**4+b*x**2+a)**2,x)
```

```
[Out] a**2*e*Integral(x, x) + a**2*Integral(d, x) + a*x**4*(a*g + 2*b*e
)/4 + a*x**3*(a*f + 2*b*d)/3 + c**2*f*x**11/11 + c**2*g*x**12/12
+ c*x**10*(2*b*g + c*e)/10 + c*x**9*(2*b*f + c*d)/9 + x**8*(a*c*g
/4 + b**2*g/8 + b*c*e/4) + x**7*(2*a*c*f/7 + b**2*f/7 + 2*b*c*d/7
) + x**6*(a*b*g/3 + a*c*e/3 + b**2*e/6) + x**5*(2*a*b*f/5 + 2*a*c
*d/5 + b**2*d/5)
```

Mathematica [A] time = 0.117559, size = 196, normalized size = 1.

$$a^2dx+\frac{1}{2}a^2ex^2+\frac{1}{7}x^7\left(2acf+b^2f+2bcd\right)+\frac{1}{5}x^5\left(2abf+2acd+b^2d\right)+\frac{1}{8}x^8\left(2acg+b^2g+2bce\right)+\frac{1}{6}x^6\left(2abg+2ace+b^2e\right)+\frac{1}{3}ax^3(af+2bd)+\frac{1}{4}ax^4(ag+2be)+\frac{1}{9}cx^9(2bf+cd)+\frac{1}{10}cx^{10}(2bg+ce)+\frac{1}{11}c^2fx^{11}+\frac{1}{12}c^2gx^{12}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x + f*x^2 + g*x^3)*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] a^2*d*x + (a^2*e*x^2)/2 + (a*(2*b*d + a*f)*x^3)/3 + (a*(2*b*e + a
*g)*x^4)/4 + ((b^2*d + 2*a*c*d + 2*a*b*f)*x^5)/5 + ((b^2*e + 2*a*
c*e + 2*a*b*g)*x^6)/6 + ((2*b*c*d + b^2*f + 2*a*c*f)*x^7)/7 + ((2
*b*c*e + b^2*g + 2*a*c*g)*x^8)/8 + (c*(c*d + 2*b*f)*x^9)/9 + (c*(
c*e + 2*b*g)*x^10)/10 + (c^2*f*x^11)/11 + (c^2*g*x^12)/12
```

Maple [A] time = 0.001, size = 183, normalized size = 0.9

$$\frac{c^2gx^{12}}{12}+\frac{c^2fx^{11}}{11}+\frac{\left(2\,gbc+c^2e\right)x^{10}}{10}+\frac{\left(2\,bcf+c^2d\right)x^9}{9}+\frac{\left(2\,bce+g\left(2\,ac+b^2\right)\right)x^8}{8}+\frac{\left(2\,bcd+f\left(2\,ac+b^2\right)\right)x^7}{7}+\frac{\left(e\left(2\,ac+b^2\right)+2\,abg\right)x^6}{6}+\frac{\left(d\left(2\,ac+b^2\right)+2\,abf\right)x^5}{5}+\frac{\left(ga^2+2\,abe\right)x^4}{4}+\frac{\left(a^2f+2\,abd\right)x^3}{3}+\frac{a^2ex^2}{2}+a^2dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((g*x^3+f*x^2+e*x+d)*(c*x^4+b*x^2+a)^2,x)
```

```
[Out] 1/12*c^2*g*x^12+1/11*c^2*f*x^11+1/10*(2*b*c*g+c^2*e)*x^10+1/9*(2*
b*c*f+c^2*d)*x^9+1/8*(2*b*c*e+g*(2*a*c+b^2))*x^8+1/7*(2*b*c*d+f*(
2*a*c+b^2))*x^7+1/6*(e*(2*a*c+b^2)+2*a*b*g)*x^6+1/5*(d*(2*a*c+b^2
)+2*a*b*f)*x^5+1/4*(a^2*g+2*a*b*e)*x^4+1/3*(a^2*f+2*a*b*d)*x^3+1/
2*a^2*e*x^2+a^2*d*x
```

Maxima [A] time = 0.699471, size = 246, normalized size = 1.26

$$\frac{1}{12}c^2gx^{12} + \frac{1}{11}c^2fx^{11} + \frac{1}{10}\left(c^2e + 2\,bcg\right)x^{10} + \frac{1}{9}\left(c^2d + 2\,bcf\right)x^9 + \frac{1}{8}\left(2\,bce + \left(b^2 + 2\,ac\right)g\right)x^8 + \frac{1}{7}\left(2\,bcd + \left(b^2 + 2\,ac\right)f\right)x^7 + \frac{1}{6}\left(2\,abg + \left(b^2 + 2\,ac\right)e\right)x^6 + \frac{1}{5}\left(2\,abf + \left(b^2 + 2\,ac\right)d\right)x^5 + \frac{1}{2}a^2ex^2 + \frac{1}{4}\left(2\,abe + a^2g\right)x^4 + a^2dx + \frac{1}{3}\left(2\,abd + a^2f\right)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2 + a)^2*(g*x^3 + f*x^2 + e*x + d),x, algorithm="maxima")
```

```
[Out] 1/12*c^2*g*x^12 + 1/11*c^2*f*x^11 + 1/10*(c^2*e + 2*b*c*g)*x^10 +
 1/9*(c^2*d + 2*b*c*f)*x^9 + 1/8*(2*b*c*e + (b^2 + 2*a*c)*g)*x^8
+ 1/7*(2*b*c*d + (b^2 + 2*a*c)*f)*x^7 + 1/6*(2*a*b*g + (b^2 + 2*a
*c)*e)*x^6 + 1/5*(2*a*b*f + (b^2 + 2*a*c)*d)*x^5 + 1/2*a^2*e*x^2
+ 1/4*(2*a*b*e + a^2*g)*x^4 + a^2*d*x + 1/3*(2*a*b*d + a^2*f)*x^3
```

Fricas [A] time = 0.239816, size = 1, normalized size = 0.01

$$\frac{1}{12}x^{12}gc^2 + \frac{1}{11}x^{11}fc^2 + \frac{1}{10}x^{10}ec^2 + \frac{1}{5}x^{10}gcb + \frac{1}{9}x^9dc^2 + \frac{2}{9}x^9fcb + \frac{1}{4}x^8ecb + \frac{1}{8}x^8gb^2 + \frac{1}{4}x^8gca + \frac{2}{7}x^7dcb + \frac{1}{7}x^7fb^2 + \frac{2}{7}x^7fca + \frac{1}{6}x^6eb^2 + \frac{1}{3}x^6eca + \frac{1}{3}x^6gba + \frac{1}{5}x^5db^2 + \frac{2}{5}x^5dca + \frac{2}{5}x^5fba + \frac{1}{2}x^4eba + \frac{1}{4}x^4ga^2 + \frac{2}{3}x^3dba + \frac{1}{3}x^3fa^2 + \frac{1}{2}x^2ea^2 + xda^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2 + a)^2*(g*x^3 + f*x^2 + e*x + d),x, algorithm="fricas")
```

```
[Out] 1/12*x^12*g*c^2 + 1/11*x^11*f*c^2 + 1/10*x^10*e*c^2 + 1/5*x^10*g*
c*b + 1/9*x^9*d*c^2 + 2/9*x^9*f*c*b + 1/4*x^8*e*c*b + 1/8*x^8*g*b
^2 + 1/4*x^8*g*c*a + 2/7*x^7*d*c*b + 1/7*x^7*f*b^2 + 2/7*x^7*f*c*
a + 1/6*x^6*e*b^2 + 1/3*x^6*e*c*a + 1/3*x^6*g*b*a + 1/5*x^5*d*b^2
 + 2/5*x^5*d*c*a + 2/5*x^5*f*b*a + 1/2*x^4*e*b*a + 1/4*x^4*g*a^2
+ 2/3*x^3*d*b*a + 1/3*x^3*f*a^2 + 1/2*x^2*e*a^2 + x*d*a^2
```

Sympy [A] time = 0.206414, size = 209, normalized size = 1.07

$$a^2dx + \frac{a^2ex^2}{2} + \frac{c^2fx^{11}}{11} + \frac{c^2gx^{12}}{12} + x^{10}\left(\frac{bcg}{5} + \frac{c^2e}{10}\right) + x^9\left(\frac{2bcf}{9} + \frac{c^2d}{9}\right) + x^8\left(\frac{acg}{4} + \frac{b^2g}{8} + \frac{bce}{4}\right) + x^7\left(\frac{2acf}{7} + \frac{b^2f}{7} + \frac{2bcd}{7}\right) + x^6\left(\frac{abg}{3} + \frac{ace}{3} + \frac{b^2e}{6}\right) + x^5\left(\frac{2abf}{5} + \frac{2acd}{5} + \frac{b^2d}{5}\right) + x^4\left(\frac{a^2g}{4} + \frac{abe}{2}\right) + x^3\left(\frac{a^2f}{3} + \frac{2abd}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((g*x**3+f*x**2+e*x+d)*(c*x**4+b*x**2+a)**2,x)
```

```
[Out] a**2*d*x + a**2*e*x**2/2 + c**2*f*x**11/11 + c**2*g*x**12/12 + x*
*10*(b*c*g/5 + c**2*e/10) + x**9*(2*b*c*f/9 + c**2*d/9) + x**8*(a
*c*g/4 + b**2*g/8 + b*c*e/4) + x**7*(2*a*c*f/7 + b**2*f/7 + 2*b*c
*d/7) + x**6*(a*b*g/3 + a*c*e/3 + b**2*e/6) + x**5*(2*a*b*f/5 + 2
*a*c*d/5 + b**2*d/5) + x**4*(a**2*g/4 + a*b*e/2) + x**3*(a**2*f/3
 + 2*a*b*d/3)
```

GIAC/XCAS [A] time = 0.286153, size = 281, normalized size = 1.43

$$\frac{1}{12}c^2gx^{12} + \frac{1}{11}c^2fx^{11} + \frac{1}{5}bcgx^{10} + \frac{1}{10}c^2x^{10}e + \frac{1}{9}c^2dx^9 + \frac{2}{9}bcfx^9 + \frac{1}{8}b^2gx^8 + \frac{1}{4}acgx^8 + \frac{1}{4}bcx^8e + \frac{2}{7}bcdx^7 + \frac{1}{7}b^2fx^7 + \frac{2}{7}acfx^7 + \frac{1}{3}abgx^6 + \frac{1}{6}b^2x^6e + \frac{1}{3}acx^6e + \frac{1}{5}b^2dx^5 + \frac{2}{5}acdx^5 + \frac{2}{5}abfx^5 + \frac{1}{4}a^2gx^4 + \frac{1}{2}abx^4e + \frac{2}{3}abdx^3 + \frac{1}{3}a^2fx^3 + \frac{1}{2}a^2x^2e + a^2dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2 + a)^2*(g*x^3 + f*x^2 + e*x + d),x, algorithm="giac")
```

```
[Out] 1/12*c^2*g*x^12 + 1/11*c^2*f*x^11 + 1/5*b*c*g*x^10 + 1/10*c^2*x^1
0*e + 1/9*c^2*d*x^9 + 2/9*b*c*f*x^9 + 1/8*b^2*g*x^8 + 1/4*a*c*g*x
^8 + 1/4*b*c*x^8*e + 2/7*b*c*d*x^7 + 1/7*b^2*f*x^7 + 2/7*a*c*f*x^
7 + 1/3*a*b*g*x^6 + 1/6*b^2*x^6*e + 1/3*a*c*x^6*e + 1/5*b^2*d*x^5
 + 2/5*a*c*d*x^5 + 2/5*a*b*f*x^5 + 1/4*a^2*g*x^4 + 1/2*a*b*x^4*e
+ 2/3*a*b*d*x^3 + 1/3*a^2*f*x^3 + 1/2*a^2*x^2*e + a^2*d*x
```

3.9 $\int \left(a+bx^2+cx^4\right)^2 \left(d+ex+fx^2+gx^3+hx^4\right) \, dx$

Optimal. Leaf size=234

$$a^2dx+\frac{1}{2}a^2ex^2+\frac{1}{9}x^9\left(2c(ah+bf)+b^2h+c^2d\right)+\frac{1}{7}x^7\left(2b(ah+cd)+2acf+b^2f\right)+\frac{1}{5}x^5\left(2abf+a(ah+2cd)+b^2d\right)+\frac{1}{8}x^8\left(2acg+b^2g+2bce\right)+\frac{1}{6}x^6\left(2abg+2ace+b^2e\right)+\frac{1}{3}ax^3(af+2bd)+\frac{1}{4}ax^4(ag+2be)+\frac{1}{10}cx^{10}(2bg+ce)+\frac{1}{11}cx^{11}(2bh+cf)+\frac{1}{12}c^2gx^{12}+\frac{1}{13}c^2hx^{13}$$

```
[Out] a^2*d*x + (a^2*e*x^2)/2 + (a*(2*b*d + a*f)*x^3)/3 + (a*(2*b*e + a
*g)*x^4)/4 + ((b^2*d + 2*a*b*f + a*(2*c*d + a*h))*x^5)/5 + ((b^2*
e + 2*a*c*e + 2*a*b*g)*x^6)/6 + ((b^2*f + 2*a*c*f + 2*b*(c*d + a*
h))*x^7)/7 + ((2*b*c*e + b^2*g + 2*a*c*g)*x^8)/8 + ((c^2*d + b^2*
h + 2*c*(b*f + a*h))*x^9)/9 + (c*(c*e + 2*b*g)*x^10)/10 + (c*(c*f
 + 2*b*h)*x^11)/11 + (c^2*g*x^12)/12 + (c^2*h*x^13)/13
```

Rubi [A] time = 0.582064, antiderivative size = 234, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.029

$$a^2dx+\frac{1}{2}a^2ex^2+\frac{1}{9}x^9\left(2c(ah+bf)+b^2h+c^2d\right)+\frac{1}{7}x^7\left(2b(ah+cd)+2acf+b^2f\right)+\frac{1}{5}x^5\left(2abf+a(ah+2cd)+b^2d\right)+\frac{1}{8}x^8\left(2acg+b^2g+2bce\right)+\frac{1}{6}x^6\left(2abg+2ace+b^2e\right)+\frac{1}{3}ax^3(af+2bd)+\frac{1}{4}ax^4(ag+2be)+\frac{1}{10}cx^{10}(2bg+ce)+\frac{1}{11}cx^{11}(2bh+cf)+\frac{1}{12}c^2gx^{12}+\frac{1}{13}c^2hx^{13}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2 + c*x^4)^2*(d + e*x + f*x^2 + g*x^3 + h*x^4),x]
```

```
[Out] a^2*d*x + (a^2*e*x^2)/2 + (a*(2*b*d + a*f)*x^3)/3 + (a*(2*b*e + a
*g)*x^4)/4 + ((b^2*d + 2*a*b*f + a*(2*c*d + a*h))*x^5)/5 + ((b^2*
e + 2*a*c*e + 2*a*b*g)*x^6)/6 + ((b^2*f + 2*a*c*f + 2*b*(c*d + a*
h))*x^7)/7 + ((2*b*c*e + b^2*g + 2*a*c*g)*x^8)/8 + ((c^2*d + b^2*
h + 2*c*(b*f + a*h))*x^9)/9 + (c*(c*e + 2*b*g)*x^10)/10 + (c*(c*f
 + 2*b*h)*x^11)/11 + (c^2*g*x^12)/12 + (c^2*h*x^13)/13
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^2e\int x\,dx + a^2\int d\,dx + \frac{ax^4\,(ag+2be)}{4} + \frac{ax^3\,(af+2bd)}{3} + \frac{c^2gx^{12}}{12} + \frac{c^2hx^{13}}{13} + \frac{cx^{11}\,(2bh+cf)}{11} + \frac{cx^{10}\,(2bg+ce)}{10} + x^9\left(\frac{2ach}{9} + \frac{b^2h}{9} + \frac{2bcf}{9} + \frac{c^2d}{9}\right) + x^8\left(\frac{acg}{4} + \frac{b^2g}{8} + \frac{bce}{4}\right) + x^7\left(\frac{2abh}{7} + \frac{2acf}{7} + \frac{b^2f}{7} + \frac{2bcd}{7}\right) + x^6\left(\frac{abg}{3} + \frac{ace}{3} + \frac{b^2e}{6}\right) + x^5\left(\frac{a^2h}{5} + \frac{2abf}{5} + \frac{2acd}{5} + \frac{b^2d}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((c*x**4+b*x**2+a)**2*(h*x**4+g*x**3+f*x**2+e*x+d),x)
```

```
[Out] a**2*e*Integral(x, x) + a**2*Integral(d, x) + a*x**4*(a*g + 2*b*e
)/4 + a*x**3*(a*f + 2*b*d)/3 + c**2*g*x**12/12 + c**2*h*x**13/13
+ c*x**11*(2*b*h + c*f)/11 + c*x**10*(2*b*g + c*e)/10 + x**9*(2*a
*c*h/9 + b**2*h/9 + 2*b*c*f/9 + c**2*d/9) + x**8*(a*c*g/4 + b**2*
g/8 + b*c*e/4) + x**7*(2*a*b*h/7 + 2*a*c*f/7 + b**2*f/7 + 2*b*c*d
/7) + x**6*(a*b*g/3 + a*c*e/3 + b**2*e/6) + x**5*(a**2*h/5 + 2*a*
b*f/5 + 2*a*c*d/5 + b**2*d/5)
```

Mathematica [A] time = 0.285563, size = 234, normalized size = 1.

$$\frac{1}{5}x^5\left(a^2h + 2abf + 2acd + b^2d\right) + a^2dx + \frac{1}{2}a^2ex^2 + \frac{1}{9}x^9\left(2ach + b^2h + 2bcf + c^2d\right) + \frac{1}{7}x^7\left(2abh + 2acf + b^2f + 2bcd\right) + \frac{1}{8}x^8\left(2acg + b^2g + 2bce\right) + \frac{1}{6}x^6\left(2abg + 2ace + b^2e\right) + \frac{1}{3}ax^3(af+2bd) + \frac{1}{4}ax^4(ag+2be) + \frac{1}{10}cx^{10}(2bg+ce) + \frac{1}{11}cx^{11}(2bh+cf) + \frac{1}{12}c^2gx^{12} + \frac{1}{13}c^2hx^{13}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2 + c*x^4)^2*(d + e*x + f*x^2 + g*x^3 + h*x^4),x]
```

```
[Out] a^2*d*x + (a^2*e*x^2)/2 + (a*(2*b*d + a*f)*x^3)/3 + (a*(2*b*e + a
*g)*x^4)/4 + ((b^2*d + 2*a*c*d + 2*a*b*f + a^2*h)*x^5)/5 + ((b^2*
e + 2*a*c*e + 2*a*b*g)*x^6)/6 + ((2*b*c*d + b^2*f + 2*a*c*f + 2*a
*b*h)*x^7)/7 + ((2*b*c*e + b^2*g + 2*a*c*g)*x^8)/8 + ((c^2*d + 2*
b*c*f + b^2*h + 2*a*c*h)*x^9)/9 + (c*(c*e + 2*b*g)*x^10)/10 + (c*
(c*f + 2*b*h)*x^11)/11 + (c^2*g*x^12)/12 + (c^2*h*x^13)/13
```

Maple [A] time = 0.001, size = 219, normalized size = 0.9

$$\frac{c^2hx^{13}}{13}+\frac{c^2gx^{12}}{12}+\frac{(2\,bch+c^2f)\,x^{11}}{11}+\frac{(2\,gbc+c^2e)\,x^{10}}{10}+\frac{((2\,ac+b^2)\,h+2\,bcf+c^2d)\,x^9}{9}+\frac{(2\,bce+g\,(2\,ac+b^2))\,x^8}{8}+\frac{(2\,abh+f\,(2\,ac+b^2)+2\,bcd)\,x^7}{7}+\frac{(e\,(2\,ac+b^2)+2\,abg)\,x^6}{6}+\frac{(a^2h+2\,abf+d\,(2\,ac+b^2))\,x^5}{5}+\frac{(ga^2+2\,abe)\,x^4}{4}+\frac{(a^2f+2\,abd)\,x^3}{3}+\frac{a^2ex^2}{2}+a^2dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2+a)^2*(h*x^4+g*x^3+f*x^2+e*x+d),x)
```

```
[Out] 1/13*c^2*h*x^13+1/12*c^2*g*x^12+1/11*(2*b*c*h+c^2*f)*x^11+1/10*(2
*b*c*g+c^2*e)*x^10+1/9*((2*a*c+b^2)*h+2*b*c*f+c^2*d)*x^9+1/8*(2*b
*c*e+g*(2*a*c+b^2))*x^8+1/7*(2*a*b*h+f*(2*a*c+b^2)+2*b*c*d)*x^7+1
/6*(e*(2*a*c+b^2)+2*a*b*g)*x^6+1/5*(a^2*h+2*a*b*f+d*(2*a*c+b^2))*
x^5+1/4*(a^2*g+2*a*b*e)*x^4+1/3*(a^2*f+2*a*b*d)*x^3+1/2*a^2*e*x^2
+a^2*d*x
```

Maxima [A] time = 0.706078, size = 294, normalized size = 1.26

$$\frac{1}{13}\,c^2hx^{13}+\frac{1}{12}\,c^2gx^{12}+\frac{1}{11}\,(c^2f+2\,bch)\,x^{11}+\frac{1}{10}\,(c^2e+2\,bcg)\,x^{10}+\frac{1}{9}\,(c^2d+2\,bcf+(b^2+2\,ac)\,h)\,x^9+\frac{1}{8}\,(2\,bce+(b^2+2\,ac)\,g)\,x^8+\frac{1}{7}\,(2\,bcd+2\,abh+(b^2+2\,ac)\,f)\,x^7+\frac{1}{6}\,(2\,abg+(b^2+2\,ac)\,e)\,x^6+\frac{1}{5}\,(2\,abf+a^2h+(b^2+2\,ac)\,d)\,x^5+\frac{1}{2}\,a^2ex^2+\frac{1}{4}\,(2\,abe+a^2g)\,x^4+a^2dx+\frac{1}{3}\,(2\,abd+a^2f)\,x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^2*(h*x^4 + g*x^3 + f*x^2 + e*x + d),x, algorithm="ma
```

```
[Out] 1/13*c^2*h*x^13 + 1/12*c^2*g*x^12 + 1/11*(c^2*f + 2*b*c*h)*x^11 +
 1/10*(c^2*e + 2*b*c*g)*x^10 + 1/9*(c^2*d + 2*b*c*f + (b^2 + 2*a*
c)*h)*x^9 + 1/8*(2*b*c*e + (b^2 + 2*a*c)*g)*x^8 + 1/7*(2*b*c*d +
2*a*b*h + (b^2 + 2*a*c)*f)*x^7 + 1/6*(2*a*b*g + (b^2 + 2*a*c)*e)*
x^6 + 1/5*(2*a*b*f + a^2*h + (b^2 + 2*a*c)*d)*x^5 + 1/2*a^2*e*x^2
 + 1/4*(2*a*b*e + a^2*g)*x^4 + a^2*d*x + 1/3*(2*a*b*d + a^2*f)*x^
3
```

Fricas [A] time = 0.241094, size = 1, normalized size = 0.

$$\frac{1}{13}x^{13}hc^2 + \frac{1}{12}x^{12}gc^2 + \frac{1}{11}x^{11}fc^2 + \frac{2}{11}x^{11}hcb + \frac{1}{10}x^{10}ec^2 + \frac{1}{5}x^{10}gcb + \frac{1}{9}x^9dc^2 + \frac{2}{9}x^9fcb + \frac{1}{9}x^9hb^2 + \frac{2}{9}x^9hca + \frac{1}{4}x^8ecb + \frac{1}{8}x^8gb^2 + \frac{1}{4}x^8gca + \frac{2}{7}x^7dcb + \frac{1}{7}x^7fb^2 + \frac{2}{7}x^7fca + \frac{2}{7}x^7hba + \frac{1}{6}x^6eb^2 + \frac{1}{3}x^6eca + \frac{1}{3}x^6gba + \frac{1}{5}x^5db^2 + \frac{2}{5}x^5dca + \frac{2}{5}x^5fba + \frac{1}{5}x^5ha^2 + \frac{1}{2}x^4eba + \frac{1}{4}x^4ga^2 + \frac{2}{3}x^3dba + \frac{1}{3}x^3fa^2 + \frac{1}{2}x^2ea^2 + xda^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2 + a)^2*(h*x^4 + g*x^3 + f*x^2 + e*x + d),x, algorithm="fr
```

```
[Out] 1/13*x^13*h*c^2 + 1/12*x^12*g*c^2 + 1/11*x^11*f*c^2 + 2/11*x^11*h
*c*b + 1/10*x^10*e*c^2 + 1/5*x^10*g*c*b + 1/9*x^9*d*c^2 + 2/9*x^9
*f*c*b + 1/9*x^9*h*b^2 + 2/9*x^9*h*c*a + 1/4*x^8*e*c*b + 1/8*x^8*
g*b^2 + 1/4*x^8*g*c*a + 2/7*x^7*d*c*b + 1/7*x^7*f*b^2 + 2/7*x^7*f
*c*a + 2/7*x^7*h*b*a + 1/6*x^6*e*b^2 + 1/3*x^6*e*c*a + 1/3*x^6*g*
b*a + 1/5*x^5*d*b^2 + 2/5*x^5*d*c*a + 2/5*x^5*f*b*a + 1/5*x^5*h*a
^2 + 1/2*x^4*e*b*a + 1/4*x^4*g*a^2 + 2/3*x^3*d*b*a + 1/3*x^3*f*a^
2 + 1/2*x^2*e*a^2 + x*d*a^2
```

Sympy [A] time = 0.231105, size = 258, normalized size = 1.1

$$a^2dx + \frac{a^2ex^2}{2} + \frac{c^2gx^{12}}{12} + \frac{c^2hx^{13}}{13} + x^{11}\left(\frac{2bch}{11} + \frac{c^2f}{11}\right) + x^{10}\left(\frac{bcg}{5} + \frac{c^2e}{10}\right) + x^9\left(\frac{2ach}{9} + \frac{b^2h}{9} + \frac{2bcf}{9} + \frac{c^2d}{9}\right) + x^8\left(\frac{acg}{4} + \frac{b^2g}{8} + \frac{bce}{4}\right) + x^7\left(\frac{2abh}{7} + \frac{2acf}{7} + \frac{b^2f}{7} + \frac{2bcd}{7}\right) + x^6\left(\frac{abg}{3} + \frac{ace}{3} + \frac{b^2e}{6}\right) + x^5\left(\frac{a^2h}{5} + \frac{2abf}{5} + \frac{2acd}{5} + \frac{b^2d}{5}\right) + x^4\left(\frac{a^2g}{4} + \frac{abe}{2}\right) + x^3\left(\frac{a^2f}{3} + \frac{2abd}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2+a)**2*(h*x**4+g*x**3+f*x**2+e*x+d),x)
```

```
[Out] a**2*d*x + a**2*e*x**2/2 + c**2*g*x**12/12 + c**2*h*x**13/13 + x*
*11*(2*b*c*h/11 + c**2*f/11) + x**10*(b*c*g/5 + c**2*e/10) + x**9
*(2*a*c*h/9 + b**2*h/9 + 2*b*c*f/9 + c**2*d/9) + x**8*(a*c*g/4 +
b**2*g/8 + b*c*e/4) + x**7*(2*a*b*h/7 + 2*a*c*f/7 + b**2*f/7 + 2*
b*c*d/7) + x**6*(a*b*g/3 + a*c*e/3 + b**2*e/6) + x**5*(a**2*h/5 +
 2*a*b*f/5 + 2*a*c*d/5 + b**2*d/5) + x**4*(a**2*g/4 + a*b*e/2) +
x**3*(a**2*f/3 + 2*a*b*d/3)
```

GIAC/XCAS [A] time = 0.287739, size = 350, normalized size = 1.5

$$\frac{1}{13}\,c^2hx^{13}+\frac{1}{12}\,c^2gx^{12}+\frac{1}{11}\,c^2fx^{11}+\frac{2}{11}\,bchx^{11}+\frac{1}{5}\,bcgx^{10}+\frac{1}{10}\,c^2x^{10}e+\frac{1}{9}\,c^2dx^9+\frac{2}{9}\,bcfx^9+\frac{1}{9}\,b^2hx^9+\frac{2}{9}\,achx^9+\frac{1}{8}\,b^2gx^8+\frac{1}{4}\,acgx^8+\frac{1}{4}\,bcx^8e+\frac{2}{7}\,bcdx^7+\frac{1}{7}\,b^2fx^7+\frac{2}{7}\,acfx^7+\frac{2}{7}\,abhx^7+\frac{1}{3}\,abgx^6+\frac{1}{6}\,b^2x^6e+\frac{1}{3}\,acx^6e+\frac{1}{5}\,b^2dx^5+\frac{2}{5}\,acdx^5+\frac{2}{5}\,abfx^5+\frac{1}{5}\,a^2hx^5+\frac{1}{4}\,a^2gx^4+\frac{1}{2}\,abx^4e+\frac{2}{3}\,abdx^3+\frac{1}{3}\,a^2fx^3+\frac{1}{2}\,a^2x^2e+a^2dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2 + a)^2*(h*x^4 + g*x^3 + f*x^2 + e*x + d),x, algorithm="gi
```

```
[Out] 1/13*c^2*h*x^13 + 1/12*c^2*g*x^12 + 1/11*c^2*f*x^11 + 2/11*b*c*h*
x^11 + 1/5*b*c*g*x^10 + 1/10*c^2*x^10*e + 1/9*c^2*d*x^9 + 2/9*b*c
*f*x^9 + 1/9*b^2*h*x^9 + 2/9*a*c*h*x^9 + 1/8*b^2*g*x^8 + 1/4*a*c*
g*x^8 + 1/4*b*c*x^8*e + 2/7*b*c*d*x^7 + 1/7*b^2*f*x^7 + 2/7*a*c*f
*x^7 + 2/7*a*b*h*x^7 + 1/3*a*b*g*x^6 + 1/6*b^2*x^6*e + 1/3*a*c*x^
6*e + 1/5*b^2*d*x^5 + 2/5*a*c*d*x^5 + 2/5*a*b*f*x^5 + 1/5*a^2*h*x
^5 + 1/4*a^2*g*x^4 + 1/2*a*b*x^4*e + 2/3*a*b*d*x^3 + 1/3*a^2*f*x^
3 + 1/2*a^2*x^2*e + a^2*d*x
```

3.10 $\int \frac{d+ex}{4-5x^2+x^4}\,dx$

Optimal. Leaf size=45

$$-\frac{1}{6}d\tanh^{-1}\left(\frac{x}{2}\right)+\frac{1}{3}d\tanh^{-1}(x)-\frac{1}{6}e\log\left(1-x^2\right)+\frac{1}{6}e\log\left(4-x^2\right)$$

```
[Out] -(d*ArcTanh[x/2])/6 + (d*ArcTanh[x])/3 - (e*Log[1 - x^2])/6 + (e*
Log[4 - x^2])/6
```

Rubi [A] time = 0.06644, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389

$$-\frac{1}{6}d\tanh^{-1}\left(\frac{x}{2}\right)+\frac{1}{3}d\tanh^{-1}(x)-\frac{1}{6}e\log\left(1-x^2\right)+\frac{1}{6}e\log\left(4-x^2\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x)/(4 - 5*x^2 + x^4),x]
```

```
[Out] -(d*ArcTanh[x/2])/6 + (d*ArcTanh[x])/3 - (e*Log[1 - x^2])/6 + (e*
Log[4 - x^2])/6
```

Rubi in Sympy [A] time = 22.0075, size = 34, normalized size = 0.76

$$-\frac{d\operatorname{atanh}\left(\frac{x}{2}\right)}{6}+\frac{d\operatorname{atanh}(x)}{3}-\frac{e\log\left(-x^2+1\right)}{6}+\frac{e\log\left(-x^2+4\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x+d)/(x**4-5*x**2+4),x)
```

```
[Out] -d*atanh(x/2)/6 + d*atanh(x)/3 - e*log(-x**2 + 1)/6 + e*log(-x**2
 + 4)/6
```

Mathematica [A] time = 0.0320466, size = 50, normalized size = 1.11

$$\frac{1}{12}(-2(d+e)\log(1-x)+(d+2e)\log(2-x)+2(d-e)\log(x+1)-(d-2e)\log(x+2))$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x)/(4 - 5*x^2 + x^4),x]
```

```
[Out] (-2*(d + e)*Log[1 - x] + (d + 2*e)*Log[2 - x] + 2*(d - e)*Log[1 +
 x] - (d - 2*e)*Log[2 + x])/12
```

Maple [A] time = 0.013, size = 58, normalized size = 1.3

$$-\frac{\ln(2+x)\,d}{12}+\frac{\ln(2+x)\,e}{6}-\frac{\ln(-1+x)\,d}{6}-\frac{\ln(-1+x)\,e}{6}+\frac{\ln(1+x)\,d}{6}-\frac{\ln(1+x)\,e}{6}+\frac{\ln(x-2)\,d}{12}+\frac{\ln(x-2)\,e}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x+d)/(x^4-5*x^2+4),x)
```

```
[Out] -1/12*ln(2+x)*d+1/6*ln(2+x)*e-1/6*ln(-1+x)*d-1/6*ln(-1+x)*e+1/6*l
n(1+x)*d-1/6*ln(1+x)*e+1/12*ln(x-2)*d+1/6*ln(x-2)*e
```

Maxima [A] time = 0.70226, size = 58, normalized size = 1.29

$$-\frac{1}{12}\,(d-2\,e)\log(x+2)+\frac{1}{6}\,(d-e)\log(x+1)-\frac{1}{6}\,(d+e)\log(x-1)+\frac{1}{12}\,(d+2\,e)\log(x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x + d)/(x^4 - 5*x^2 + 4),x, algorithm="maxima")
```

```
[Out] -1/12*(d - 2*e)*log(x + 2) + 1/6*(d - e)*log(x + 1) - 1/6*(d + e)
*log(x - 1) + 1/12*(d + 2*e)*log(x - 2)
```

Fricas [A] time = 0.271535, size = 58, normalized size = 1.29

$$-\frac{1}{12}\,(d-2\,e)\log(x+2)+\frac{1}{6}\,(d-e)\log(x+1)-\frac{1}{6}\,(d+e)\log(x-1)+\frac{1}{12}\,(d+2\,e)\log(x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x + d)/(x^4 - 5*x^2 + 4),x, algorithm="fricas")
```

```
[Out] -1/12*(d - 2*e)*log(x + 2) + 1/6*(d - e)*log(x + 1) - 1/6*(d + e)
*log(x - 1) + 1/12*(d + 2*e)*log(x - 2)
```

Sympy [A] time = 6.81026, size = 515, normalized size = 11.44

$$-\frac{(d-2e)\log\left(x+\frac{-35d^4e+\frac{51d^4(d-2e)}{2}-180d^2e^3-90d^2e^2(d-2e)+41d^2e(d-2e)^2-\frac{15d^2(d-2e)^3}{2}+320e^5-96e^4(d-2e)-80e^3(d-2e)^2+24e^2(d-2e)^3}{9d^5-160d^3e^2+256de^4}\right)}{12}+\frac{(d-e)\log\left(x+\frac{-35d^4e-51d^4(d-e)-180d^2e^3+180d^2e^2(d-e)+164d^2e(d-e)^2+60d^2(d-e)^3+320e^5+192e^4(d-e)-320e^3(d-e)^2-192e^2(d-e)^3}{9d^5-160d^3e^2+256de^4}\right)}{6}-\frac{(d+e)\log\left(x+\frac{-35d^4e+51d^4(d+e)-180d^2e^3-180d^2e^2(d+e)+164d^2e(d+e)^2-60d^2(d+e)^3+320e^5-192e^4(d+e)-320e^3(d+e)^2+192e^2(d+e)^3}{9d^5-160d^3e^2+256de^4}\right)}{6}+\frac{(d+2e)\log\left(x+\frac{-35d^4e-\frac{51d^4(d+2e)}{2}-180d^2e^3+90d^2e^2(d+2e)+41d^2e(d+2e)^2+\frac{15d^2(d+2e)^3}{2}+320e^5+96e^4(d+2e)-80e^3(d+2e)^2-24e^2(d+2e)^3}{9d^5-160d^3e^2+256de^4}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x+d)/(x**4-5*x**2+4),x)
```

```
[Out] -(d - 2*e)*log(x + (-35*d**4*e + 51*d**4*(d - 2*e)/2 - 180*d**2*e
**3 - 90*d**2*e**2*(d - 2*e) + 41*d**2*e*(d - 2*e)**2 - 15*d**2*(
d - 2*e)**3/2 + 320*e**5 - 96*e**4*(d - 2*e) - 80*e**3*(d - 2*e)*
*2 + 24*e**2*(d - 2*e)**3)/(9*d**5 - 160*d**3*e**2 + 256*d*e**4))
/12 + (d - e)*log(x + (-35*d**4*e - 51*d**4*(d - e) - 180*d**2*e*
*3 + 180*d**2*e**2*(d - e) + 164*d**2*e*(d - e)**2 + 60*d**2*(d -
 e)**3 + 320*e**5 + 192*e**4*(d - e) - 320*e**3*(d - e)**2 - 192*
e**2*(d - e)**3)/(9*d**5 - 160*d**3*e**2 + 256*d*e**4))/6 - (d +
e)*log(x + (-35*d**4*e + 51*d**4*(d + e) - 180*d**2*e**3 - 180*d*
*2*e**2*(d + e) + 164*d**2*e*(d + e)**2 - 60*d**2*(d + e)**3 + 32
0*e**5 - 192*e**4*(d + e) - 320*e**3*(d + e)**2 + 192*e**2*(d + e
)**3)/(9*d**5 - 160*d**3*e**2 + 256*d*e**4))/6 + (d + 2*e)*log(x
+ (-35*d**4*e - 51*d**4*(d + 2*e)/2 - 180*d**2*e**3 + 90*d**2*e**
2*(d + 2*e) + 41*d**2*e*(d + 2*e)**2 + 15*d**2*(d + 2*e)**3/2 + 3
20*e**5 + 96*e**4*(d + 2*e) - 80*e**3*(d + 2*e)**2 - 24*e**2*(d +
 2*e)**3)/(9*d**5 - 160*d**3*e**2 + 256*d*e**4))/12
```

GIAC/XCAS [A] time = 0.302092, size = 69, normalized size = 1.53

$$-\frac{1}{12}(d-2e)\ln(|x+2|)+\frac{1}{6}(d-e)\ln(|x+1|)-\frac{1}{6}(d+e)\ln(|x-1|)+\frac{1}{12}(d+2e)\ln(|x-2|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x + d)/(x^4 - 5*x^2 + 4),x, algorithm="giac")
```

```
[Out] -1/12*(d - 2*e)*ln(abs(x + 2)) + 1/6*(d - e)*ln(abs(x + 1)) - 1/6
*(d + e)*ln(abs(x - 1)) + 1/12*(d + 2*e)*ln(abs(x - 2))
```

3.11 $\int \frac{d+ex+fx^2}{4-5x^2+x^4}\,dx$

Optimal. Leaf size=51

$$-\frac{1}{6}(d+4f)\tanh^{-1}\left(\frac{x}{2}\right)+\frac{1}{3}(d+f)\tanh^{-1}(x)-\frac{1}{6}e\log\left(1-x^2\right)+\frac{1}{6}e\log\left(4-x^2\right)$$

```
[Out] -((d + 4*f)*ArcTanh[x/2])/6 + ((d + f)*ArcTanh[x])/3 - (e*Log[1 -
 x^2])/6 + (e*Log[4 - x^2])/6
```

Rubi [A] time = 0.116943, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304

$$-\frac{1}{6}(d+4f)\tanh^{-1}\left(\frac{x}{2}\right)+\frac{1}{3}(d+f)\tanh^{-1}(x)-\frac{1}{6}e\log\left(1-x^2\right)+\frac{1}{6}e\log\left(4-x^2\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x + f*x^2)/(4 - 5*x^2 + x^4),x]
```

```
[Out] -((d + 4*f)*ArcTanh[x/2])/6 + ((d + f)*ArcTanh[x])/3 - (e*Log[1 -
 x^2])/6 + (e*Log[4 - x^2])/6
```

Rubi in Sympy [A] time = 23.5199, size = 42, normalized size = 0.82

$$-\frac{e\log\left(-x^2+1\right)}{6}+\frac{e\log\left(-x^2+4\right)}{6}-\left(\frac{d}{6}+\frac{2f}{3}\right)\operatorname{atanh}\left(\frac{x}{2}\right)+\left(\frac{d}{3}+\frac{f}{3}\right)\operatorname{atanh}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**2+e*x+d)/(x**4-5*x**2+4),x)
```

```
[Out] -e*log(-x**2 + 1)/6 + e*log(-x**2 + 4)/6 - (d/6 + 2*f/3)*atanh(x/
2) + (d/3 + f/3)*atanh(x)
```

Mathematica [A] time = 0.0502888, size = 58, normalized size = 1.14

$$\frac{1}{12}(-2\log(1-x)(d+e+f)+\log(2-x)(d+2e+4f)+2\log(x+1)(d-e+f)-\log(x+2)(d-2e+4f))$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x + f*x^2)/(4 - 5*x^2 + x^4),x]
```

```
[Out] (-2*(d + e + f)*Log[1 - x] + (d + 2*e + 4*f)*Log[2 - x] + 2*(d -
e + f)*Log[1 + x] - (d - 2*e + 4*f)*Log[2 + x])/12
```

Maple [B] time = 0.011, size = 86, normalized size = 1.7

$$-\frac{\ln(2+x)d}{12}+\frac{\ln(2+x)e}{6}-\frac{\ln(2+x)f}{3}-\frac{\ln(-1+x)d}{6}-\frac{\ln(-1+x)e}{6}-\frac{\ln(-1+x)f}{6}+\frac{\ln(1+x)d}{6}-\frac{\ln(1+x)e}{6}+\frac{\ln(1+x)f}{6}+\frac{\ln(x-2)d}{12}+\frac{\ln(x-2)e}{6}+\frac{\ln(x-2)f}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x^2+e*x+d)/(x^4-5*x^2+4),x)
```

```
[Out] -1/12*ln(2+x)*d+1/6*ln(2+x)*e-1/3*ln(2+x)*f-1/6*ln(-1+x)*d-1/6*ln
(-1+x)*e-1/6*ln(-1+x)*f+1/6*ln(1+x)*d-1/6*ln(1+x)*e+1/6*ln(1+x)*f
+1/12*ln(x-2)*d+1/6*ln(x-2)*e+1/3*ln(x-2)*f
```

Maxima [A] time = 0.702025, size = 69, normalized size = 1.35

$$-\frac{1}{12}(d-2e+4f)\log(x+2)+\frac{1}{6}(d-e+f)\log(x+1)-\frac{1}{6}(d+e+f)\log(x-1)+\frac{1}{12}(d+2e+4f)\log(x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^2 + e*x + d)/(x^4 - 5*x^2 + 4),x, algorithm="maxima")
```

```
[Out] -1/12*(d - 2*e + 4*f)*log(x + 2) + 1/6*(d - e + f)*log(x + 1) - 1
/6*(d + e + f)*log(x - 1) + 1/12*(d + 2*e + 4*f)*log(x - 2)
```

Fricas [A] time = 0.324102, size = 69, normalized size = 1.35

$$-\frac{1}{12}(d-2e+4f)\log(x+2)+\frac{1}{6}(d-e+f)\log(x+1)-\frac{1}{6}(d+e+f)\log(x-1)+\frac{1}{12}(d+2e+4f)\log(x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^2 + e*x + d)/(x^4 - 5*x^2 + 4),x, algorithm="fricas")
```

```
[Out] -1/12*(d - 2*e + 4*f)*log(x + 2) + 1/6*(d - e + f)*log(x + 1) - 1
/6*(d + e + f)*log(x - 1) + 1/12*(d + 2*e + 4*f)*log(x - 2)
```

Sympy [A] time = 88.8178, size = 2195, normalized size = 43.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x**2+e*x+d)/(x**4-5*x**2+4),x)
```

```
[Out] -(d - 2*e + 4*f)*log(x + (-35*d**5*e + 51*d**5*(d - 2*e + 4*f)/2
- 820*d**4*e*f + 90*d**4*f*(d - 2*e + 4*f) - 180*d**3*e**3 - 90*d
**3*e**2*(d - 2*e + 4*f) - 4100*d**3*e*f**2 + 41*d**3*e*(d - 2*e
+ 4*f)**2 + 42*d**3*f**2*(d - 2*e + 4*f) - 15*d**3*(d - 2*e + 4*f
)**3/2 - 432*d**2*e**2*f*(d - 2*e + 4*f) - 8000*d**2*e*f**3 + 240
*d**2*e*f*(d - 2*e + 4*f)**2 - 240*d**2*f**3*(d - 2*e + 4*f) - 12
*d**2*f*(d - 2*e + 4*f)**3 + 320*d*e**5 - 96*d*e**4*(d - 2*e + 4*
f) + 720*d*e**3*f**2 - 80*d*e**3*(d - 2*e + 4*f)**2 - 1080*d*e**2
*f**2*(d - 2*e + 4*f) + 24*d*e**2*(d - 2*e + 4*f)**3 - 6400*d*e*f
**4 + 492*d*e*f**2*(d - 2*e + 4*f)**2 - 576*d*f**4*(d - 2*e + 4*f
) + 30*d*f**2*(d - 2*e + 4*f)**3 + 512*e**5*f - 128*e**3*f*(d - 2
*e + 4*f)**2 - 576*e**2*f**3*(d - 2*e + 4*f) - 1472*e*f**5 + 320*
e*f**3*(d - 2*e + 4*f)**2 - 480*f**5*(d - 2*e + 4*f) + 48*f**3*(d
 - 2*e + 4*f)**3)/(9*d**6 + 45*d**5*f - 160*d**4*e**2 - 36*d**4*f
**2 - 1312*d**3*e**2*f - 360*d**3*f**3 + 256*d**2*e**4 - 3840*d**
2*e**2*f**2 - 144*d**2*f**4 + 1280*d*e**4*f - 5248*d*e**2*f**3 +
720*d*f**5 + 1024*e**4*f**2 - 2560*e**2*f**4 + 576*f**6))/12 + (d
 - e + f)*log(x + (-35*d**5*e - 51*d**5*(d - e + f) - 820*d**4*e*
f - 180*d**4*f*(d - e + f) - 180*d**3*e**3 + 180*d**3*e**2*(d - e
 + f) - 4100*d**3*e*f**2 + 164*d**3*e*(d - e + f)**2 - 84*d**3*f*
*2*(d - e + f) + 60*d**3*(d - e + f)**3 + 864*d**2*e**2*f*(d - e
+ f) - 8000*d**2*e*f**3 + 960*d**2*e*f*(d - e + f)**2 + 480*d**2*
f**3*(d - e + f) + 96*d**2*f*(d - e + f)**3 + 320*d*e**5 + 192*d*
e**4*(d - e + f) + 720*d*e**3*f**2 - 320*d*e**3*(d - e + f)**2 +
2160*d*e**2*f**2*(d - e + f) - 192*d*e**2*(d - e + f)**3 - 6400*d
*e*f**4 + 1968*d*e*f**2*(d - e + f)**2 + 1152*d*f**4*(d - e + f)
- 240*d*f**2*(d - e + f)**3 + 512*e**5*f - 512*e**3*f*(d - e + f)
**2 + 1152*e**2*f**3*(d - e + f) - 1472*e*f**5 + 1280*e*f**3*(d -
 e + f)**2 + 960*f**5*(d - e + f) - 384*f**3*(d - e + f)**3)/(9*d
**6 + 45*d**5*f - 160*d**4*e**2 - 36*d**4*f**2 - 1312*d**3*e**2*f
 - 360*d**3*f**3 + 256*d**2*e**4 - 3840*d**2*e**2*f**2 - 144*d**2
*f**4 + 1280*d*e**4*f - 5248*d*e**2*f**3 + 720*d*f**5 + 1024*e**4
*f**2 - 2560*e**2*f**4 + 576*f**6))/6 - (d + e + f)*log(x + (-35*
```

```
d**5*e + 51*d**5*(d + e + f) - 820*d**4*e*f + 180*d**4*f*(d + e +
 f) - 180*d**3*e**3 - 180*d**3*e**2*(d + e + f) - 4100*d**3*e*f**
2 + 164*d**3*e*(d + e + f)**2 + 84*d**3*f**2*(d + e + f) - 60*d**
3*(d + e + f)**3 - 864*d**2*e**2*f*(d + e + f) - 8000*d**2*e*f**3
 + 960*d**2*e*f*(d + e + f)**2 - 480*d**2*f**3*(d + e + f) - 96*d
**2*f*(d + e + f)**3 + 320*d*e**5 - 192*d*e**4*(d + e + f) + 720*
d*e**3*f**2 - 320*d*e**3*(d + e + f)**2 - 2160*d*e**2*f**2*(d + e
 + f) + 192*d*e**2*(d + e + f)**3 - 6400*d*e*f**4 + 1968*d*e*f**2
*(d + e + f)**2 - 1152*d*f**4*(d + e + f) + 240*d*f**2*(d + e + f
)**3 + 512*e**5*f - 512*e**3*f*(d + e + f)**2 - 1152*e**2*f**3*(d
 + e + f) - 1472*e*f**5 + 1280*e*f**3*(d + e + f)**2 - 960*f**5*(
d + e + f) + 384*f**3*(d + e + f)**3)/(9*d**6 + 45*d**5*f - 160*d
**4*e**2 - 36*d**4*f**2 - 1312*d**3*e**2*f - 360*d**3*f**3 + 256*
d**2*e**4 - 3840*d**2*e**2*f**2 - 144*d**2*f**4 + 1280*d*e**4*f -
 5248*d*e**2*f**3 + 720*d*f**5 + 1024*e**4*f**2 - 2560*e**2*f**4
+ 576*f**6))/6 + (d + 2*e + 4*f)*log(x + (-35*d**5*e - 51*d**5*(d
 + 2*e + 4*f)/2 - 820*d**4*e*f - 90*d**4*f*(d + 2*e + 4*f) - 180*
d**3*e**3 + 90*d**3*e**2*(d + 2*e + 4*f) - 4100*d**3*e*f**2 + 41*
d**3*e*(d + 2*e + 4*f)**2 - 42*d**3*f**2*(d + 2*e + 4*f) + 15*d**
3*(d + 2*e + 4*f)**3/2 + 432*d**2*e**2*f*(d + 2*e + 4*f) - 8000*d
**2*e*f**3 + 240*d**2*e*f*(d + 2*e + 4*f)**2 + 240*d**2*f**3*(d +
 2*e + 4*f) + 12*d**2*f*(d + 2*e + 4*f)**3 + 320*d*e**5 + 96*d*e*
*4*(d + 2*e + 4*f) + 720*d*e**3*f**2 - 80*d*e**3*(d + 2*e + 4*f)*
*2 + 1080*d*e**2*f**2*(d + 2*e + 4*f) - 24*d*e**2*(d + 2*e + 4*f)
**3 - 6400*d*e*f**4 + 492*d*e*f**2*(d + 2*e + 4*f)**2 + 576*d*f**
4*(d + 2*e + 4*f) - 30*d*f**2*(d + 2*e + 4*f)**3 + 512*e**5*f - 1
28*e**3*f*(d + 2*e + 4*f)**2 + 576*e**2*f**3*(d + 2*e + 4*f) - 14
72*e*f**5 + 320*e*f**3*(d + 2*e + 4*f)**2 + 480*f**5*(d + 2*e + 4
*f) - 48*f**3*(d + 2*e + 4*f)**3)/(9*d**6 + 45*d**5*f - 160*d**4*
e**2 - 36*d**4*f**2 - 1312*d**3*e**2*f - 360*d**3*f**3 + 256*d**2
*e**4 - 3840*d**2*e**2*f**2 - 144*d**2*f**4 + 1280*d*e**4*f - 524
8*d*e**2*f**3 + 720*d*f**5 + 1024*e**4*f**2 - 2560*e**2*f**4 + 57
6*f**6))/12
```

GIAC/XCAS [A] time = 0.290333, size = 80, normalized size = 1.57

$$-\frac{1}{12}\,(d+4\,f-2\,e)\ln\left(|x+2|\right)+\frac{1}{6}\,(d+f-e)\ln\left(|x+1|\right)-\frac{1}{6}\,(d+f+e)\ln\left(|x-1|\right)+\frac{1}{12}\,(d+4\,f+2\,e)\ln\left(|x-2|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^2 + e*x + d)/(x^4 - 5*x^2 + 4),x, algorithm="giac")
```

```
[Out] -1/12*(d + 4*f - 2*e)*ln(abs(x + 2)) + 1/6*(d + f - e)*ln(abs(x +
 1)) - 1/6*(d + f + e)*ln(abs(x - 1)) + 1/12*(d + 4*f + 2*e)*ln(a
bs(x - 2))
```

3.12 $\int \frac{d+ex+fx^2+gx^3}{4-5x^2+x^4}\,dx$

Optimal. Leaf size=57

$$-\frac{1}{6}(d+4f)\tanh^{-1}\left(\frac{x}{2}\right)+\frac{1}{3}(d+f)\tanh^{-1}(x)-\frac{1}{6}(e+g)\log\left(1-x^2\right)+\frac{1}{6}(e+4g)\log\left(4-x^2\right)$$

```
[Out] -((d + 4*f)*ArcTanh[x/2])/6 + ((d + f)*ArcTanh[x])/3 - ((e + g)*L
og[1 - x^2])/6 + ((e + 4*g)*Log[4 - x^2])/6
```

Rubi [A] time = 0.175102, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$

$$-\frac{1}{6}(d+4f)\tanh^{-1}\left(\frac{x}{2}\right)+\frac{1}{3}(d+f)\tanh^{-1}(x)-\frac{1}{6}(e+g)\log\left(1-x^2\right)+\frac{1}{6}(e+4g)\log\left(4-x^2\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x + f*x^2 + g*x^3)/(4 - 5*x^2 + x^4),x]
```

```
[Out] -((d + 4*f)*ArcTanh[x/2])/6 + ((d + f)*ArcTanh[x])/3 - ((e + g)*L
og[1 - x^2])/6 + ((e + 4*g)*Log[4 - x^2])/6
```

Rubi in Sympy [A] time = 33.6666, size = 51, normalized size = 0.89

$$-\left(\frac{d}{6}+\frac{2f}{3}\right)\operatorname{atanh}\left(\frac{x}{2}\right)+\left(\frac{d}{3}+\frac{f}{3}\right)\operatorname{atanh}(x)-\left(\frac{e}{6}+\frac{g}{6}\right)\log\left(-x^2+1\right)+\left(\frac{e}{6}+\frac{2g}{3}\right)\log\left(-x^2+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((g*x**3+f*x**2+e*x+d)/(x**4-5*x**2+4),x)
```

```
[Out] -(d/6 + 2*f/3)*atanh(x/2) + (d/3 + f/3)*atanh(x) - (e/6 + g/6)*lo
g(-x**2 + 1) + (e/6 + 2*g/3)*log(-x**2 + 4)
```

Mathematica [A] time = 0.0589006, size = 68, normalized size = 1.19

$$\frac{1}{12}(-2\log(1-x)(d+e+f+g)+\log(2-x)(d+2e+4f+8g)+2\log(x+1)(d-e+f-g)-\log(x+2)(d-2e+4f-8g))$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x + f*x^2 + g*x^3)/(4 - 5*x^2 + x^4),x]
```

```
[Out] (-2*(d + e + f + g)*Log[1 - x] + (d + 2*e + 4*f + 8*g)*Log[2 - x]
 + 2*(d - e + f - g)*Log[1 + x] - (d - 2*e + 4*f - 8*g)*Log[2 + x
])/12
```

Maple [B] time = 0.013, size = 114, normalized size = 2.

$$-\frac{\ln(2+x)\,d}{12}+\frac{\ln(2+x)\,e}{6}-\frac{\ln(2+x)\,f}{3}+\frac{2\,\ln(2+x)\,g}{3}-\frac{\ln(-1+x)\,d}{6}-\frac{\ln(-1+x)\,e}{6} \\ -\frac{\ln(-1+x)\,f}{6}-\frac{\ln(-1+x)\,g}{6}+\frac{\ln(1+x)\,d}{6}-\frac{\ln(1+x)\,e}{6}+\frac{\ln(1+x)\,f}{6} \\ -\frac{\ln(1+x)\,g}{6}+\frac{\ln(x-2)\,d}{12}+\frac{\ln(x-2)\,e}{6}+\frac{\ln(x-2)\,f}{3}+\frac{2\,\ln(x-2)\,g}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4),x)
```

```
[Out] -1/12*ln(2+x)*d+1/6*ln(2+x)*e-1/3*ln(2+x)*f+2/3*ln(2+x)*g-1/6*ln(
-1+x)*d-1/6*ln(-1+x)*e-1/6*ln(-1+x)*f-1/6*ln(-1+x)*g+1/6*ln(1+x)*
d-1/6*ln(1+x)*e+1/6*ln(1+x)*f-1/6*ln(1+x)*g+1/12*ln(x-2)*d+1/6*ln
(x-2)*e+1/3*ln(x-2)*f+2/3*ln(x-2)*g
```

Maxima [A] time = 0.706383, size = 82, normalized size = 1.44

$$-\frac{1}{12}\,(d-2\,e+4\,f-8\,g)\log(x+2)+\frac{1}{6}\,(d-e+f-g)\log(x+1) \\ -\frac{1}{6}\,(d+e+f+g)\log(x-1)+\frac{1}{12}\,(d+2\,e+4\,f+8\,g)\log(x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3 + f*x^2 + e*x + d)/(x^4 - 5*x^2 + 4),x, algorithm="maxima")
```

```
[Out] -1/12*(d - 2*e + 4*f - 8*g)*log(x + 2) + 1/6*(d - e + f - g)*log(
x + 1) - 1/6*(d + e + f + g)*log(x - 1) + 1/12*(d + 2*e + 4*f + 8
*g)*log(x - 2)
```

Fricas [A] time = 0.48402, size = 82, normalized size = 1.44

$$-\frac{1}{12}\left(d-2e+4f-8g\right)\log\left(x+2\right)+\frac{1}{6}\left(d-e+f-g\right)\log\left(x+1\right)-\frac{1}{6}\left(d+e+f+g\right)\log\left(x-1\right)+\frac{1}{12}\left(d+2e+4f+8g\right)\log\left(x-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3 + f*x^2 + e*x + d)/(x^4 - 5*x^2 + 4),x, algorithm="fricas")
```

```
[Out] -1/12*(d - 2*e + 4*f - 8*g)*log(x + 2) + 1/6*(d - e + f - g)*log(
x + 1) - 1/6*(d + e + f + g)*log(x - 1) + 1/12*(d + 2*e + 4*f + 8
*g)*log(x - 2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((g*x**3+f*x**2+e*x+d)/(x**4-5*x**2+4),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.291984, size = 93, normalized size = 1.63

$$-\frac{1}{12}\left(d+4f-8g-2e\right)\ln\left(|x+2|\right)+\frac{1}{6}\left(d+f-g-e\right)\ln\left(|x+1|\right)-\frac{1}{6}\left(d+f+g+e\right)\ln\left(|x-1|\right)+\frac{1}{12}\left(d+4f+8g+2e\right)\ln\left(|x-2|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3 + f*x^2 + e*x + d)/(x^4 - 5*x^2 + 4),x, algorithm="giac")
```

```
[Out] -1/12*(d + 4*f - 8*g - 2*e)*ln(abs(x + 2)) + 1/6*(d + f - g - e)*
ln(abs(x + 1)) - 1/6*(d + f + g + e)*ln(abs(x - 1)) + 1/12*(d + 4
*f + 8*g + 2*e)*ln(abs(x - 2))
```

3.13 $\int \frac{d+ex+fx^2+gx^3+hx^4}{4-5x^2+x^4}\,dx$

Optimal. Leaf size=64

$$-\frac{1}{6}\tanh^{-1}\left(\frac{x}{2}\right)(d+4f+16h)+\frac{1}{3}\tanh^{-1}(x)(d+f+h)-\frac{1}{6}(e+g)\log\left(1-x^2\right)+\frac{1}{6}(e+4g)\log\left(4-x^2\right)+hx$$

```
[Out] h*x - ((d + 4*f + 16*h)*ArcTanh[x/2])/6 + ((d + f + h)*ArcTanh[x]
)/3 - ((e + g)*Log[1 - x^2])/6 + ((e + 4*g)*Log[4 - x^2])/6
```

Rubi [A] time = 0.315363, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212

$$-\frac{1}{6}\tanh^{-1}\left(\frac{x}{2}\right)(d+4f+16h)+\frac{1}{3}\tanh^{-1}(x)(d+f+h)-\frac{1}{6}(e+g)\log\left(1-x^2\right)+\frac{1}{6}(e+4g)\log\left(4-x^2\right)+hx$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x + f*x^2 + g*x^3 + h*x^4)/(4 - 5*x^2 + x^4),x]
```

```
[Out] h*x - ((d + 4*f + 16*h)*ArcTanh[x/2])/6 + ((d + f + h)*ArcTanh[x]
)/3 - ((e + g)*Log[1 - x^2])/6 + ((e + 4*g)*Log[4 - x^2])/6
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\left(\frac{e}{6}+\frac{g}{6}\right)\log\left(-x^2+1\right)+\left(\frac{e}{6}+\frac{2g}{3}\right)\log\left(-x^2+4\right)-\left(\frac{d}{6}+\frac{2f}{3}+\frac{8h}{3}\right)\operatorname{atanh}\left(\frac{x}{2}\right)+\left(\frac{d}{3}+\frac{f}{3}+\frac{h}{3}\right)\operatorname{atanh}(x)+\int h\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((h*x**4+g*x**3+f*x**2+e*x+d)/(x**4-5*x**2+4),x)
```

```
[Out] -(e/6 + g/6)*log(-x**2 + 1) + (e/6 + 2*g/3)*log(-x**2 + 4) - (d/6
 + 2*f/3 + 8*h/3)*atanh(x/2) + (d/3 + f/3 + h/3)*atanh(x) + Integ
ral(h, x)
```

Mathematica [A] time = 0.0879934, size = 81, normalized size = 1.27

$$\frac{1}{12}(-2\log(1-x)(d+e+f+g+h)+\log(2-x)(d+2(e+2f+4g+8h)) \\ +2\log(x+1)(d-e+f-g+h)-\log(x+2)(d-2e+4f-8g+16h)+12hx)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x + f*x^2 + g*x^3 + h*x^4)/(4 - 5*x^2 + x^4),x]
```

```
[Out] (12*h*x - 2*(d + e + f + g + h)*Log[1 - x] + (d + 2*(e + 2*f + 4*
g + 8*h))*Log[2 - x] + 2*(d - e + f - g + h)*Log[1 + x] - (d - 2*
e + 4*f - 8*g + 16*h)*Log[2 + x])/12
```

Maple [B] time = 0.014, size = 145, normalized size = 2.3

$$hx-\frac{\ln(2+x)\,d}{12}+\frac{\ln(2+x)\,e}{6}-\frac{\ln(2+x)\,f}{3}+\frac{2\ln(2+x)\,g}{3}-\frac{4\ln(2+x)\,h}{3} \\ -\frac{\ln(-1+x)\,d}{6}-\frac{\ln(-1+x)\,e}{6}-\frac{\ln(-1+x)\,f}{6}-\frac{\ln(-1+x)\,g}{6}-\frac{\ln(-1+x)\,h}{6} \\ +\frac{\ln(1+x)\,d}{6}-\frac{\ln(1+x)\,e}{6}+\frac{\ln(1+x)\,f}{6}-\frac{\ln(1+x)\,g}{6}+\frac{\ln(1+x)\,h}{6} \\ +\frac{\ln(x-2)\,d}{12}+\frac{\ln(x-2)\,e}{6}+\frac{\ln(x-2)\,f}{3}+\frac{2\ln(x-2)\,g}{3}+\frac{4\ln(x-2)\,h}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4),x)
```

```
[Out] h*x-1/12*ln(2+x)*d+1/6*ln(2+x)*e-1/3*ln(2+x)*f+2/3*ln(2+x)*g-4/3*
ln(2+x)*h-1/6*ln(-1+x)*d-1/6*ln(-1+x)*e-1/6*ln(-1+x)*f-1/6*ln(-1+
x)*g-1/6*ln(-1+x)*h+1/6*ln(1+x)*d-1/6*ln(1+x)*e+1/6*ln(1+x)*f-1/6
*ln(1+x)*g+1/6*ln(1+x)*h+1/12*ln(x-2)*d+1/6*ln(x-2)*e+1/3*ln(x-2)
*f+2/3*ln(x-2)*g+4/3*ln(x-2)*h
```

Maxima [A] time = 0.704997, size = 97, normalized size = 1.52

$$hx-\frac{1}{12}(d-2e+4f-8g+16h)\log(x+2)+\frac{1}{6}(d-e+f-g+h)\log(x+1) \\ -\frac{1}{6}(d+e+f+g+h)\log(x-1)+\frac{1}{12}(d+2e+4f+8g+16h)\log(x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^4 + g*x^3 + f*x^2 + e*x + d)/(x^4 - 5*x^2 + 4),x, algorithm="maxima
```

```
[Out] h*x - 1/12*(d - 2*e + 4*f - 8*g + 16*h)*log(x + 2) + 1/6*(d - e +
 f - g + h)*log(x + 1) - 1/6*(d + e + f + g + h)*log(x - 1) + 1/1
2*(d + 2*e + 4*f + 8*g + 16*h)*log(x - 2)
```

Fricas [A] time = 1.51834, size = 97, normalized size = 1.52

$$hx - \frac{1}{12}\left(d - 2e + 4f - 8g + 16h\right)\log\left(x + 2\right) + \frac{1}{6}\left(d - e + f - g + h\right)\log\left(x + 1\right) - \frac{1}{6}\left(d + e + f + g + h\right)\log\left(x - 1\right) + \frac{1}{12}\left(d + 2e + 4f + 8g + 16h\right)\log\left(x - 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^4 + g*x^3 + f*x^2 + e*x + d)/(x^4 - 5*x^2 + 4),x, algorithm="fricas
```

```
[Out] h*x - 1/12*(d - 2*e + 4*f - 8*g + 16*h)*log(x + 2) + 1/6*(d - e +
 f - g + h)*log(x + 1) - 1/6*(d + e + f + g + h)*log(x - 1) + 1/1
2*(d + 2*e + 4*f + 8*g + 16*h)*log(x - 2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x**4+g*x**3+f*x**2+e*x+d)/(x**4-5*x**2+4),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.297523, size = 108, normalized size = 1.69

$$hx - \frac{1}{12}\left(d + 4f - 8g + 16h - 2e\right)\ln\left(|x + 2|\right) + \frac{1}{6}\left(d + f - g + h - e\right)\ln\left(|x + 1|\right) - \frac{1}{6}\left(d + f + g + h + e\right)\ln\left(|x - 1|\right) + \frac{1}{12}\left(d + 4f + 8g + 16h + 2e\right)\ln\left(|x - 2|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^4 + g*x^3 + f*x^2 + e*x + d)/(x^4 - 5*x^2 + 4),x, algorithm="giac")
```

```
[Out] h*x - 1/12*(d + 4*f - 8*g + 16*h - 2*e)*ln(abs(x + 2)) + 1/6*(d +
 f - g + h - e)*ln(abs(x + 1)) - 1/6*(d + f + g + h + e)*ln(abs(x
 - 1)) + 1/12*(d + 4*f + 8*g + 16*h + 2*e)*ln(abs(x - 2))
```

3.14 $\int \frac{d+ex+fx^2+gx^3+hx^4+ix^5}{4-5x^2+x^4}\,dx$

Optimal. Leaf size=76

$$-\frac{1}{6}\tanh^{-1}\left(\frac{x}{2}\right)(d+4f+16h)+\frac{1}{3}\tanh^{-1}(x)(d+f+h)$$
$$-\frac{1}{6}\log\left(1-x^2\right)(e+g+i)+\frac{1}{6}\log\left(4-x^2\right)(e+4g+16i)+hx+\frac{ix^2}{2}$$

```
[Out] h*x + (i*x^2)/2 - ((d + 4*f + 16*h)*ArcTanh[x/2])/6 + ((d + f + h
)*ArcTanh[x])/3 - ((e + g + i)*Log[1 - x^2])/6 + ((e + 4*g + 16*i
)*Log[4 - x^2])/6
```

Rubi [A] time = 0.364424, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21

$$-\frac{1}{6}\tanh^{-1}\left(\frac{x}{2}\right)(d+4f+16h)+\frac{1}{3}\tanh^{-1}(x)(d+f+h)$$
$$-\frac{1}{6}\log\left(1-x^2\right)(e+g+i)+\frac{1}{6}\log\left(4-x^2\right)(e+4g+16i)+hx+\frac{ix^2}{2}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5)/(4 - 5*x^2 + x^4),x]
```

```
[Out] h*x + (i*x^2)/2 - ((d + 4*f + 16*h)*ArcTanh[x/2])/6 + ((d + f + h
)*ArcTanh[x])/3 - ((e + g + i)*Log[1 - x^2])/6 + ((e + 4*g + 16*i
)*Log[4 - x^2])/6
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\left(\frac{d}{6}+\frac{2f}{3}+\frac{8h}{3}\right)\operatorname{atanh}\left(\frac{x}{2}\right)+\left(\frac{d}{3}+\frac{f}{3}+\frac{h}{3}\right)\operatorname{atanh}(x)$$
$$-\left(\frac{e}{6}+\frac{g}{6}+\frac{i}{6}\right)\log\left(-x^2+1\right)+\left(\frac{e}{6}+\frac{2g}{3}+\frac{8i}{3}\right)\log\left(-x^2+4\right)+\int h\,dx+\frac{\int^{x^2} i\,dx}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((i*x**5+h*x**4+g*x**3+f*x**2+e*x+d)/(x**4-5*x**2+4),x)
```

```
[Out] -(d/6 + 2*f/3 + 8*h/3)*atanh(x/2) + (d/3 + f/3 + h/3)*atanh(x) -
(e/6 + g/6 + i/6)*log(-x**2 + 1) + (e/6 + 2*g/3 + 8*i/3)*log(-x**
```

```
2 + 4) + Integral(h, x) + Integral(i, (x, x**2))/2
```

Mathematica [A] time = 0.134652, size = 98, normalized size = 1.29

$$\frac{1}{12}\left(-2\log(1-x)(d+e+f+g+h+i)+\log(2-x)(d+2e+4(f+2g+4h+8i)) + 2\log(x+1)(d-e+f-g+h-i)-\log(x+2)(d-2(e-2f+4g-8h+16i))+12hx+6ix^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5)/(4 - 5*x^2 + x^4),x]
```

```
[Out] (12*h*x + 6*i*x^2 - 2*(d + e + f + g + h + i)*Log[1 - x] + (d + 2
*e + 4*(f + 2*g + 4*h + 8*i))*Log[2 - x] + 2*(d - e + f - g + h -
 i)*Log[1 + x] - (d - 2*(e - 2*f + 4*g - 8*h + 16*i))*Log[2 + x])
/12
```

Maple [B] time = 0.014, size = 179, normalized size = 2.4

$$hx+\frac{\ln(1+x)d}{6}-\frac{\ln(1+x)e}{6}-\frac{\ln(-1+x)d}{6}-\frac{\ln(-1+x)e}{6}+\frac{8\ln(x-2)i}{3}-\frac{\ln(1+x)i}{6} -\frac{\ln(-1+x)i}{6}+\frac{8\ln(2+x)i}{3}+\frac{4\ln(x-2)h}{3}+\frac{\ln(1+x)h}{6}-\frac{4\ln(2+x)h}{3}-\frac{\ln(-1+x)h}{6} -\frac{\ln(1+x)g}{6}+\frac{2\ln(x-2)g}{3}-\frac{\ln(-1+x)g}{6}+\frac{2\ln(2+x)g}{3}+\frac{\ln(x-2)d}{12}+\frac{\ln(x-2)e}{6} +\frac{\ln(2+x)e}{6}+\frac{\ln(x-2)f}{3}-\frac{\ln(2+x)d}{12}+\frac{\ln(1+x)f}{6}-\frac{\ln(-1+x)f}{6}-\frac{\ln(2+x)f}{3}+\frac{ix^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4),x)
```

```
[Out] h*x+1/6*ln(1+x)*d-1/6*ln(1+x)*e-1/6*ln(-1+x)*d-1/6*ln(-1+x)*e+8/3
*ln(x-2)*i-1/6*ln(1+x)*i-1/6*ln(-1+x)*i+8/3*ln(2+x)*i+4/3*ln(x-2)
*h+1/6*ln(1+x)*h-4/3*ln(2+x)*h-1/6*ln(-1+x)*h-1/6*ln(1+x)*g+2/3*l
n(x-2)*g-1/6*ln(-1+x)*g+2/3*ln(2+x)*g+1/12*ln(x-2)*d+1/6*ln(x-2)*
e+1/6*ln(2+x)*e+1/3*ln(x-2)*f-1/12*ln(2+x)*d+1/6*ln(1+x)*f-1/6*ln
(-1+x)*f-1/3*ln(2+x)*f+1/2*i*x^2
```

Maxima [A] time = 0.70752, size = 119, normalized size = 1.57

$$\frac{1}{2}\,ix^{2}+hx-\frac{1}{12}\,(d-2\,e+4\,f-8\,g+16\,h-32\,i)\log\left(x+2\right)+\frac{1}{6}\,(d-e+f-g+h-i)\log\left(x+1\right)-\frac{1}{6}\,(d+e+f+g+h+i)\log\left(x-1\right)+\frac{1}{12}\,(d+2\,e+4\,f+8\,g+16\,h+32\,i)\log\left(x-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^5 + h*x^4 + g*x^3 + f*x^2 + e*x + d)/(x^4 - 5*x^2 + 4),x, algorithm=
```

```
[Out] 1/2*i*x^2 + h*x - 1/12*(d - 2*e + 4*f - 8*g + 16*h - 32*i)*log(x
+ 2) + 1/6*(d - e + f - g + h - i)*log(x + 1) - 1/6*(d + e + f +
g + h + i)*log(x - 1) + 1/12*(d + 2*e + 4*f + 8*g + 16*h + 32*i)*
log(x - 2)
```

Fricas [A] time = 6.91263, size = 119, normalized size = 1.57

$$\frac{1}{2}\,ix^{2}+hx-\frac{1}{12}\,(d-2\,e+4\,f-8\,g+16\,h-32\,i)\log\left(x+2\right)+\frac{1}{6}\,(d-e+f-g+h-i)\log\left(x+1\right)-\frac{1}{6}\,(d+e+f+g+h+i)\log\left(x-1\right)+\frac{1}{12}\,(d+2\,e+4\,f+8\,g+16\,h+32\,i)\log\left(x-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^5 + h*x^4 + g*x^3 + f*x^2 + e*x + d)/(x^4 - 5*x^2 + 4),x, algorithm=
```

```
[Out] 1/2*i*x^2 + h*x - 1/12*(d - 2*e + 4*f - 8*g + 16*h - 32*i)*log(x
+ 2) + 1/6*(d - e + f - g + h - i)*log(x + 1) - 1/6*(d + e + f +
g + h + i)*log(x - 1) + 1/12*(d + 2*e + 4*f + 8*g + 16*h + 32*i)*
log(x - 2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x**5+h*x**4+g*x**3+f*x**2+e*x+d)/(x**4-5*x**2+4),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.301763, size = 130, normalized size = 1.71

$$\frac{1}{2}\,ix^2 + hx - \frac{1}{12}\,(d + 4\,f - 8\,g + 16\,h - 32\,i - 2\,e)\ln\left(|x + 2|\right) + \frac{1}{6}\,(d + f - g + h - i - e)\ln\left(|x + 1|\right) - \frac{1}{6}\,(d + f + g + h + i + e)\ln\left(|x - 1|\right) + \frac{1}{12}\,(d + 4\,f + 8\,g + 16\,h + 32\,i + 2\,e)\ln\left(|x - 2|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^5 + h*x^4 + g*x^3 + f*x^2 + e*x + d)/(x^4 - 5*x^2 + 4),x, algorithm=
```

```
[Out] 1/2*i*x^2 + h*x - 1/12*(d + 4*f - 8*g + 16*h - 32*i - 2*e)*ln(abs
(x + 2)) + 1/6*(d + f - g + h - i - e)*ln(abs(x + 1)) - 1/6*(d +
f + g + h + i + e)*ln(abs(x - 1)) + 1/12*(d + 4*f + 8*g + 16*h +
32*i + 2*e)*ln(abs(x - 2))
```

3.15 $\int \frac{d+ex}{1+x^2+x^4}\,dx$

Optimal. Leaf size=92

$$-\frac{1}{4}d\log\left(x^2-x+1\right)+\frac{1}{4}d\log\left(x^2+x+1\right)-\frac{d\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{d\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{e\tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] -(d*ArcTan[(1 - 2*x)/Sqrt[3]])/(2*Sqrt[3]) + (d*ArcTan[(1 + 2*x)/
Sqrt[3]])/(2*Sqrt[3]) + (e*ArcTan[(1 + 2*x^2)/Sqrt[3]])/Sqrt[3] -
 (d*Log[1 - x + x^2])/4 + (d*Log[1 + x + x^2])/4
```

Rubi [A] time = 0.158581, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$

$$-\frac{1}{4}d\log\left(x^2-x+1\right)+\frac{1}{4}d\log\left(x^2+x+1\right)-\frac{d\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{d\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{e\tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x)/(1 + x^2 + x^4),x]
```

```
[Out] -(d*ArcTan[(1 - 2*x)/Sqrt[3]])/(2*Sqrt[3]) + (d*ArcTan[(1 + 2*x)/
Sqrt[3]])/(2*Sqrt[3]) + (e*ArcTan[(1 + 2*x^2)/Sqrt[3]])/Sqrt[3] -
 (d*Log[1 - x + x^2])/4 + (d*Log[1 + x + x^2])/4
```

Rubi in Sympy [A] time = 21.704, size = 95, normalized size = 1.03

$$-\frac{d\log\left(x^2-x+1\right)}{4}+\frac{d\log\left(x^2+x+1\right)}{4}+\frac{\sqrt{3}d\,\mathrm{atan}\left(\sqrt{3}\left(\frac{2x}{3}-\frac{1}{3}\right)\right)}{6}+\frac{\sqrt{3}d\,\mathrm{atan}\left(\sqrt{3}\left(\frac{2x}{3}+\frac{1}{3}\right)\right)}{6}+\frac{\sqrt{3}e\,\mathrm{atan}\left(\sqrt{3}\left(\frac{2x^2}{3}+\frac{1}{3}\right)\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x+d)/(x**4+x**2+1),x)
```

```
[Out] -d*log(x**2 - x + 1)/4 + d*log(x**2 + x + 1)/4 + sqrt(3)*d*atan(s
qrt(3)*(2*x/3 - 1/3))/6 + sqrt(3)*d*atan(sqrt(3)*(2*x/3 + 1/3))/6
```

```
+ sqrt(3)*e*atan(sqrt(3)*(2*x**2/3 + 1/3))/3
```

Mathematica [C] time = 0.325108, size = 98, normalized size = 1.07

$$\frac{1}{6}i\left(\sqrt{6-6i\sqrt{3}}d\tan^{-1}\left(\frac{1}{2}\left(\sqrt{3}-i\right)x\right)-\sqrt{6+6i\sqrt{3}}d\tan^{-1}\left(\frac{1}{2}\left(\sqrt{3}+i\right)x\right)+2i\sqrt{3}e\tan^{-1}\left(\frac{\sqrt{3}}{2x^2+1}\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(d + e*x)/(1 + x^2 + x^4),x]
```

```
[Out] (I/6)*(Sqrt[6 - (6*I)*Sqrt[3]]*d*ArcTan[((-I + Sqrt[3])*x)/2] - S
qrt[6 + (6*I)*Sqrt[3]]*d*ArcTan[((I + Sqrt[3])*x)/2] + (2*I)*Sqrt
[3]*e*ArcTan[Sqrt[3]/(1 + 2*x^2)])
```

Maple [A] time = 0.008, size = 92, normalized size = 1.

$$\frac{d\ln\left(x^2+x+1\right)}{4}+\frac{d\sqrt{3}}{6}\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)-\frac{\sqrt{3}e}{3}\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)-\frac{d\ln\left(x^2-x+1\right)}{4}+\frac{d\sqrt{3}}{6}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)+\frac{\sqrt{3}e}{3}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x+d)/(x^4+x^2+1),x)
```

```
[Out] 1/4*d*ln(x^2+x+1)+1/6*d*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)-1/3*3
^(1/2)*arctan(1/3*(1+2*x)*3^(1/2))*e-1/4*d*ln(x^2-x+1)+1/6*3^(1/2
)*arctan(1/3*(2*x-1)*3^(1/2))*d+1/3*3^(1/2)*arctan(1/3*(2*x-1)*3^
(1/2))*e
```

Maxima [A] time = 0.782779, size = 88, normalized size = 0.96

$$\frac{1}{6}\sqrt{3}(d-2e)\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{6}\sqrt{3}(d+2e)\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{4}d\log\left(x^2+x+1\right)-\frac{1}{4}d\log\left(x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x + d)/(x^4 + x^2 + 1),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(3)*(d - 2*e)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqrt(3)
*(d + 2*e)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/4*d*log(x^2 + x + 1)
 - 1/4*d*log(x^2 - x + 1)
```

Fricas [A] time = 0.267046, size = 93, normalized size = 1.01

$$\frac{1}{12}\sqrt{3}\left(\sqrt{3}d\log\left(x^2+x+1\right)-\sqrt{3}d\log\left(x^2-x+1\right)+2\left(d-2e\right)\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+2\left(d+2e\right)\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x + d)/(x^4 + x^2 + 1),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(3)*(sqrt(3)*d*log(x^2 + x + 1) - sqrt(3)*d*log(x^2 - x
+ 1) + 2*(d - 2*e)*arctan(1/3*sqrt(3)*(2*x + 1)) + 2*(d + 2*e)*ar
ctan(1/3*sqrt(3)*(2*x - 1)))
```

Sympy [A] time = 6.45709, size = 923, normalized size = 10.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x+d)/(x**4+x**2+1),x)
```

```
[Out] (-d/4 - sqrt(3)*I*(d + 2*e)/12)*log(x + (-7*d**4*e + 6*d**4*(-d/4
 - sqrt(3)*I*(d + 2*e)/12) - 15*d**2*e**3 - 18*d**2*e**2*(-d/4 -
sqrt(3)*I*(d + 2*e)/12) + 60*d**2*e*(-d/4 - sqrt(3)*I*(d + 2*e)/1
2)**2 + 72*d**2*(-d/4 - sqrt(3)*I*(d + 2*e)/12)**3 + 4*e**5 + 24*
e**4*(-d/4 - sqrt(3)*I*(d + 2*e)/12) + 48*e**3*(-d/4 - sqrt(3)*I*
(d + 2*e)/12)**2 + 288*e**2*(-d/4 - sqrt(3)*I*(d + 2*e)/12)**3)/(
3*d**5 - 8*d**3*e**2 - 16*d*e**4)) + (-d/4 + sqrt(3)*I*(d + 2*e)/
12)*log(x + (-7*d**4*e + 6*d**4*(-d/4 + sqrt(3)*I*(d + 2*e)/12) -
 15*d**2*e**3 - 18*d**2*e**2*(-d/4 + sqrt(3)*I*(d + 2*e)/12) + 60
*d**2*e*(-d/4 + sqrt(3)*I*(d + 2*e)/12)**2 + 72*d**2*(-d/4 + sqrt
(3)*I*(d + 2*e)/12)**3 + 4*e**5 + 24*e**4*(-d/4 + sqrt(3)*I*(d +
2*e)/12) + 48*e**3*(-d/4 + sqrt(3)*I*(d + 2*e)/12)**2 + 288*e**2*
(-d/4 + sqrt(3)*I*(d + 2*e)/12)**3)/(3*d**5 - 8*d**3*e**2 - 16*d*
e**4)) + (d/4 - sqrt(3)*I*(d - 2*e)/12)*log(x + (-7*d**4*e + 6*d*
*4*(d/4 - sqrt(3)*I*(d - 2*e)/12) - 15*d**2*e**3 - 18*d**2*e**2*(
d/4 - sqrt(3)*I*(d - 2*e)/12) + 60*d**2*e*(d/4 - sqrt(3)*I*(d - 2
*e)/12)**2 + 72*d**2*(d/4 - sqrt(3)*I*(d - 2*e)/12)**3 + 4*e**5 +
```

```
 24*e**4*(d/4 - sqrt(3)*I*(d - 2*e)/12) + 48*e**3*(d/4 - sqrt(3)*
I*(d - 2*e)/12)**2 + 288*e**2*(d/4 - sqrt(3)*I*(d - 2*e)/12)**3)/
(3*d**5 - 8*d**3*e**2 - 16*d*e**4)) + (d/4 + sqrt(3)*I*(d - 2*e)/
12)*log(x + (-7*d**4*e + 6*d**4*(d/4 + sqrt(3)*I*(d - 2*e)/12) -
15*d**2*e**3 - 18*d**2*e**2*(d/4 + sqrt(3)*I*(d - 2*e)/12) + 60*d
**2*e*(d/4 + sqrt(3)*I*(d - 2*e)/12)**2 + 72*d**2*(d/4 + sqrt(3)*
I*(d - 2*e)/12)**3 + 4*e**5 + 24*e**4*(d/4 + sqrt(3)*I*(d - 2*e)/
12) + 48*e**3*(d/4 + sqrt(3)*I*(d - 2*e)/12)**2 + 288*e**2*(d/4 +
 sqrt(3)*I*(d - 2*e)/12)**3)/(3*d**5 - 8*d**3*e**2 - 16*d*e**4))
```

GIAC/XCAS [A] time = 0.281183, size = 90, normalized size = 0.98

$$\frac{1}{6}\sqrt{3}(d-2e)\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{6}\sqrt{3}(d+2e)\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{4}d\ln\left(x^2+x+1\right)-\frac{1}{4}d\ln\left(x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x + d)/(x^4 + x^2 + 1),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(3)*(d - 2*e)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqrt(3)
*(d + 2*e)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/4*d*ln(x^2 + x + 1)
- 1/4*d*ln(x^2 - x + 1)
```

3.16 $\int \frac{d+ex+fx^2}{1+x^2+x^4}\,dx$

Optimal. Leaf size=104

$$-\frac{1}{4}(d-f)\log\left(x^2-x+1\right)+\frac{1}{4}(d-f)\log\left(x^2+x+1\right)-\frac{(d+f)\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{(d+f)\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{e\tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] -((d + f)*ArcTan[(1 - 2*x)/Sqrt[3]])/(2*Sqrt[3]) + ((d + f)*ArcTa
n[(1 + 2*x)/Sqrt[3]])/(2*Sqrt[3]) + (e*ArcTan[(1 + 2*x^2)/Sqrt[3]
])/Sqrt[3] - ((d - f)*Log[1 - x + x^2])/4 + ((d - f)*Log[1 + x +
x^2])/4
```

Rubi [A] time = 0.18929, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381

$$-\frac{1}{4}(d-f)\log\left(x^2-x+1\right)+\frac{1}{4}(d-f)\log\left(x^2+x+1\right)-\frac{(d+f)\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{(d+f)\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{e\tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x + f*x^2)/(1 + x^2 + x^4),x]
```

```
[Out] -((d + f)*ArcTan[(1 - 2*x)/Sqrt[3]])/(2*Sqrt[3]) + ((d + f)*ArcTa
n[(1 + 2*x)/Sqrt[3]])/(2*Sqrt[3]) + (e*ArcTan[(1 + 2*x^2)/Sqrt[3]
])/Sqrt[3] - ((d - f)*Log[1 - x + x^2])/4 + ((d - f)*Log[1 + x +
x^2])/4
```

Rubi in Sympy [A] time = 33.2623, size = 105, normalized size = 1.01

$$\frac{\sqrt{3}e\,\mathrm{atan}\left(\sqrt{3}\left(\frac{2x^2}{3}+\frac{1}{3}\right)\right)}{3}-\left(\frac{d}{4}-\frac{f}{4}\right)\log\left(x^2-x+1\right)+\left(\frac{d}{4}-\frac{f}{4}\right)\log\left(x^2+x+1\right)+\frac{\sqrt{3}(d+f)\,\mathrm{atan}\left(\sqrt{3}\left(\frac{2x}{3}-\frac{1}{3}\right)\right)}{6}+\frac{\sqrt{3}(d+f)\,\mathrm{atan}\left(\sqrt{3}\left(\frac{2x}{3}+\frac{1}{3}\right)\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**2+e*x+d)/(x**4+x**2+1),x)
```

```
[Out] sqrt(3)*e*atan(sqrt(3)*(2*x**2/3 + 1/3))/3 - (d/4 - f/4)*log(x**2
 - x + 1) + (d/4 - f/4)*log(x**2 + x + 1) + sqrt(3)*(d + f)*atan(
sqrt(3)*(2*x/3 - 1/3))/6 + sqrt(3)*(d + f)*atan(sqrt(3)*(2*x/3 +
1/3))/6
```

Mathematica [C] time = 0.270526, size = 121, normalized size = 1.16

$$\frac{\left(2id+\left(\sqrt{3}-i\right)f\right)\tan^{-1}\left(\frac{1}{2}\left(\sqrt{3}-i\right)x\right)}{\sqrt{6+6i\sqrt{3}}}+\frac{\left(\left(\sqrt{3}+i\right)f-2id\right)\tan^{-1}\left(\frac{1}{2}\left(\sqrt{3}+i\right)x\right)}{\sqrt{6-6i\sqrt{3}}}-\frac{e\tan^{-1}\left(\frac{\sqrt{3}}{2x^2+1}\right)}{\sqrt{3}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(d + e*x + f*x^2)/(1 + x^2 + x^4),x]
```

```
[Out] (((2*I)*d + (-I + Sqrt[3])*f)*ArcTan[((-I + Sqrt[3])*x)/2])/Sqrt[
6 + (6*I)*Sqrt[3]] + (((-2*I)*d + (I + Sqrt[3])*f)*ArcTan[((I + S
qrt[3])*x)/2])/Sqrt[6 - (6*I)*Sqrt[3]] - (e*ArcTan[Sqrt[3]/(1 + 2
*x^2)])/Sqrt[3]
```

Maple [A] time = 0.007, size = 148, normalized size = 1.4

$$\frac{d\ln\left(x^2+x+1\right)}{4}-\frac{\ln\left(x^2+x+1\right)f}{4}+\frac{d\sqrt{3}}{6}\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)-\frac{\sqrt{3}e}{3}\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)$$
$$+\frac{\sqrt{3}f}{6}\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)+\frac{\ln\left(x^2-x+1\right)f}{4}-\frac{d\ln\left(x^2-x+1\right)}{4}$$
$$+\frac{d\sqrt{3}}{6}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)+\frac{\sqrt{3}e}{3}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)+\frac{\sqrt{3}f}{6}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x^2+e*x+d)/(x^4+x^2+1),x)
```

```
[Out] 1/4*d*ln(x^2+x+1)-1/4*ln(x^2+x+1)*f+1/6*d*arctan(1/3*(1+2*x)*3^(1
/2))*3^(1/2)-1/3*3^(1/2)*arctan(1/3*(1+2*x)*3^(1/2))*e+1/6*3^(1/2
)*arctan(1/3*(1+2*x)*3^(1/2))*f+1/4*ln(x^2-x+1)*f-1/4*d*ln(x^2-x+
1)+1/6*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))*d+1/3*3^(1/2)*arctan(1
/3*(2*x-1)*3^(1/2))*e+1/6*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))*f
```

Maxima [A] time = 0.790007, size = 101, normalized size = 0.97

$$\frac{1}{6}\sqrt{3}(d-2e+f)\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{6}\sqrt{3}(d+2e+f)\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{4}(d-f)\log\left(x^2+x+1\right)-\frac{1}{4}(d-f)\log\left(x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^2 + e*x + d)/(x^4 + x^2 + 1),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(3)*(d - 2*e + f)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqr
t(3)*(d + 2*e + f)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/4*(d - f)*lo
g(x^2 + x + 1) - 1/4*(d - f)*log(x^2 - x + 1)
```

Fricas [A] time = 0.289862, size = 107, normalized size = 1.03

$$\frac{1}{12}\sqrt{3}\left(\sqrt{3}(d-f)\log\left(x^2+x+1\right)-\sqrt{3}(d-f)\log\left(x^2-x+1\right)+2(d-2e+f)\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+2(d+2e+f)\text{arc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^2 + e*x + d)/(x^4 + x^2 + 1),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(3)*(sqrt(3)*(d - f)*log(x^2 + x + 1) - sqrt(3)*(d - f)*
log(x^2 - x + 1) + 2*(d - 2*e + f)*arctan(1/3*sqrt(3)*(2*x + 1))
+ 2*(d + 2*e + f)*arctan(1/3*sqrt(3)*(2*x - 1)))
```

Sympy [A] time = 81.8656, size = 3589, normalized size = 34.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x**2+e*x+d)/(x**4+x**2+1),x)
```

```
[Out] (-d/4 + f/4 - sqrt(3)*I*(d + 2*e + f)/12)*log(x + (-7*d**5*e + 6*
d**5*(-d/4 + f/4 - sqrt(3)*I*(d + 2*e + f)/12) + 25*d**4*e*f + 18
*d**4*f*(-d/4 + f/4 - sqrt(3)*I*(d + 2*e + f)/12) - 15*d**3*e**3
- 18*d**3*e**2*(-d/4 + f/4 - sqrt(3)*I*(d + 2*e + f)/12) - 25*d**
3*e*f**2 + 60*d**3*e*(-d/4 + f/4 - sqrt(3)*I*(d + 2*e + f)/12)**2
 - 42*d**3*f**2*(-d/4 + f/4 - sqrt(3)*I*(d + 2*e + f)/12) + 72*d*
```

```
*3*(-d/4 + f/4 - sqrt(3)*I*(d + 2*e + f)/12)**3 + 108*d**2*e**2*f
*(-d/4 + f/4 - sqrt(3)*I*(d + 2*e + f)/12) + 20*d**2*e*f**3 - 144
*d**2*e*f*(-d/4 + f/4 - sqrt(3)*I*(d + 2*e + f)/12)**2 - 12*d**2*
f**3*(-d/4 + f/4 - sqrt(3)*I*(d + 2*e + f)/12) - 144*d**2*f*(-d/4
 + f/4 - sqrt(3)*I*(d + 2*e + f)/12)**3 + 4*d*e**5 + 24*d*e**4*(-
d/4 + f/4 - sqrt(3)*I*(d + 2*e + f)/12) + 15*d*e**3*f**2 + 48*d*e
**3*(-d/4 + f/4 - sqrt(3)*I*(d + 2*e + f)/12)**2 - 54*d*e**2*f**2
*(-d/4 + f/4 - sqrt(3)*I*(d + 2*e + f)/12) + 288*d*e**2*(-d/4 + f
/4 - sqrt(3)*I*(d + 2*e + f)/12)**3 - 20*d*e*f**4 + 180*d*e*f**2*
(-d/4 + f/4 - sqrt(3)*I*(d + 2*e + f)/12)**2 + 36*d*f**4*(-d/4 +
f/4 - sqrt(3)*I*(d + 2*e + f)/12) - 72*d*f**2*(-d/4 + f/4 - sqrt(
3)*I*(d + 2*e + f)/12)**3 - 8*e**5*f - 96*e**3*f*(-d/4 + f/4 - sq
rt(3)*I*(d + 2*e + f)/12)**2 + 36*e**2*f**3*(-d/4 + f/4 - sqrt(3)
*I*(d + 2*e + f)/12) + 11*e*f**5 - 48*e*f**3*(-d/4 + f/4 - sqrt(3
)*I*(d + 2*e + f)/12)**2 - 6*f**5*(-d/4 + f/4 - sqrt(3)*I*(d + 2*
e + f)/12) + 144*f**3*(-d/4 + f/4 - sqrt(3)*I*(d + 2*e + f)/12)**
3)/(3*d**6 - 3*d**5*f - 8*d**4*e**2 - 3*d**4*f**2 + 40*d**3*e**2*
f + 6*d**3*f**3 - 16*d**2*e**4 - 48*d**2*e**2*f**2 - 3*d**2*f**4
+ 16*d*e**4*f + 40*d*e**2*f**3 - 3*d*f**5 - 16*e**4*f**2 - 8*e**2
*f**4 + 3*f**6)) + (-d/4 + f/4 + sqrt(3)*I*(d + 2*e + f)/12)*log(
x + (-7*d**5*e + 6*d**5*(-d/4 + f/4 + sqrt(3)*I*(d + 2*e + f)/12)
 + 25*d**4*e*f + 18*d**4*f*(-d/4 + f/4 + sqrt(3)*I*(d + 2*e + f)/
12) - 15*d**3*e**3 - 18*d**3*e**2*(-d/4 + f/4 + sqrt(3)*I*(d + 2*
e + f)/12) - 25*d**3*e*f**2 + 60*d**3*e*(-d/4 + f/4 + sqrt(3)*I*(
d + 2*e + f)/12)**2 - 42*d**3*f**2*(-d/4 + f/4 + sqrt(3)*I*(d + 2
*e + f)/12) + 72*d**3*(-d/4 + f/4 + sqrt(3)*I*(d + 2*e + f)/12)**
3 + 108*d**2*e**2*f*(-d/4 + f/4 + sqrt(3)*I*(d + 2*e + f)/12) + 2
0*d**2*e*f**3 - 144*d**2*e*f*(-d/4 + f/4 + sqrt(3)*I*(d + 2*e + f
)/12)**2 - 12*d**2*f**3*(-d/4 + f/4 + sqrt(3)*I*(d + 2*e + f)/12)
 - 144*d**2*f*(-d/4 + f/4 + sqrt(3)*I*(d + 2*e + f)/12)**3 + 4*d*
e**5 + 24*d*e**4*(-d/4 + f/4 + sqrt(3)*I*(d + 2*e + f)/12) + 15*d
*e**3*f**2 + 48*d*e**3*(-d/4 + f/4 + sqrt(3)*I*(d + 2*e + f)/12)*
*2 - 54*d*e**2*f**2*(-d/4 + f/4 + sqrt(3)*I*(d + 2*e + f)/12) + 2
88*d*e**2*(-d/4 + f/4 + sqrt(3)*I*(d + 2*e + f)/12)**3 - 20*d*e*f
**4 + 180*d*e*f**2*(-d/4 + f/4 + sqrt(3)*I*(d + 2*e + f)/12)**2 +
 36*d*f**4*(-d/4 + f/4 + sqrt(3)*I*(d + 2*e + f)/12) - 72*d*f**2*
(-d/4 + f/4 + sqrt(3)*I*(d + 2*e + f)/12)**3 - 8*e**5*f - 96*e**3
*f*(-d/4 + f/4 + sqrt(3)*I*(d + 2*e + f)/12)**2 + 36*e**2*f**3*(-
d/4 + f/4 + sqrt(3)*I*(d + 2*e + f)/12) + 11*e*f**5 - 48*e*f**3*(
-d/4 + f/4 + sqrt(3)*I*(d + 2*e + f)/12)**2 - 6*f**5*(-d/4 + f/4
+ sqrt(3)*I*(d + 2*e + f)/12) + 144*f**3*(-d/4 + f/4 + sqrt(3)*I*
(d + 2*e + f)/12)**3)/(3*d**6 - 3*d**5*f - 8*d**4*e**2 - 3*d**4*f
**2 + 40*d**3*e**2*f + 6*d**3*f**3 - 16*d**2*e**4 - 48*d**2*e**2*
f**2 - 3*d**2*f**4 + 16*d*e**4*f + 40*d*e**2*f**3 - 3*d*f**5 - 16
*e**4*f**2 - 8*e**2*f**4 + 3*f**6)) + (d/4 - f/4 - sqrt(3)*I*(d -
 2*e + f)/12)*log(x + (-7*d**5*e + 6*d**5*(d/4 - f/4 - sqrt(3)*I*
(d - 2*e + f)/12) + 25*d**4*e*f + 18*d**4*f*(d/4 - f/4 - sqrt(3)*
I*(d - 2*e + f)/12) - 15*d**3*e**3 - 18*d**3*e**2*(d/4 - f/4 - sq
rt(3)*I*(d - 2*e + f)/12) - 25*d**3*e*f**2 + 60*d**3*e*(d/4 - f/4
 - sqrt(3)*I*(d - 2*e + f)/12)**2 - 42*d**3*f**2*(d/4 - f/4 - sqr
t(3)*I*(d - 2*e + f)/12) + 72*d**3*(d/4 - f/4 - sqrt(3)*I*(d - 2*
e + f)/12)**3 + 108*d**2*e**2*f*(d/4 - f/4 - sqrt(3)*I*(d - 2*e +
 f)/12) + 20*d**2*e*f**3 - 144*d**2*e*f*(d/4 - f/4 - sqrt(3)*I*(d
 - 2*e + f)/12)**2 - 12*d**2*f**3*(d/4 - f/4 - sqrt(3)*I*(d - 2*e
```

```
 + f)/12) - 144*d**2*f*(d/4 - f/4 - sqrt(3)*I*(d - 2*e + f)/12)**
3 + 4*d*e**5 + 24*d*e**4*(d/4 - f/4 - sqrt(3)*I*(d - 2*e + f)/12)
 + 15*d*e**3*f**2 + 48*d*e**3*(d/4 - f/4 - sqrt(3)*I*(d - 2*e + f
)/12)**2 - 54*d*e**2*f**2*(d/4 - f/4 - sqrt(3)*I*(d - 2*e + f)/12
) + 288*d*e**2*(d/4 - f/4 - sqrt(3)*I*(d - 2*e + f)/12)**3 - 20*d
*e*f**4 + 180*d*e*f**2*(d/4 - f/4 - sqrt(3)*I*(d - 2*e + f)/12)**
2 + 36*d*f**4*(d/4 - f/4 - sqrt(3)*I*(d - 2*e + f)/12) - 72*d*f**
2*(d/4 - f/4 - sqrt(3)*I*(d - 2*e + f)/12)**3 - 8*e**5*f - 96*e**
3*f*(d/4 - f/4 - sqrt(3)*I*(d - 2*e + f)/12)**2 + 36*e**2*f**3*(d
/4 - f/4 - sqrt(3)*I*(d - 2*e + f)/12) + 11*e*f**5 - 48*e*f**3*(d
/4 - f/4 - sqrt(3)*I*(d - 2*e + f)/12)**2 - 6*f**5*(d/4 - f/4 - s
qrt(3)*I*(d - 2*e + f)/12) + 144*f**3*(d/4 - f/4 - sqrt(3)*I*(d -
 2*e + f)/12)**3)/(3*d**6 - 3*d**5*f - 8*d**4*e**2 - 3*d**4*f**2
+ 40*d**3*e**2*f + 6*d**3*f**3 - 16*d**2*e**4 - 48*d**2*e**2*f**2
 - 3*d**2*f**4 + 16*d*e**4*f + 40*d*e**2*f**3 - 3*d*f**5 - 16*e**
4*f**2 - 8*e**2*f**4 + 3*f**6)) + (d/4 - f/4 + sqrt(3)*I*(d - 2*e
 + f)/12)*log(x + (-7*d**5*e + 6*d**5*(d/4 - f/4 + sqrt(3)*I*(d -
 2*e + f)/12) + 25*d**4*e*f + 18*d**4*f*(d/4 - f/4 + sqrt(3)*I*(d
 - 2*e + f)/12) - 15*d**3*e**3 - 18*d**3*e**2*(d/4 - f/4 + sqrt(3
)*I*(d - 2*e + f)/12) - 25*d**3*e*f**2 + 60*d**3*e*(d/4 - f/4 + s
qrt(3)*I*(d - 2*e + f)/12)**2 - 42*d**3*f**2*(d/4 - f/4 + sqrt(3)
*I*(d - 2*e + f)/12) + 72*d**3*(d/4 - f/4 + sqrt(3)*I*(d - 2*e +
f)/12)**3 + 108*d**2*e**2*f*(d/4 - f/4 + sqrt(3)*I*(d - 2*e + f)/
12) + 20*d**2*e*f**3 - 144*d**2*e*f*(d/4 - f/4 + sqrt(3)*I*(d - 2
*e + f)/12)**2 - 12*d**2*f**3*(d/4 - f/4 + sqrt(3)*I*(d - 2*e + f
)/12) - 144*d**2*f*(d/4 - f/4 + sqrt(3)*I*(d - 2*e + f)/12)**3 +
4*d*e**5 + 24*d*e**4*(d/4 - f/4 + sqrt(3)*I*(d - 2*e + f)/12) + 1
5*d*e**3*f**2 + 48*d*e**3*(d/4 - f/4 + sqrt(3)*I*(d - 2*e + f)/12
)**2 - 54*d*e**2*f**2*(d/4 - f/4 + sqrt(3)*I*(d - 2*e + f)/12) +
288*d*e**2*(d/4 - f/4 + sqrt(3)*I*(d - 2*e + f)/12)**3 - 20*d*e*f
**4 + 180*d*e*f**2*(d/4 - f/4 + sqrt(3)*I*(d - 2*e + f)/12)**2 +
36*d*f**4*(d/4 - f/4 + sqrt(3)*I*(d - 2*e + f)/12) - 72*d*f**2*(d
/4 - f/4 + sqrt(3)*I*(d - 2*e + f)/12)**3 - 8*e**5*f - 96*e**3*f*
(d/4 - f/4 + sqrt(3)*I*(d - 2*e + f)/12)**2 + 36*e**2*f**3*(d/4 -
 f/4 + sqrt(3)*I*(d - 2*e + f)/12) + 11*e*f**5 - 48*e*f**3*(d/4 -
 f/4 + sqrt(3)*I*(d - 2*e + f)/12)**2 - 6*f**5*(d/4 - f/4 + sqrt(
3)*I*(d - 2*e + f)/12) + 144*f**3*(d/4 - f/4 + sqrt(3)*I*(d - 2*e
 + f)/12)**3)/(3*d**6 - 3*d**5*f - 8*d**4*e**2 - 3*d**4*f**2 + 40
*d**3*e**2*f + 6*d**3*f**3 - 16*d**2*e**4 - 48*d**2*e**2*f**2 - 3
*d**2*f**4 + 16*d*e**4*f + 40*d*e**2*f**3 - 3*d*f**5 - 16*e**4*f*
*2 - 8*e**2*f**4 + 3*f**6))
```

GIAC/XCAS [A] time = 0.273901, size = 104, normalized size = 1.

$$\frac{1}{6}\sqrt{3}(d+f-2e)\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{6}\sqrt{3}(d+f+2e)\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{4}(d-f)\ln\left(x^2+x+1\right)-\frac{1}{4}(d-f)\ln\left(x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^2 + e*x + d)/(x^4 + x^2 + 1),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(3)*(d + f - 2*e)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqr
t(3)*(d + f + 2*e)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/4*(d - f)*ln
(x^2 + x + 1) - 1/4*(d - f)*ln(x^2 - x + 1)
```

3.17 $\int \frac{d+ex+fx^2+gx^3}{1+x^2+x^4}\,dx$

Optimal. Leaf size=127

$$-\frac{1}{4}(d-f)\log\left(x^2-x+1\right)+\frac{1}{4}(d-f)\log\left(x^2+x+1\right)-\frac{(d+f)\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{(d+f)\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{(2e-g)\tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{1}{4}g\log\left(x^4+x^2+1\right)$$

```
[Out] -((d + f)*ArcTan[(1 - 2*x)/Sqrt[3]])/(2*Sqrt[3]) + ((d + f)*ArcTa
n[(1 + 2*x)/Sqrt[3]])/(2*Sqrt[3]) + ((2*e - g)*ArcTan[(1 + 2*x^2)
/Sqrt[3]])/(2*Sqrt[3]) - ((d - f)*Log[1 - x + x^2])/4 + ((d - f)*
Log[1 + x + x^2])/4 + (g*Log[1 + x^2 + x^4])/4
```

Rubi [A] time = 0.238745, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269

$$-\frac{1}{4}(d-f)\log\left(x^2-x+1\right)+\frac{1}{4}(d-f)\log\left(x^2+x+1\right)-\frac{(d+f)\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{(d+f)\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{(2e-g)\tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{1}{4}g\log\left(x^4+x^2+1\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x + f*x^2 + g*x^3)/(1 + x^2 + x^4),x]
```

```
[Out] -((d + f)*ArcTan[(1 - 2*x)/Sqrt[3]])/(2*Sqrt[3]) + ((d + f)*ArcTa
n[(1 + 2*x)/Sqrt[3]])/(2*Sqrt[3]) + ((2*e - g)*ArcTan[(1 + 2*x^2)
/Sqrt[3]])/(2*Sqrt[3]) - ((d - f)*Log[1 - x + x^2])/4 + ((d - f)*
Log[1 + x + x^2])/4 + (g*Log[1 + x^2 + x^4])/4
```

Rubi in Sympy [A] time = 43.0819, size = 122, normalized size = 0.96

$$\frac{g\log\left(x^4+x^2+1\right)}{4}-\left(\frac{d}{4}-\frac{f}{4}\right)\log\left(x^2-x+1\right)+\left(\frac{d}{4}-\frac{f}{4}\right)\log\left(x^2+x+1\right)+\frac{\sqrt{3}\,(d+f)\operatorname{atan}\left(\sqrt{3}\left(\frac{2x}{3}-\frac{1}{3}\right)\right)}{6}+\frac{\sqrt{3}\,(d+f)\operatorname{atan}\left(\sqrt{3}\left(\frac{2x}{3}+\frac{1}{3}\right)\right)}{6}+\frac{\sqrt{3}\left(e-\frac{g}{2}\right)\operatorname{atan}\left(\sqrt{3}\left(\frac{2x^2}{3}+\frac{1}{3}\right)\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((g*x**3+f*x**2+e*x+d)/(x**4+x**2+1),x)
```

```
[Out] g*log(x**4 + x**2 + 1)/4 - (d/4 - f/4)*log(x**2 - x + 1) + (d/4 -
 f/4)*log(x**2 + x + 1) + sqrt(3)*(d + f)*atan(sqrt(3)*(2*x/3 - 1
/3))/6 + sqrt(3)*(d + f)*atan(sqrt(3)*(2*x/3 + 1/3))/6 + sqrt(3)*
(e - g/2)*atan(sqrt(3)*(2*x**2/3 + 1/3))/3
```

Mathematica [C] time = 1.42212, size = 150, normalized size = 1.18

$$\frac{2\left(\sqrt{2+2i\sqrt{3}}\left(\left(\sqrt{3}+i\right)f-2id\right)\tan^{-1}\left(\frac{1}{2}\left(\sqrt{3}+i\right)x\right)+(2g-4e)\tan^{-1}\left(\frac{\sqrt{3}}{2x^2+1}\right)+\sqrt{3}g\log\left(x^4+x^2+1\right)\right)+2\sqrt{2-2i\sqrt{3}}\left(2\cdots\right.}{8\sqrt{3}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(d + e*x + f*x^2 + g*x^3)/(1 + x^2 + x^4),x]
```

```
[Out] (2*Sqrt[2 - (2*I)*Sqrt[3]]*((2*I)*d + (-I + Sqrt[3])*f)*ArcTan[((
-I + Sqrt[3])*x)/2] + 2*(Sqrt[2 + (2*I)*Sqrt[3]]*((-2*I)*d + (I +
 Sqrt[3])*f)*ArcTan[((I + Sqrt[3])*x)/2] + (-4*e + 2*g)*ArcTan[Sq
rt[3]/(1 + 2*x^2)] + Sqrt[3]*g*Log[1 + x^2 + x^4]))/(8*Sqrt[3])
```

Maple [A] time = 0.007, size = 204, normalized size = 1.6

$$\frac{d\ln\left(x^2+x+1\right)}{4}-\frac{\ln\left(x^2+x+1\right)f}{4}+\frac{\ln\left(x^2+x+1\right)g}{4}+\frac{d\sqrt{3}}{6}\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)-\frac{\sqrt{3}e}{3}\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)+\frac{\sqrt{3}f}{6}\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)+\frac{\sqrt{3}g}{6}\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)+\frac{\ln\left(x^2-x+1\right)f}{4}-\frac{d\ln\left(x^2-x+1\right)}{4}+\frac{\ln\left(x^2-x+1\right)g}{4}+\frac{d\sqrt{3}}{6}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)+\frac{\sqrt{3}e}{3}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)+\frac{\sqrt{3}f}{6}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)-\frac{\sqrt{3}g}{6}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((g*x^3+f*x^2+e*x+d)/(x^4+x^2+1),x)
```

```
[Out] 1/4*d*ln(x^2+x+1)-1/4*ln(x^2+x+1)*f+1/4*ln(x^2+x+1)*g+1/6*d*arcta
n(1/3*(1+2*x)*3^(1/2))*3^(1/2)-1/3*3^(1/2)*arctan(1/3*(1+2*x)*3^(
1/2))*e+1/6*3^(1/2)*arctan(1/3*(1+2*x)*3^(1/2))*f+1/6*3^(1/2)*arc
tan(1/3*(1+2*x)*3^(1/2))*g+1/4*ln(x^2-x+1)*f-1/4*d*ln(x^2-x+1)+1/
4*ln(x^2-x+1)*g+1/6*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))*d+1/3*3^(
```

```
1/2)*arctan(1/3*(2*x-1)*3^(1/2))*e+1/6*3^(1/2)*arctan(1/3*(2*x-1)
*3^(1/2))*f-1/6*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))*g
```

Maxima [A] time = 0.78524, size = 112, normalized size = 0.88

$$\frac{1}{6}\sqrt{3}(d-2e+f+g)\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{6}\sqrt{3}(d+2e+f-g)\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{4}(d-f+g)\log\left(x^2+x+1\right)-\frac{1}{4}(d-f-g)\log\left(x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3 + f*x^2 + e*x + d)/(x^4 + x^2 + 1),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(3)*(d - 2*e + f + g)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6
*sqrt(3)*(d + 2*e + f - g)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/4*(d
 - f + g)*log(x^2 + x + 1) - 1/4*(d - f - g)*log(x^2 - x + 1)
```

Fricas [A] time = 0.450368, size = 117, normalized size = 0.92

$$\frac{1}{12}\sqrt{3}\left(\sqrt{3}(d-f+g)\log\left(x^2+x+1\right)-\sqrt{3}(d-f-g)\log\left(x^2-x+1\right)+2(d-2e+f+g)\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+2(d+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3 + f*x^2 + e*x + d)/(x^4 + x^2 + 1),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(3)*(sqrt(3)*(d - f + g)*log(x^2 + x + 1) - sqrt(3)*(d -
 f - g)*log(x^2 - x + 1) + 2*(d - 2*e + f + g)*arctan(1/3*sqrt(3)
*(2*x + 1)) + 2*(d + 2*e + f - g)*arctan(1/3*sqrt(3)*(2*x - 1)))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((g*x**3+f*x**2+e*x+d)/(x**4+x**2+1),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.275451, size = 115, normalized size = 0.91

$$\frac{1}{6}\sqrt{3}(d+f+g-2e)\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{6}\sqrt{3}(d+f-g+2e)\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{4}(d-f+g)\ln\left(x^2+x+1\right)-\frac{1}{4}(d-f-g)\ln\left(x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3 + f*x^2 + e*x + d)/(x^4 + x^2 + 1),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(3)*(d + f + g - 2*e)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6
*sqrt(3)*(d + f - g + 2*e)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/4*(d
 - f + g)*ln(x^2 + x + 1) - 1/4*(d - f - g)*ln(x^2 - x + 1)
```

3.18 $\int \frac{d+ex+fx^2+gx^3+hx^4}{1+x^2+x^4}\,dx$

Optimal. Leaf size=136

$$-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)(d+f-2h)}{2\sqrt{3}}+\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)(d+f-2h)}{2\sqrt{3}}-\frac{1}{4}(d-f)\log\left(x^2-x+1\right)+\frac{1}{4}(d-f)\log\left(x^2+x+1\right)+\frac{(2e-g)\tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{1}{4}g\log\left(x^4+x^2+1\right)+hx$$

```
[Out] h*x - ((d + f - 2*h)*ArcTan[(1 - 2*x)/Sqrt[3]])/(2*Sqrt[3]) + ((d
 + f - 2*h)*ArcTan[(1 + 2*x)/Sqrt[3]])/(2*Sqrt[3]) + ((2*e - g)*A
rcTan[(1 + 2*x^2)/Sqrt[3]])/(2*Sqrt[3]) - ((d - f)*Log[1 - x + x^
2])/4 + ((d - f)*Log[1 + x + x^2])/4 + (g*Log[1 + x^2 + x^4])/4
```

Rubi [A] time = 0.303389, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.258

$$-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)(d+f-2h)}{2\sqrt{3}}+\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)(d+f-2h)}{2\sqrt{3}}-\frac{1}{4}(d-f)\log\left(x^2-x+1\right)+\frac{1}{4}(d-f)\log\left(x^2+x+1\right)+\frac{(2e-g)\tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{1}{4}g\log\left(x^4+x^2+1\right)+hx$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x + f*x^2 + g*x^3 + h*x^4)/(1 + x^2 + x^4),x]
```


```
[Out] h*x - ((d + f - 2*h)*ArcTan[(1 - 2*x)/Sqrt[3]])/(2*Sqrt[3]) + ((d
 + f - 2*h)*ArcTan[(1 + 2*x)/Sqrt[3]])/(2*Sqrt[3]) + ((2*e - g)*A
rcTan[(1 + 2*x^2)/Sqrt[3]])/(2*Sqrt[3]) - ((d - f)*Log[1 - x + x^
2])/4 + ((d - f)*Log[1 + x + x^2])/4 + (g*Log[1 + x^2 + x^4])/4
```

Rubi in Sympy [A] time = 66.0655, size = 133, normalized size = 0.98

$$\frac{g\log\left(x^4+x^2+1\right)}{4}+hx-\left(\frac{d}{4}-\frac{f}{4}\right)\log\left(x^2-x+1\right)+\left(\frac{d}{4}-\frac{f}{4}\right)\log\left(x^2+x+1\right)+\frac{\sqrt{3}\left(e-\frac{g}{2}\right)\operatorname{atan}\left(\sqrt{3}\left(\frac{2x^2}{3}+\frac{1}{3}\right)\right)}{3}+\frac{\sqrt{3}(d+f-2h)\operatorname{atan}\left(\sqrt{3}\left(\frac{2x}{3}-\frac{1}{3}\right)\right)}{6}+\frac{\sqrt{3}(d+f-2h)\operatorname{atan}\left(\sqrt{3}\left(\frac{2x}{3}+\frac{1}{3}\right)\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((h*x**4+g*x**3+f*x**2+e*x+d)/(x**4+x**2+1),x)
```

```
[Out] g*log(x**4 + x**2 + 1)/4 + h*x - (d/4 - f/4)*log(x**2 - x + 1) +
(d/4 - f/4)*log(x**2 + x + 1) + sqrt(3)*(e - g/2)*atan(sqrt(3)*(2
*x**2/3 + 1/3))/3 + sqrt(3)*(d + f - 2*h)*atan(sqrt(3)*(2*x/3 - 1
/3))/6 + sqrt(3)*(d + f - 2*h)*atan(sqrt(3)*(2*x/3 + 1/3))/6
```

Mathematica [C] time = 1.19334, size = 165, normalized size = 1.21

$$\frac{1}{24}\left(4\tan^{-1}\left(\frac{1}{2}\left(\sqrt{3}-i\right)x\right)\left(\left(\sqrt{3}+3i\right)d+\left(\sqrt{3}-3i\right)f-2\sqrt{3}h\right)+4\tan^{-1}\left(\frac{1}{2}\left(\sqrt{3}+i\right)x\right)\left(\left(\sqrt{3}-3i\right)d+\left(\sqrt{3}+3i\right)f-2\sqrt{3}h\right)-8\sqrt{3}e\tan^{-1}\left(\frac{\sqrt{3}}{2x^2+1}\right)+4\sqrt{3}g\tan^{-1}\left(\frac{\sqrt{3}}{2x^2+1}\right)+6g\log\left(x^4+x^2+1\right)+24hx\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x + f*x^2 + g*x^3 + h*x^4)/(1 + x^2 + x^4),x]
```

```
[Out] (24*h*x + 4*((3*I + Sqrt[3])*d + (-3*I + Sqrt[3])*f - 2*Sqrt[3]*h
)*ArcTan[((-I + Sqrt[3])*x)/2] + 4*((-3*I + Sqrt[3])*d + (3*I + S
qrt[3])*f - 2*Sqrt[3]*h)*ArcTan[((I + Sqrt[3])*x)/2] - 8*Sqrt[3]*
e*ArcTan[Sqrt[3]/(1 + 2*x^2)] + 4*Sqrt[3]*g*ArcTan[Sqrt[3]/(1 + 2
*x^2)] + 6*g*Log[1 + x^2 + x^4])/24
```

Maple [B] time = 0.006, size = 241, normalized size = 1.8

$$hx+\frac{d\ln\left(x^2+x+1\right)}{4}-\frac{\ln\left(x^2+x+1\right)f}{4}+\frac{\ln\left(x^2+x+1\right)g}{4}+\frac{d\sqrt{3}}{6}\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)-\frac{\sqrt{3}e}{3}\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)+\frac{\sqrt{3}f}{6}\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)+\frac{\sqrt{3}g}{6}\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)-\frac{\sqrt{3}h}{3}\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)+\frac{\ln\left(x^2-x+1\right)f}{4}-\frac{d\ln\left(x^2-x+1\right)}{4}+\frac{\ln\left(x^2-x+1\right)g}{4}+\frac{d\sqrt{3}}{6}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)+\frac{\sqrt{3}e}{3}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)+\frac{\sqrt{3}f}{6}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)-\frac{\sqrt{3}g}{6}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)-\frac{\sqrt{3}h}{3}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((h*x^4+g*x^3+f*x^2+e*x+d)/(x^4+x^2+1),x)
```

```
[Out] h*x+1/4*d*ln(x^2+x+1)-1/4*ln(x^2+x+1)*f+1/4*ln(x^2+x+1)*g+1/6*d*a
rctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)-1/3*3^(1/2)*arctan(1/3*(1+2*x)
*3^(1/2))*e+1/6*3^(1/2)*arctan(1/3*(1+2*x)*3^(1/2))*f+1/6*3^(1/2)
*arctan(1/3*(1+2*x)*3^(1/2))*g-1/3*3^(1/2)*arctan(1/3*(1+2*x)*3^(
1/2))*h+1/4*ln(x^2-x+1)*f-1/4*d*ln(x^2-x+1)+1/4*ln(x^2-x+1)*g+1/6
*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))*d+1/3*3^(1/2)*arctan(1/3*(2*
x-1)*3^(1/2))*e+1/6*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))*f-1/6*3^(
1/2)*arctan(1/3*(2*x-1)*3^(1/2))*g-1/3*3^(1/2)*arctan(1/3*(2*x-1)
*3^(1/2))*h
```

Maxima [A] time = 0.779341, size = 124, normalized size = 0.91

$$\frac{1}{6}\sqrt{3}(d-2e+f+g-2h)\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{6}\sqrt{3}(d+2e+f-g-2h)\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+hx+\frac{1}{4}(d-f+g)\log\left(x^2+x+1\right)-\frac{1}{4}(d-f-g)\log\left(x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^4 + g*x^3 + f*x^2 + e*x + d)/(x^4 + x^2 + 1),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(3)*(d - 2*e + f + g - 2*h)*arctan(1/3*sqrt(3)*(2*x + 1))
 + 1/6*sqrt(3)*(d + 2*e + f - g - 2*h)*arctan(1/3*sqrt(3)*(2*x -
1)) + h*x + 1/4*(d - f + g)*log(x^2 + x + 1) - 1/4*(d - f - g)*lo
g(x^2 - x + 1)
```

Fricas [A] time = 1.39739, size = 135, normalized size = 0.99

$$\frac{1}{12}\sqrt{3}\left(4\sqrt{3}hx+\sqrt{3}(d-f+g)\log\left(x^2+x+1\right)-\sqrt{3}(d-f-g)\log\left(x^2-x+1\right)+2(d-2e+f+g-2h)\arctan\left(\frac{1}{3}\sqrt{3}(2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^4 + g*x^3 + f*x^2 + e*x + d)/(x^4 + x^2 + 1),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(3)*(4*sqrt(3)*h*x + sqrt(3)*(d - f + g)*log(x^2 + x + 1
) - sqrt(3)*(d - f - g)*log(x^2 - x + 1) + 2*(d - 2*e + f + g - 2
*h)*arctan(1/3*sqrt(3)*(2*x + 1)) + 2*(d + 2*e + f - g - 2*h)*arc
tan(1/3*sqrt(3)*(2*x - 1)))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((h*x**4+g*x**3+f*x**2+e*x+d)/(x**4+x**2+1),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.273887, size = 127, normalized size = 0.93

$$\frac{1}{6}\sqrt{3}(d+f+g-2h-2e)\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{6}\sqrt{3}(d+f-g-2h+2e)\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+hx+\frac{1}{4}(d-f+g)\ln\left(x^2+x+1\right)-\frac{1}{4}(d-f-g)\ln\left(x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((h*x^4 + g*x^3 + f*x^2 + e*x + d)/(x^4 + x^2 + 1),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(3)*(d + f + g - 2*h - 2*e)*arctan(1/3*sqrt(3)*(2*x + 1))
 + 1/6*sqrt(3)*(d + f - g - 2*h + 2*e)*arctan(1/3*sqrt(3)*(2*x -
1)) + h*x + 1/4*(d - f + g)*ln(x^2 + x + 1) - 1/4*(d - f - g)*ln(
x^2 - x + 1)
```

3.19 $\int \frac{d+ex+fx^2+gx^3+hx^4+ix^5}{1+x^2+x^4}\,dx$

Optimal. Leaf size=151

$$-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)(d+f-2h)}{2\sqrt{3}}+\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)(d+f-2h)}{2\sqrt{3}}-\frac{1}{4}(d-f)\log\left(x^2-x+1\right)+\frac{1}{4}(d-f)\log\left(x^2+x+1\right)+\frac{\tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right)(2e-g-i)}{2\sqrt{3}}+\frac{1}{4}(g-i)\log\left(x^4+x^2+1\right)+hx+\frac{ix^2}{2}$$

```
[Out] h*x + (i*x^2)/2 - ((d + f - 2*h)*ArcTan[(1 - 2*x)/Sqrt[3]])/(2*Sq
rt[3]) + ((d + f - 2*h)*ArcTan[(1 + 2*x)/Sqrt[3]])/(2*Sqrt[3]) +
((2*e - g - i)*ArcTan[(1 + 2*x^2)/Sqrt[3]])/(2*Sqrt[3]) - ((d - f
)*Log[1 - x + x^2])/4 + ((d - f)*Log[1 + x + x^2])/4 + ((g - i)*L
og[1 + x^2 + x^4])/4
```

Rubi [A] time = 0.352666, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 9, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)(d+f-2h)}{2\sqrt{3}}+\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)(d+f-2h)}{2\sqrt{3}}-\frac{1}{4}(d-f)\log\left(x^2-x+1\right)+\frac{1}{4}(d-f)\log\left(x^2+x+1\right)+\frac{\tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right)(2e-g-i)}{2\sqrt{3}}+\frac{1}{4}(g-i)\log\left(x^4+x^2+1\right)+hx+\frac{ix^2}{2}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5)/(1 + x^2 + x^4),x]
```

```
[Out] h*x + (i*x^2)/2 - ((d + f - 2*h)*ArcTan[(1 - 2*x)/Sqrt[3]])/(2*Sq
rt[3]) + ((d + f - 2*h)*ArcTan[(1 + 2*x)/Sqrt[3]])/(2*Sqrt[3]) +
((2*e - g - i)*ArcTan[(1 + 2*x^2)/Sqrt[3]])/(2*Sqrt[3]) - ((d - f
)*Log[1 - x + x^2])/4 + ((d - f)*Log[1 + x + x^2])/4 + ((g - i)*L
og[1 + x^2 + x^4])/4
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$hx - \left(\frac{d}{4} - \frac{f}{4}\right)\log\left(x^2 - x + 1\right) + \left(\frac{d}{4} - \frac{f}{4}\right)\log\left(x^2 + x + 1\right) + \left(\frac{g}{4} - \frac{i}{4}\right)\log\left(x^4 + x^2 + 1\right) + \frac{\sqrt{3}\,(d + f - 2h)\,\mathrm{atan}\left(\sqrt{3}\left(\frac{2x}{3} - \frac{1}{3}\right)\right)}{6} + \frac{\sqrt{3}\,(d + f - 2h)\,\mathrm{atan}\left(\sqrt{3}\left(\frac{2x}{3} + \frac{1}{3}\right)\right)}{6} + \frac{\sqrt{3}\,(2e - g - i)\,\mathrm{atan}\left(\sqrt{3}\left(\frac{2x^2}{3} + \frac{1}{3}\right)\right)}{6} + \frac{\int^{x^2} i\,dx}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((i*x**5+h*x**4+g*x**3+f*x**2+e*x+d)/(x**4+x**2+1),x)
```

```
[Out] h*x - (d/4 - f/4)*log(x**2 - x + 1) + (d/4 - f/4)*log(x**2 + x +
1) + (g/4 - i/4)*log(x**4 + x**2 + 1) + sqrt(3)*(d + f - 2*h)*ata
n(sqrt(3)*(2*x/3 - 1/3))/6 + sqrt(3)*(d + f - 2*h)*atan(sqrt(3)*(
2*x/3 + 1/3))/6 + sqrt(3)*(2*e - g - i)*atan(sqrt(3)*(2*x**2/3 +
1/3))/6 + Integral(i, (x, x**2))/2
```

Mathematica [C] time = 1.44361, size = 187, normalized size = 1.24

$$\frac{1}{12}\left(\left(1 + i\sqrt{3}\right)\tan^{-1}\left(\frac{1}{2}\left(\sqrt{3} - i\right)x\right)\left(2\sqrt{3}d - \left(\sqrt{3} + 3i\right)f - \left(\sqrt{3} - 3i\right)h\right) + \left(\sqrt{3} + i\right)\tan^{-1}\left(\frac{1}{2}\left(\sqrt{3} + i\right)x\right)\left(-2i\sqrt{3}d + \left(3 + i\sqrt{3}\right)f + i\left(\sqrt{3} + 3i\right)h\right) - 2\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}}{2x^2 + 1}\right)(2e - g - i) + 3(g - i)\log\left(x^4 + x^2 + 1\right) + 6x(2h + ix)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5)/(1 + x^2 + x^4),x]
```

```
[Out] (6*x*(2*h + i*x) + (1 + I*Sqrt[3])*(2*Sqrt[3]*d - (3*I + Sqrt[3])
*f - (-3*I + Sqrt[3])*h)*ArcTan[((-I + Sqrt[3])*x)/2] + (I + Sqrt
[3])*((-2*I)*Sqrt[3]*d + (3 + I*Sqrt[3])*f + I*(3*I + Sqrt[3])*h)
*ArcTan[((I + Sqrt[3])*x)/2] - 2*Sqrt[3]*(2*e - g - i)*ArcTan[Sqr
t[3]/(1 + 2*x^2)] + 3*(g - i)*Log[1 + x^2 + x^4])/12
```

Maple [B] time = 0.011, size = 303, normalized size = 2.

$$\frac{ix^2}{2}+hx+\frac{d\ln\left(x^2+x+1\right)}{4}-\frac{\ln\left(x^2+x+1\right)f}{4}+\frac{\ln\left(x^2+x+1\right)g}{4}-\frac{\ln\left(x^2+x+1\right)i}{4}+\frac{d\sqrt{3}}{6}\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)-\frac{\sqrt{3}e}{3}\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)+\frac{\sqrt{3}f}{6}\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)+\frac{\sqrt{3}g}{6}\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)-\frac{\sqrt{3}h}{3}\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)+\frac{\sqrt{3}i}{6}\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)+\frac{\ln\left(x^2-x+1\right)g}{4}-\frac{\ln\left(x^2-x+1\right)i}{4}+\frac{\ln\left(x^2-x+1\right)f}{4}-\frac{d\ln\left(x^2-x+1\right)}{4}+\frac{d\sqrt{3}}{6}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)+\frac{\sqrt{3}e}{3}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)+\frac{\sqrt{3}f}{6}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)-\frac{\sqrt{3}g}{6}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)-\frac{\sqrt{3}h}{3}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)-\frac{\sqrt{3}i}{6}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4+x^2+1),x)
```

```
[Out] 1/2*i*x^2+h*x+1/4*d*ln(x^2+x+1)-1/4*ln(x^2+x+1)*f+1/4*ln(x^2+x+1)
*g-1/4*ln(x^2+x+1)*i+1/6*d*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)-1/
3*3^(1/2)*arctan(1/3*(1+2*x)*3^(1/2))*e+1/6*3^(1/2)*arctan(1/3*(1
+2*x)*3^(1/2))*f+1/6*3^(1/2)*arctan(1/3*(1+2*x)*3^(1/2))*g-1/3*3^
(1/2)*arctan(1/3*(1+2*x)*3^(1/2))*h+1/6*3^(1/2)*arctan(1/3*(1+2*x
)*3^(1/2))*i+1/4*ln(x^2-x+1)*g-1/4*ln(x^2-x+1)*i+1/4*ln(x^2-x+1)*
f-1/4*d*ln(x^2-x+1)+1/6*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))*d+1/3
*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))*e+1/6*3^(1/2)*arctan(1/3*(2*
x-1)*3^(1/2))*f-1/6*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))*g-1/3*3^(
1/2)*arctan(1/3*(2*x-1)*3^(1/2))*h-1/6*3^(1/2)*arctan(1/3*(2*x-1)
*3^(1/2))*i
```

Maxima [A] time = 0.795338, size = 143, normalized size = 0.95

$$\frac{1}{2}\,ix^2+\frac{1}{6}\,\sqrt{3}(d-2\,e+f+g-2\,h+i)\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right)+\frac{1}{6}\,\sqrt{3}(d+2\,e+f-g-2\,h-i)\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right)+hx+\frac{1}{4}\,(d-f+g-i)\log\left(x^2+x+1\right)-\frac{1}{4}\,(d-f-g+i)\log\left(x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^5 + h*x^4 + g*x^3 + f*x^2 + e*x + d)/(x^4 + x^2 + 1),x, algorithm="m
```

```
[Out] 1/2*i*x^2 + 1/6*sqrt(3)*(d - 2*e + f + g - 2*h + i)*arctan(1/3*sq
rt(3)*(2*x + 1)) + 1/6*sqrt(3)*(d + 2*e + f - g - 2*h - i)*arctan
(1/3*sqrt(3)*(2*x - 1)) + h*x + 1/4*(d - f + g - i)*log(x^2 + x +
 1) - 1/4*(d - f - g + i)*log(x^2 - x + 1)
```

Fricas [A] time = 5.72668, size = 157, normalized size = 1.04

$$\frac{1}{12}\sqrt{3}\left(\sqrt{3}(d-f+g-i)\log\left(x^2+x+1\right)-\sqrt{3}(d-f-g+i)\log\left(x^2-x+1\right)+2\left(d-2e+f+g-2h+i\right)\arctan\left(\frac{1}{3}\sqrt{3}(2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^5 + h*x^4 + g*x^3 + f*x^2 + e*x + d)/(x^4 + x^2 + 1),x, algorithm="f
```

```
[Out] 1/12*sqrt(3)*(sqrt(3)*(d - f + g - i)*log(x^2 + x + 1) - sqrt(3)*
(d - f - g + i)*log(x^2 - x + 1) + 2*(d - 2*e + f + g - 2*h + i)*
arctan(1/3*sqrt(3)*(2*x + 1)) + 2*(d + 2*e + f - g - 2*h - i)*arc
tan(1/3*sqrt(3)*(2*x - 1)) + 2*sqrt(3)*(i*x^2 + 2*h*x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x**5+h*x**4+g*x**3+f*x**2+e*x+d)/(x**4+x**2+1),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.274007, size = 146, normalized size = 0.97

$$\frac{1}{2}ix^2+\frac{1}{6}\sqrt{3}(d+f+g-2h+i-2e)\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{6}\sqrt{3}(d+f-g-2h-i+2e)\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+hx+\frac{1}{4}(d-f+g-i)\ln\left(x^2+x+1\right)-\frac{1}{4}(d-f-g+i)\ln\left(x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^5 + h*x^4 + g*x^3 + f*x^2 + e*x + d)/(x^4 + x^2 + 1),x, algorithm="g

[Out] 1/2*i*x^2 + 1/6*sqrt(3)*(d + f + g - 2*h + i - 2*e)*arctan(1/3*sq
rt(3)*(2*x + 1)) + 1/6*sqrt(3)*(d + f - g - 2*h - i + 2*e)*arctan
(1/3*sqrt(3)*(2*x - 1)) + h*x + 1/4*(d - f + g - i)*ln(x^2 + x +
1) - 1/4*(d - f - g + i)*ln(x^2 - x + 1)
```

3.20 $\int \frac{d+ex}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=189

$$\frac{\sqrt{2}\sqrt{c}d\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2}\sqrt{c}d\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}-\frac{e\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

```
[Out] (Sqrt[2]*Sqrt[c]*d*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 -
 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqr
t[2]*Sqrt[c]*d*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a
*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - (e*ArcTa
nh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/Sqrt[b^2 - 4*a*c]
```

Rubi [A] time = 0.459179, antiderivative size = 189, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35

$$\frac{\sqrt{2}\sqrt{c}d\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2}\sqrt{c}d\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}-\frac{e\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x)/(a + b*x^2 + c*x^4),x]
```

```
[Out] (Sqrt[2]*Sqrt[c]*d*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 -
 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqr
t[2]*Sqrt[c]*d*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a
*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - (e*ArcTa
nh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/Sqrt[b^2 - 4*a*c]
```

Rubi in Sympy [A] time = 42.9693, size = 177, normalized size = 0.94

$$-\frac{\sqrt{2}\sqrt{c}d\,\mathrm{atan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{-4ac+b^2}}}\right)}{\sqrt{b+\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}}+\frac{\sqrt{2}\sqrt{c}d\,\mathrm{atan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{-4ac+b^2}}}\right)}{\sqrt{b-\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}}-\frac{e\,\mathrm{atanh}\left(\frac{b+2cx^2}{\sqrt{-4ac+b^2}}\right)}{\sqrt{-4ac+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] -sqrt(2)*sqrt(c)*d*atan(sqrt(2)*sqrt(c)*x/sqrt(b + sqrt(-4*a*c +
b**2)))/(sqrt(b + sqrt(-4*a*c + b**2))*sqrt(-4*a*c + b**2)) + sqr
t(2)*sqrt(c)*d*atan(sqrt(2)*sqrt(c)*x/sqrt(b - sqrt(-4*a*c + b**2
)))/(sqrt(b - sqrt(-4*a*c + b**2))*sqrt(-4*a*c + b**2)) - e*atanh
((b + 2*c*x**2)/sqrt(-4*a*c + b**2))/sqrt(-4*a*c + b**2)
```

Mathematica [A] time = 0.705602, size = 194, normalized size = 1.03

$$\frac{\frac{2\sqrt{2}\sqrt{c}d\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}}-\frac{2\sqrt{2}\sqrt{c}d\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{\sqrt{b^2-4ac}+b}}+e\left(\log\left(\sqrt{b^2-4ac}-b-2cx^2\right)-\log\left(\sqrt{b^2-4ac}+b+2cx^2\right)\right)}{2\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x)/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((2*Sqrt[2]*Sqrt[c]*d*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^
2 - 4*a*c]]])/Sqrt[b - Sqrt[b^2 - 4*a*c]] - (2*Sqrt[2]*Sqrt[c]*d*
ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/Sqrt[b +
 Sqrt[b^2 - 4*a*c]] + e*(Log[-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2] -
Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2]))/(2*Sqrt[b^2 - 4*a*c])
```

Maple [A] time = 0.039, size = 231, normalized size = 1.2

$$\frac{e}{8\,ac-2\,b^2}\sqrt{-4\,ac+b^2}\ln\left(2\,cx^2+\sqrt{-4\,ac+b^2}+b\right)+2\,\frac{\sqrt{-4\,ac+b^2}cd\sqrt{2}}{(8\,ac-2\,b^2)\sqrt{\left(b+\sqrt{-4\,ac+b^2}\right)c}}\arctan\left(\frac{cx\sqrt{2}}{\sqrt{\left(b+\sqrt{-4\,ac+b^2}\right)c}}\right)-\frac{e}{8\,ac-2\,b^2}\sqrt{-4\,ac+b^2}\ln\left(-2\,cx^2+\sqrt{-4\,ac+b^2}-b\right)+2\,\frac{\sqrt{-4\,ac+b^2}cd\sqrt{2}}{(8\,ac-2\,b^2)\sqrt{\left(-b+\sqrt{-4\,ac+b^2}\right)c}}Artanh\left(\frac{cx\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4\,ac+b^2}\right)c}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x+d)/(c*x^4+b*x^2+a),x)
```

```
[Out] (-4*a*c+b^2)^(1/2)/(8*a*c-2*b^2)*e*ln(2*c*x^2+(-4*a*c+b^2)^(1/2)+
b)+2*c*(-4*a*c+b^2)^(1/2)/(8*a*c-2*b^2)*d*2^(1/2)/((b+(-4*a*c+b^2
)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(
1/2))-(-4*a*c+b^2)^(1/2)/(8*a*c-2*b^2)*e*ln(-2*c*x^2+(-4*a*c+b^2)
^(1/2)-b)+2*c*(-4*a*c+b^2)^(1/2)/(8*a*c-2*b^2)*d*2^(1/2)/((-b+(-4
*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(
1/2))*c)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex + d}{cx^4 + bx^2 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x + d)/(c*x^4 + b*x^2 + a),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)/(c*x^4 + b*x^2 + a), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x + d)/(c*x^4 + b*x^2 + a),x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.647762, size = 1, normalized size = 0.01

$$Done$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x + d)/(c*x^4 + b*x^2 + a),x, algorithm="giac")
```

```
[Out] Done
```

3.21 $\int \frac{d+ex+fx^2}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=211

$$\frac{\left(\frac{2cd-bf}{\sqrt{b^2-4ac}}+f\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(f-\frac{2cd-bf}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}-\frac{e\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

```
[Out] ((f + (2*c*d - b*f)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)
/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b - Sqrt[b^2
 - 4*a*c]]) + ((f - (2*c*d - b*f)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt
[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqr
t[b + Sqrt[b^2 - 4*a*c]]) - (e*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4
*a*c]])/Sqrt[b^2 - 4*a*c]
```

Rubi [A] time = 0.55072, antiderivative size = 211, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.28$

$$\frac{\left(\frac{2cd-bf}{\sqrt{b^2-4ac}}+f\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(f-\frac{2cd-bf}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}-\frac{e\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x + f*x^2)/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((f + (2*c*d - b*f)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)
/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b - Sqrt[b^2
 - 4*a*c]]) + ((f - (2*c*d - b*f)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt
[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqr
t[b + Sqrt[b^2 - 4*a*c]]) - (e*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4
*a*c]])/Sqrt[b^2 - 4*a*c]
```

Rubi in Sympy [A] time = 53.8761, size = 221, normalized size = 1.05

$$-\frac{e\,\mathrm{atanh}\left(\frac{b+2cx^2}{\sqrt{-4ac+b^2}}\right)}{\sqrt{-4ac+b^2}}+\frac{\sqrt{2}\left(bf-2cd+f\sqrt{-4ac+b^2}\right)\mathrm{atan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{-4ac+b^2}}}\right)}{2\sqrt{c}\sqrt{b+\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}}$$
$$-\frac{\sqrt{2}\left(bf-2cd-f\sqrt{-4ac+b^2}\right)\mathrm{atan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{-4ac+b^2}}}\right)}{2\sqrt{c}\sqrt{b-\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**2+e*x+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] -e*atanh((b + 2*c*x**2)/sqrt(-4*a*c + b**2))/sqrt(-4*a*c + b**2)
+ sqrt(2)*(b*f - 2*c*d + f*sqrt(-4*a*c + b**2))*atan(sqrt(2)*sqrt
(c)*x/sqrt(b + sqrt(-4*a*c + b**2)))/(2*sqrt(c)*sqrt(b + sqrt(-4*
a*c + b**2))*sqrt(-4*a*c + b**2)) - sqrt(2)*(b*f - 2*c*d - f*sqrt
(-4*a*c + b**2))*atan(sqrt(2)*sqrt(c)*x/sqrt(b - sqrt(-4*a*c + b*
*2)))/(2*sqrt(c)*sqrt(b - sqrt(-4*a*c + b**2))*sqrt(-4*a*c + b**2
))
```

Mathematica [A] time = 0.433217, size = 234, normalized size = 1.11

$$\frac{\frac{\sqrt{2}\left(f\left(\sqrt{b^2-4ac}-b\right)+2cd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\left(f\left(\sqrt{b^2-4ac}+b\right)-2cd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}+e\log\left(\sqrt{b^2-4ac}-b-2cx^2\right)-e\log\left(\sqrt{b^2}\right.}{2\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x + f*x^2)/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((Sqrt[2]*(2*c*d + (-b + Sqrt[b^2 - 4*a*c])*f)*ArcTan[(Sqrt[2]*Sq
rt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*Sqrt[b - Sqrt[b^2
 - 4*a*c]]) + (Sqrt[2]*(-2*c*d + (b + Sqrt[b^2 - 4*a*c])*f)*ArcTa
n[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*Sqrt
[b + Sqrt[b^2 - 4*a*c]]) + e*Log[-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2
] - e*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(2*Sqrt[b^2 - 4*a*c])
```

Maple [B] time = 0.03, size = 616, normalized size = 2.9

$$\frac{e}{8\,ac-2\,b^{2}}\sqrt{-4\,ac+b^{2}}\ln\left(2\,c{x}^{2}+\sqrt{-4\,ac+b^{2}}+b\right)$$
$$+2\,{\frac{c\sqrt{2}fa}{\left(4\,ac-{b}^{2}\right)\sqrt{\left(b+\sqrt{-4\,ac+{b}^{2}}\right)c}}\arctan\left({\frac{cx\sqrt{2}}{\sqrt{\left(b+\sqrt{-4\,ac+{b}^{2}}\right)c}}}\right)}$$
$$-{\frac{\sqrt{2}f{b}^{2}}{8\,ac-2\,{b}^{2}}\arctan\left(cx\sqrt{2}{\frac{1}{\sqrt{\left(b+\sqrt{-4\,ac+{b}^{2}}\right)c}}}\right){\frac{1}{\sqrt{\left(b+\sqrt{-4\,ac+{b}^{2}}\right)c}}}}$$
$$-{\frac{b\sqrt{2}f}{8\,ac-2\,{b}^{2}}\sqrt{-4\,ac+{b}^{2}}\arctan\left(cx\sqrt{2}{\frac{1}{\sqrt{\left(b+\sqrt{-4\,ac+{b}^{2}}\right)c}}}\right){\frac{1}{\sqrt{\left(b+\sqrt{-4\,ac+{b}^{2}}\right)c}}}}$$
$$+{\frac{c\sqrt{2}d}{4\,ac-{b}^{2}}\sqrt{-4\,ac+{b}^{2}}\arctan\left(cx\sqrt{2}{\frac{1}{\sqrt{\left(b+\sqrt{-4\,ac+{b}^{2}}\right)c}}}\right){\frac{1}{\sqrt{\left(b+\sqrt{-4\,ac+{b}^{2}}\right)c}}}}$$
$$-{\frac{e}{8\,ac-2\,{b}^{2}}\sqrt{-4\,ac+{b}^{2}}\ln\left(-2\,c{x}^{2}+\sqrt{-4\,ac+{b}^{2}}-b\right)}$$
$$-2\,{\frac{c\sqrt{2}fa}{\left(4\,ac-{b}^{2}\right)\sqrt{\left(-b+\sqrt{-4\,ac+{b}^{2}}\right)c}}{\it Artanh}\left({\frac{cx\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4\,ac+{b}^{2}}\right)c}}}\right)}$$
$$+{\frac{\sqrt{2}f{b}^{2}}{8\,ac-2\,{b}^{2}}{\it Artanh}\left(cx\sqrt{2}{\frac{1}{\sqrt{\left(-b+\sqrt{-4\,ac+{b}^{2}}\right)c}}}\right){\frac{1}{\sqrt{\left(-b+\sqrt{-4\,ac+{b}^{2}}\right)c}}}}$$
$$-{\frac{b\sqrt{2}f}{8\,ac-2\,{b}^{2}}\sqrt{-4\,ac+{b}^{2}}{\it Artanh}\left(cx\sqrt{2}{\frac{1}{\sqrt{\left(-b+\sqrt{-4\,ac+{b}^{2}}\right)c}}}\right){\frac{1}{\sqrt{\left(-b+\sqrt{-4\,ac+{b}^{2}}\right)c}}}}$$
$$+{\frac{c\sqrt{2}d}{4\,ac-{b}^{2}}\sqrt{-4\,ac+{b}^{2}}{\it Artanh}\left(cx\sqrt{2}{\frac{1}{\sqrt{\left(-b+\sqrt{-4\,ac+{b}^{2}}\right)c}}}\right){\frac{1}{\sqrt{\left(-b+\sqrt{-4\,ac+{b}^{2}}\right)c}}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x^2+e*x+d)/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/2*(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)*e*ln(2*c*x^2+(-4*a*c+b^2)^(1/2
)+b)+2*c/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arc
tan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*f*a-1/2/(4*a*c-
b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/
((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*f*b^2-1/2*(-4*a*c+b^2)^(1/2)/(4
*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(
1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b*f+c*(-4*a*c+b^2)^(1/2)/(
4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^
(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*d-1/2*(-4*a*c+b^2)^(1/2)/
(4*a*c-b^2)*e*ln(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)-2*c/(4*a*c-b^2)*2
^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b
+(-4*a*c+b^2)^(1/2))*c)^(1/2))*f*a+1/2/(4*a*c-b^2)*2^(1/2)/((-b+(
-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)
^(1/2))*c)^(1/2))*f*b^2-1/2*(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)*2^(1/2
)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*
a*c+b^2)^(1/2))*c)^(1/2))*b*f+c*(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)*2^
(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+
(-4*a*c+b^2)^(1/2))*c)^(1/2))*d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx^2 + ex + d}{cx^4 + bx^2 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2 + e*x + d)/(c*x^4 + b*x^2 + a),x, algorithm="maxima")
```

```
[Out] integrate((f*x^2 + e*x + d)/(c*x^4 + b*x^2 + a), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2 + e*x + d)/(c*x^4 + b*x^2 + a),x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x**2+e*x+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 1.05305, size = 1, normalized size = 0.

$$Done$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^2 + e*x + d)/(c*x^4 + b*x^2 + a),x, algorithm="giac")
```

```
[Out] Done
```

3.22 $\int \frac{d+ex+fx^2+gx^3}{a+bx^2+cx^4} \, dx$

Optimal. Leaf size=245

$$\frac{\left(\frac{2cd-bf}{\sqrt{b^2-4ac}}+f\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(f-\frac{2cd-bf}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}-\frac{(2ce-bg)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c\sqrt{b^2-4ac}}+\frac{g\log\left(a+bx^2+cx^4\right)}{4c}$$

```
[Out] ((f + (2*c*d - b*f)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)
/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b - Sqrt[b^2
 - 4*a*c]]) + ((f - (2*c*d - b*f)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt
[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqr
t[b + Sqrt[b^2 - 4*a*c]]) - ((2*c*e - b*g)*ArcTanh[(b + 2*c*x^2)/
Sqrt[b^2 - 4*a*c]])/(2*c*Sqrt[b^2 - 4*a*c]) + (g*Log[a + b*x^2 +
c*x^4])/(4*c)
```

Rubi [A] time = 0.515882, antiderivative size = 245, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$

$$\frac{\left(\frac{2cd-bf}{\sqrt{b^2-4ac}}+f\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(f-\frac{2cd-bf}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}-\frac{(2ce-bg)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c\sqrt{b^2-4ac}}+\frac{g\log\left(a+bx^2+cx^4\right)}{4c}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x + f*x^2 + g*x^3)/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((f + (2*c*d - b*f)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)
/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b - Sqrt[b^2
 - 4*a*c]]) + ((f - (2*c*d - b*f)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt
[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqr
t[b + Sqrt[b^2 - 4*a*c]]) - ((2*c*e - b*g)*ArcTanh[(b + 2*c*x^2)/
Sqrt[b^2 - 4*a*c]])/(2*c*Sqrt[b^2 - 4*a*c]) + (g*Log[a + b*x^2 +
c*x^4])/(4*c)
```

Rubi in Sympy [A] time = 69.6605, size = 250, normalized size = 1.02

$$\frac{g\log\left(a+bx^2+cx^4\right)}{4c}+\frac{(bg-2ce)\operatorname{atanh}\left(\frac{b+2cx^2}{\sqrt{-4ac+b^2}}\right)}{2c\sqrt{-4ac+b^2}}+\frac{\sqrt{2}\left(bf-2cd+f\sqrt{-4ac+b^2}\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{-4ac+b^2}}}\right)}{2\sqrt{c}\sqrt{b+\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}}-\frac{\sqrt{2}\left(bf-2cd-f\sqrt{-4ac+b^2}\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{-4ac+b^2}}}\right)}{2\sqrt{c}\sqrt{b-\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((g*x**3+f*x**2+e*x+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] g*log(a + b*x**2 + c*x**4)/(4*c) + (b*g - 2*c*e)*atanh((b + 2*c*x
**2)/sqrt(-4*a*c + b**2))/(2*c*sqrt(-4*a*c + b**2)) + sqrt(2)*(b*
f - 2*c*d + f*sqrt(-4*a*c + b**2))*atan(sqrt(2)*sqrt(c)*x/sqrt(b
+ sqrt(-4*a*c + b**2)))/(2*sqrt(c)*sqrt(b + sqrt(-4*a*c + b**2))*
sqrt(-4*a*c + b**2)) - sqrt(2)*(b*f - 2*c*d - f*sqrt(-4*a*c + b**
2))*atan(sqrt(2)*sqrt(c)*x/sqrt(b - sqrt(-4*a*c + b**2)))/(2*sqrt
(c)*sqrt(b - sqrt(-4*a*c + b**2))*sqrt(-4*a*c + b**2))
```

Mathematica [A] time = 0.57325, size = 280, normalized size = 1.14

$$\frac{\frac{2\sqrt{2}\sqrt{c}\left(f\left(\sqrt{b^2-4ac}-b\right)+2cd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}}+\frac{2\sqrt{2}\sqrt{c}\left(f\left(\sqrt{b^2-4ac}+b\right)-2cd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{\sqrt{b^2-4ac}+b}}+\left(g\left(\sqrt{b^2-4ac}-b\right)+2ce\right)\log\left(\sqrt{b^2}\cdots\right.}{4c\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x + f*x^2 + g*x^3)/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((2*Sqrt[2]*Sqrt[c]*(2*c*d + (-b + Sqrt[b^2 - 4*a*c])*f)*ArcTan[(
Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/Sqrt[b - Sqrt[b^
2 - 4*a*c]] + (2*Sqrt[2]*Sqrt[c]*(-2*c*d + (b + Sqrt[b^2 - 4*a*c]
)*f)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/Sqr
t[b + Sqrt[b^2 - 4*a*c]] + (2*c*e + (-b + Sqrt[b^2 - 4*a*c])*g)*L
og[-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2] + (-2*c*e + (b + Sqrt[b^2 -
4*a*c])*g)*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(4*c*Sqrt[b^2 -
4*a*c])
```

Maple [B] time = 0.031, size = 866, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/(4*a*c-b^2)*ln(2*c*x^2+(-4*a*c+b^2)^(1/2)+b)*g*a-1/4/(4*a*c-b^2
)/c*ln(2*c*x^2+(-4*a*c+b^2)^(1/2)+b)*g*b^2-1/4*(-4*a*c+b^2)^(1/2)
/(4*a*c-b^2)/c*ln(2*c*x^2+(-4*a*c+b^2)^(1/2)+b)*b*g+1/2*(-4*a*c+b
^2)^(1/2)/(4*a*c-b^2)*e*ln(2*c*x^2+(-4*a*c+b^2)^(1/2)+b)+2*c/(4*a
*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/
2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*f*a-1/2/(4*a*c-b^2)*2^(1/2)/
((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b
^2)^(1/2))*c)^(1/2))*f*b^2-1/2*(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)*2^(
1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*
a*c+b^2)^(1/2))*c)^(1/2))*b*f+c*(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)*2^
(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4
*a*c+b^2)^(1/2))*c)^(1/2))*d+1/(4*a*c-b^2)*ln(-2*c*x^2+(-4*a*c+b^
2)^(1/2)-b)*g*a-1/4/(4*a*c-b^2)/c*ln(-2*c*x^2+(-4*a*c+b^2)^(1/2)-
b)*g*b^2+1/4*(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)/c*ln(-2*c*x^2+(-4*a*c
+b^2)^(1/2)-b)*b*g-1/2*(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)*e*ln(-2*c*x
^2+(-4*a*c+b^2)^(1/2)-b)-2*c/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2
)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)
^(1/2))*f*a+1/2/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(
1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*f*b^2
-1/2*(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/
2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2
))*b*f+c*(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)
^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^
(1/2))*d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{gx^3 + fx^2 + ex + d}{cx^4 + bx^2 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((g*x^3 + f*x^2 + e*x + d)/(c*x^4 + b*x^2 + a),x, algorithm="maxima")
```

```
[Out] integrate((g*x^3 + f*x^2 + e*x + d)/(c*x^4 + b*x^2 + a), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3 + f*x^2 + e*x + d)/(c*x^4 + b*x^2 + a),x, algorithm="fricas")

[Out] Exception raised: NotImplementedError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**3+f*x**2+e*x+d)/(c*x**4+b*x**2+a),x)

[Out] Timed out
```

GIAC/XCAS [A] time = 1.28829, size = 1, normalized size = 0.

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3 + f*x^2 + e*x + d)/(c*x^4 + b*x^2 + a),x, algorithm="giac")

[Out] Done
```

3.23 $\int \frac{d+ex+fx^2+gx^3+hx^4}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=290

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{-c(2ah+bf)+b^2h+2c^2d}{\sqrt{b^2-4ac}}-bh+cf\right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-\frac{-2ach+b^2h-bcf+2c^2d}{\sqrt{b^2-4ac}}-bh+cf\right)}{\sqrt{2}c^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}-\frac{(2ce-bg)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c\sqrt{b^2-4ac}}+\frac{g\log\left(a+bx^2+cx^4\right)}{4c}+\frac{hx}{c}$$

```
[Out] (h*x)/c + ((c*f - b*h + (2*c^2*d + b^2*h - c*(b*f + 2*a*h))/Sqrt[
b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*
c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((c*f - b*h
 - (2*c^2*d - b*c*f + b^2*h - 2*a*c*h)/Sqrt[b^2 - 4*a*c])*ArcTan[
(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2
)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - ((2*c*e - b*g)*ArcTanh[(b + 2*c*
x^2)/Sqrt[b^2 - 4*a*c]])/(2*c*Sqrt[b^2 - 4*a*c]) + (g*Log[a + b*x
^2 + c*x^4])/(4*c)
```

Rubi [A] time = 1.51883, antiderivative size = 290, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.257

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{-c(2ah+bf)+b^2h+2c^2d}{\sqrt{b^2-4ac}}-bh+cf\right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-\frac{-2ach+b^2h-bcf+2c^2d}{\sqrt{b^2-4ac}}-bh+cf\right)}{\sqrt{2}c^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}-\frac{(2ce-bg)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c\sqrt{b^2-4ac}}+\frac{g\log\left(a+bx^2+cx^4\right)}{4c}+\frac{hx}{c}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x + f*x^2 + g*x^3 + h*x^4)/(a + b*x^2 + c*x^4),x]
```

```
[Out] (h*x)/c + ((c*f - b*h + (2*c^2*d + b^2*h - c*(b*f + 2*a*h))/Sqrt[
b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*
c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((c*f - b*h
```

```
 - (2*c^2*d - b*c*f + b^2*h - 2*a*c*h)/Sqrt[b^2 - 4*a*c])*ArcTan[
(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2
)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - ((2*c*e - b*g)*ArcTanh[(b + 2*c*
x^2)/Sqrt[b^2 - 4*a*c]])/(2*c*Sqrt[b^2 - 4*a*c]) + (g*Log[a + b*x
^2 + c*x^4])/(4*c)
```

Rubi in Sympy [A] time = 143.637, size = 286, normalized size = 0.99

$$\frac{g\log\left(a+bx^2+cx^4\right)}{4c}+\frac{hx}{c}+\frac{(bg-2ce)\operatorname{atanh}\left(\frac{b+2cx^2}{\sqrt{-4ac+b^2}}\right)}{2c\sqrt{-4ac+b^2}} - \frac{\sqrt{2}\left(b\left(bh-cf\right)-2c\left(ah-cd\right)+\sqrt{-4ac+b^2}\left(bh-cf\right)\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{-4ac+b^2}}}\right)}{2c^{\frac{3}{2}}\sqrt{b+\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}} + \frac{\sqrt{2}\left(b\left(bh-cf\right)-2c\left(ah-cd\right)-\sqrt{-4ac+b^2}\left(bh-cf\right)\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{-4ac+b^2}}}\right)}{2c^{\frac{3}{2}}\sqrt{b-\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((h*x**4+g*x**3+f*x**2+e*x+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] g*log(a + b*x**2 + c*x**4)/(4*c) + h*x/c + (b*g - 2*c*e)*atanh((b
 + 2*c*x**2)/sqrt(-4*a*c + b**2))/(2*c*sqrt(-4*a*c + b**2)) - sqr
t(2)*(b*(b*h - c*f) - 2*c*(a*h - c*d) + sqrt(-4*a*c + b**2)*(b*h
- c*f))*atan(sqrt(2)*sqrt(c)*x/sqrt(b + sqrt(-4*a*c + b**2)))/(2*
c**(3/2)*sqrt(b + sqrt(-4*a*c + b**2))*sqrt(-4*a*c + b**2)) + sqr
t(2)*(b*(b*h - c*f) - 2*c*(a*h - c*d) - sqrt(-4*a*c + b**2)*(b*h
- c*f))*atan(sqrt(2)*sqrt(c)*x/sqrt(b - sqrt(-4*a*c + b**2)))/(2*
c**(3/2)*sqrt(b - sqrt(-4*a*c + b**2))*sqrt(-4*a*c + b**2))
```

Mathematica [A] time = 1.07815, size = 383, normalized size = 1.32

$$\frac{\frac{2\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(c\left(f\sqrt{b^2-4ac}-2ah-bf\right)+bh\left(b-\sqrt{b^2-4ac}\right)+2c^2d\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{2\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-c\left(f\sqrt{b^2-4ac}+2ah+bf\right)+bh\left(\sqrt{b^2-4ac}+b\right)+2c^2d\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}}{4c^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x + f*x^2 + g*x^3 + h*x^4)/(a + b*x^2 + c*x^4),x]
```

```
[Out] (4*Sqrt[c]*h*x + (2*Sqrt[2]*(2*c^2*d + b*(b - Sqrt[b^2 - 4*a*c])*
h + c*(-(b*f) + Sqrt[b^2 - 4*a*c]*f - 2*a*h))*ArcTan[(Sqrt[2]*Sqr
```

```
t[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b -
 Sqrt[b^2 - 4*a*c]]) - (2*Sqrt[2]*(2*c^2*d + b*(b + Sqrt[b^2 - 4*
a*c])*h - c*(b*f + Sqrt[b^2 - 4*a*c]*f + 2*a*h))*ArcTan[(Sqrt[2]*
Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[
b + Sqrt[b^2 - 4*a*c]]) + (Sqrt[c]*(2*c*e + (-b + Sqrt[b^2 - 4*a*
c])*g)*Log[-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2])/Sqrt[b^2 - 4*a*c] +
 (Sqrt[c]*(-2*c*e + (b + Sqrt[b^2 - 4*a*c])*g)*Log[b + Sqrt[b^2 -
 4*a*c] + 2*c*x^2])/Sqrt[b^2 - 4*a*c])/(4*c^(3/2))
```

Maple [B] time = 0.039, size = 1132, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a),x)
```

```
[Out] h*x/c-1/4*(-4*a*c+b^2)/(4*a*c-b^2)/c*ln(2*c*x^2+(-4*a*c+b^2)^(1/2
)+b)*g-1/4*(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)/c*ln(2*c*x^2+(-4*a*c+b^
2)^(1/2)+b)*b*g+1/2*(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)*e*ln(2*c*x^2+(
-4*a*c+b^2)^(1/2)+b)+1/2*(-4*a*c+b^2)/(4*a*c-b^2)/c*2^(1/2)/((b+(
-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(
1/2))*c)^(1/2))*b*h-1/2*(-4*a*c+b^2)/(4*a*c-b^2)*2^(1/2)/((b+(-4*
a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2
))*c)^(1/2))*f-(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c
+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*
c)^(1/2))*a*h+1/2*(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)/c*2^(1/2)/((b+(-
4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1
/2))*c)^(1/2))*b^2*h-1/2*(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)*2^(1/2)/(
(b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^
2)^(1/2))*c)^(1/2))*b*f+c*(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)*2^(1/2)/
((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b
^2)^(1/2))*c)^(1/2))*d-1/4*(-4*a*c+b^2)/(4*a*c-b^2)/c*ln(-2*c*x^2
+(-4*a*c+b^2)^(1/2)-b)*g+1/4*(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)/c*ln(
-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)*b*g-1/2*(-4*a*c+b^2)^(1/2)/(4*a*c-
b^2)*e*ln(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)-1/2*(-4*a*c+b^2)/(4*a*c-
b^2)/c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1
/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b*h+1/2*(-4*a*c+b^2)/(4*a*
c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1
/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*f-(-4*a*c+b^2)^(1/2)/(4*a*
c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1
/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*h+1/2*(-4*a*c+b^2)^(1/2)
/(4*a*c-b^2)/c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(
c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^2*h-1/2*(-4*a*c+
b^2)^(1/2)/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*
arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b*f+c*(-4*
a*c+b^2)^(1/2)/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1
/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{hx}{c}+\frac{\int\frac{cgx^3+cex+(cf-bh)x^2+cd-ah}{cx^4+bx^2+a}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^4 + g*x^3 + f*x^2 + e*x + d)/(c*x^4 + b*x^2 + a),x, algorithm="maxi
```

```
[Out] h*x/c + integrate((c*g*x^3 + c*e*x + (c*f - b*h)*x^2 + c*d - a*h)
/(c*x^4 + b*x^2 + a), x)/c
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^4 + g*x^3 + f*x^2 + e*x + d)/(c*x^4 + b*x^2 + a),x, algorithm="fric
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((h*x**4+g*x**3+f*x**2+e*x+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 1.69702, size = 1, normalized size = 0.

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^4 + g*x^3 + f*x^2 + e*x + d)/(c*x^4 + b*x^2 + a),x, algorithm="giac

[Out] Done
```

3.24 $\int \frac{d+ex+fx^2+gx^3+hx^4+ix^5}{a+bx^2+cx^4} \, dx$

Optimal. Leaf size=321

$$-\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)\left(-2aci+b^2i-bcg+2c^2e\right)}{2c^2\sqrt{b^2-4ac}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{-c(2ah+bf)+b^2h+2c^2d}{\sqrt{b^2-4ac}}-bh+cf\right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-\frac{-2ach+b^2h-bcf+2c^2d}{\sqrt{b^2-4ac}}-bh+cf\right)}{\sqrt{2}c^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{(cg-bi)\log\left(a+bx^2+cx^4\right)}{4c^2}+\frac{hx}{c}+\frac{ix^2}{2c}$$

```
[Out] (h*x)/c + (i*x^2)/(2*c) + ((c*f - b*h + (2*c^2*d + b^2*h - c*(b*f
 + 2*a*h))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b -
 Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]
]) + ((c*f - b*h - (2*c^2*d - b*c*f + b^2*h - 2*a*c*h)/Sqrt[b^2 -
 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])
/(Sqrt[2]*c^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - ((2*c^2*e - b*c*
g + b^2*i - 2*a*c*i)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2
*c^2*Sqrt[b^2 - 4*a*c]) + ((c*g - b*i)*Log[a + b*x^2 + c*x^4])/(4
*c^2)
```

Rubi [A] time = 1.34101, antiderivative size = 321, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$-\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)\left(-2aci+b^2i-bcg+2c^2e\right)}{2c^2\sqrt{b^2-4ac}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{-c(2ah+bf)+b^2h+2c^2d}{\sqrt{b^2-4ac}}-bh+cf\right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-\frac{-2ach+b^2h-bcf+2c^2d}{\sqrt{b^2-4ac}}-bh+cf\right)}{\sqrt{2}c^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{(cg-bi)\log\left(a+bx^2+cx^4\right)}{4c^2}+\frac{hx}{c}+\frac{ix^2}{2c}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5)/(a + b*x^2 + c*x^4),x]
```

```
[Out] (h*x)/c + (i*x^2)/(2*c) + ((c*f - b*h + (2*c^2*d + b^2*h - c*(b*f
 + 2*a*h))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b -
```

```
 Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]
]) + ((c*f - b*h - (2*c^2*d - b*c*f + b^2*h - 2*a*c*h)/Sqrt[b^2 -
 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])
/(Sqrt[2]*c^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - ((2*c^2*e - b*c*
g + b^2*i - 2*a*c*i)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2
*c^2*Sqrt[b^2 - 4*a*c]) + ((c*g - b*i)*Log[a + b*x^2 + c*x^4])/(4
*c^2)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((i*x**5+h*x**4+g*x**3+f*x**2+e*x+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 1.56646, size = 441, normalized size = 1.37

$$\frac{\frac{2\sqrt{2}\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(c\left(f\sqrt{b^2-4ac}-2ah-bf\right)+bh\left(b-\sqrt{b^2-4ac}\right)+2c^2d\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{2\sqrt{2}\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-c\left(f\sqrt{b^2-4ac}+2ah+bf\right)+bh\left(\sqrt{b^2-4ac}+b\right)+\right.}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5)/(a + b*x^2 + c*x^4),x]
```

```
[Out] (4*c*h*x + 2*c*i*x^2 + (2*Sqrt[2]*Sqrt[c]*(2*c^2*d + b*(b - Sqrt[
b^2 - 4*a*c])*h + c*(-(b*f) + Sqrt[b^2 - 4*a*c]*f - 2*a*h))*ArcTa
n[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4
*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (2*Sqrt[2]*Sqrt[c]*(2*c^2*d
+ b*(b + Sqrt[b^2 - 4*a*c])*h - c*(b*f + Sqrt[b^2 - 4*a*c]*f + 2*
a*h))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(S
qrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + ((2*c^2*e + b*(b
- Sqrt[b^2 - 4*a*c])*i + c*(-(b*g) + Sqrt[b^2 - 4*a*c]*g - 2*a*i)
)*Log[-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2])/Sqrt[b^2 - 4*a*c] - ((2*
c^2*e + b*(b + Sqrt[b^2 - 4*a*c])*i - c*(b*g + Sqrt[b^2 - 4*a*c]*
g + 2*a*i))*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/Sqrt[b^2 - 4*a*
c])/(4*c^2)
```

Maple [B] time = 0.041, size = 1435, normalized size = 4.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/2*(-4*a*c+b^2)/(4*a*c-b^2)/c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)
^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b*h-(
-4*a*c+b^2)^(1/2)/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)
^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*h
-(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c
)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*h+
1/2*i*x^2/c+1/4*(-4*a*c+b^2)/(4*a*c-b^2)/c^2*ln(-2*c*x^2+(-4*a*c+
b^2)^(1/2)-b)*b*i+1/2*(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)/c*ln(-2*c*x^
2+(-4*a*c+b^2)^(1/2)-b)*a*i-1/4*(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)/c^
2*ln(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)*b^2*i+1/4*(-4*a*c+b^2)/(4*a*c
-b^2)/c^2*ln(2*c*x^2+(-4*a*c+b^2)^(1/2)+b)*b*i-1/2*(-4*a*c+b^2)^(
1/2)/(4*a*c-b^2)/c*ln(2*c*x^2+(-4*a*c+b^2)^(1/2)+b)*a*i+1/4*(-4*a
*c+b^2)^(1/2)/(4*a*c-b^2)/c^2*ln(2*c*x^2+(-4*a*c+b^2)^(1/2)+b)*b^
2*i+h*x/c-1/2*(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+
b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c
)^(1/2))*b*f+c*(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c
+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*
c)^(1/2))*d-1/2*(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a
*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/
2))*c)^(1/2))*b*f+1/2*(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)/c*2^(1/2)/((
-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+
b^2)^(1/2))*c)^(1/2))*b^2*h+1/2*(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)*e*
ln(2*c*x^2+(-4*a*c+b^2)^(1/2)+b)-1/2*(-4*a*c+b^2)^(1/2)/(4*a*c-b^
2)*e*ln(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)+1/2*(-4*a*c+b^2)^(1/2)/(4*
a*c-b^2)/c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^
(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^2*h-1/2*(-4*a*c+b^2)/(4
*a*c-b^2)/c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x
*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b*h-1/2*(-4*a*c+b^2)/
(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2
^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*f+1/2*(-4*a*c+b^2)/(4*a*
c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1
/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*f+1/4*(-4*a*c+b^2)^(1/2)/(
4*a*c-b^2)/c*ln(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)*b*g-1/4*(-4*a*c+b^
2)/(4*a*c-b^2)/c*ln(2*c*x^2+(-4*a*c+b^2)^(1/2)+b)*g-1/4*(-4*a*c+b
^2)/(4*a*c-b^2)/c*ln(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)*g-1/4*(-4*a*c
+b^2)^(1/2)/(4*a*c-b^2)/c*ln(2*c*x^2+(-4*a*c+b^2)^(1/2)+b)*b*g+c*
(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c
)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{ix^2 + 2\,hx}{2\,c} - \frac{-\int \frac{(cg-bi)x^3+(cf-bh)x^2+cd-ah+(ce-ai)x}{cx^4+bx^2+a}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^5 + h*x^4 + g*x^3 + f*x^2 + e*x + d)/(c*x^4 + b*x^2 + a),x, algorith
```

```
[Out] 1/2*(i*x^2 + 2*h*x)/c - integrate(-((c*g - b*i)*x^3 + (c*f - b*h)
*x^2 + c*d - a*h + (c*e - a*i)*x)/(c*x^4 + b*x^2 + a), x)/c
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^5 + h*x^4 + g*x^3 + f*x^2 + e*x + d)/(c*x^4 + b*x^2 + a),x, algorith
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x**5+h*x**4+g*x**3+f*x**2+e*x+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 2.0337, size = 1, normalized size = 0.

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^5 + h*x^4 + g*x^3 + f*x^2 + e*x + d)/(c*x^4 + b*x^2 + a),x, algorith
```

```
[Out] Done
```

3.25 $\int \frac{d+ex+fx^2+gx^3+hx^4+jx^5+kx^6+lx^7+mx^8}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=545

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{c^2\left(2a^2m+3abk+b^2h\right)-b^2c(4am+bk)-c^3(2ah+bf)+b^4m+2c^4d}{\sqrt{b^2-4ac}}-c^2(ak+bh)+bc(2am+bk)+b^3(-m)+c^3f\right)}{\sqrt{2}c^{7/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-\frac{c^2\left(2a^2m+3abk+b^2h\right)-b^2c(4am+bk)-c^3(2ah+bf)+b^4m+2c^4d}{\sqrt{b^2-4ac}}-c^2(ak+bh)+bc(2am+bk)+b^3(-m)+c^3f\right)}{\sqrt{2}c^{7/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{\log\left(a+bx^2+cx^4\right)\left(-c(al+bj)+b^2l+c^2g\right)}{4c^3}+\frac{x\left(-c(am+bk)+b^2m+c^2h\right)}{c^3}-\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)\left(-c^2(2aj+bg)+bc(3al+bj)+b^3(-l)+2c^3e\right)}{2c^3\sqrt{b^2-4ac}}+\frac{x^2(cj-bl)}{2c^2}+\frac{x^3(ck-bm)}{3c^2}+\frac{lx^4}{4c}+\frac{mx^5}{5c}$$

```
[Out] ((c^2*h + b^2*m - c*(b*k + a*m))*x)/c^3 + ((c*j - b*l)*x^2)/(2*c^
2) + ((c*k - b*m)*x^3)/(3*c^2) + (l*x^4)/(4*c) + (m*x^5)/(5*c) +
((c^3*f - c^2*(b*h + a*k) - b^3*m + b*c*(b*k + 2*a*m) + (2*c^4*d
- c^3*(b*f + 2*a*h) + b^4*m - b^2*c*(b*k + 4*a*m) + c^2*(b^2*h +
3*a*b*k + 2*a^2*m))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)
/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(7/2)*Sqrt[b - Sqrt[b^2
 - 4*a*c]]) + ((c^3*f - c^2*(b*h + a*k) - b^3*m + b*c*(b*k + 2*a*
m) - (2*c^4*d - c^3*(b*f + 2*a*h) + b^4*m - b^2*c*(b*k + 4*a*m) +
 c^2*(b^2*h + 3*a*b*k + 2*a^2*m))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt
[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(7/2)*Sqr
t[b + Sqrt[b^2 - 4*a*c]]) - ((2*c^3*e - c^2*(b*g + 2*a*j) - b^3*l
 + b*c*(b*j + 3*a*l))*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(
2*c^3*Sqrt[b^2 - 4*a*c]) + ((c^2*g + b^2*l - c*(b*j + a*l))*Log[a
 + b*x^2 + c*x^4])/(4*c^3)
```

Rubi [A] time = 10.7722, antiderivative size = 545, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 55, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{c^2\left(2a^2m+3abk+b^2h\right)-b^2c(4am+bk)-c^3(2ah+bf)+b^4m+2c^4d}{\sqrt{b^2-4ac}}-c^2(ak+bh)+bc(2am+bk)+b^3(-m)+c^3f\right)}{\sqrt{2}c^{7/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-\frac{c^2\left(2a^2m+3abk+b^2h\right)-b^2c(4am+bk)-c^3(2ah+bf)+b^4m+2c^4d}{\sqrt{b^2-4ac}}-c^2(ak+bh)+bc(2am+bk)+b^3(-m)+c^3f\right)}{\sqrt{2}c^{7/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{\log\left(a+bx^2+cx^4\right)\left(-c(al+bj)+b^2l+c^2g\right)}{4c^3}+\frac{x\left(-c(am+bk)+b^2m+c^2h\right)}{c^3}-\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)\left(-c^2(2aj+bg)+bc(3al+bj)+b^3(-l)+2c^3e\right)}{2c^3\sqrt{b^2-4ac}}+\frac{x^2(cj-bl)}{2c^2}+\frac{x^3(ck-bm)}{3c^2}+\frac{lx^4}{4c}+\frac{mx^5}{5c}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x + f*x^2 + g*x^3 + h*x^4 + j*x^5 + k*x^6 + l*x^7 + m*x^8)/(a + b*x^2 + c*
```

```
[Out] ((c^2*h + b^2*m - c*(b*k + a*m))*x)/c^3 + ((c*j - b*l)*x^2)/(2*c^
2) + ((c*k - b*m)*x^3)/(3*c^2) + (l*x^4)/(4*c) + (m*x^5)/(5*c) +
((c^3*f - c^2*(b*h + a*k) - b^3*m + b*c*(b*k + 2*a*m) + (2*c^4*d
- c^3*(b*f + 2*a*h) + b^4*m - b^2*c*(b*k + 4*a*m) + c^2*(b^2*h +
3*a*b*k + 2*a^2*m))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)
/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(7/2)*Sqrt[b - Sqrt[b^2
 - 4*a*c]]) + ((c^3*f - c^2*(b*h + a*k) - b^3*m + b*c*(b*k + 2*a*
m) - (2*c^4*d - c^3*(b*f + 2*a*h) + b^4*m - b^2*c*(b*k + 4*a*m) +
 c^2*(b^2*h + 3*a*b*k + 2*a^2*m))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt
[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(7/2)*Sqr
t[b + Sqrt[b^2 - 4*a*c]]) - ((2*c^3*e - c^2*(b*g + 2*a*j) - b^3*l
 + b*c*(b*j + 3*a*l))*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(
2*c^3*Sqrt[b^2 - 4*a*c]) + ((c^2*g + b^2*l - c*(b*j + a*l))*Log[a
 + b*x^2 + c*x^4])/(4*c^3)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((m*x**8+l*x**7+k*x**6+j*x**5+h*x**4+g*x**3+f*x**2+e*x+d)/(c*x**
```

```
[Out] Timed out
```

Mathematica [A] time = 3.52472, size = 816, normalized size = 1.5

$$\frac{mx^5}{5c}+\frac{lx^4}{4c}+\frac{(ck-bm)x^3}{3c^2}+\frac{(cj-bl)x^2}{2c^2}+\frac{\left(mb^2+c^2h-c(bk+am)\right)x}{c^3}$$
$$+\frac{\left(2dc^4+\left(-bf+\sqrt{b^2-4ac}f-2ah\right)c^3+\left(2ma^2+3bka-\sqrt{b^2-4ac}ka+b^2h-b\sqrt{b^2-4ac}h\right)c^2+b\left(-kb^2+\sqrt{b^2-4ac}kb\right.}{\sqrt{2}c^{7/2}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}$$
$$-\frac{\left(2dc^4-\left(bf+\sqrt{b^2-4ac}f+2ah\right)c^3+\left(2ma^2+3bka+\sqrt{b^2-4ac}ka+b^2h+b\sqrt{b^2-4ac}h\right)c^2-b\left(kb^2+\sqrt{b^2-4ac}kb+\right.}{\sqrt{2}c^{7/2}\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}}$$
$$+\frac{\left(2ec^3+\left(-bg+\sqrt{b^2-4ac}g-2aj\right)c^2+\left(jb^2-\sqrt{b^2-4ac}jb+3alb-a\sqrt{b^2-4ac}l\right)c+b^2\left(\sqrt{b^2-4ac}-b\right)l\right)\log\left(-2cx^2\right.}{4c^3\sqrt{b^2-4ac}}$$
$$+\frac{\left(-2ec^3+\left(bg+\sqrt{b^2-4ac}g+2aj\right)c^2-\left(jb^2+\sqrt{b^2-4ac}jb+3alb+a\sqrt{b^2-4ac}l\right)c+b^2\left(b+\sqrt{b^2-4ac}\right)l\right)\log\left(2cx^2+b\right.}{4c^3\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2 + g*x^3 + h*x^4 + j*x^5 + k*x^6 + l*x^7 + m*x^8)/(a + b*x
```

```
[Out] ((c^2*h + b^2*m - c*(b*k + a*m))*x)/c^3 + ((c*j - b*l)*x^2)/(2*c^
2) + ((c*k - b*m)*x^3)/(3*c^2) + (l*x^4)/(4*c) + (m*x^5)/(5*c) +
((2*c^4*d + c^3*(-(b*f) + Sqrt[b^2 - 4*a*c]*f - 2*a*h) + b^3*(b -
 Sqrt[b^2 - 4*a*c])*m + c^2*(b^2*h - b*Sqrt[b^2 - 4*a*c]*h + 3*a*
b*k - a*Sqrt[b^2 - 4*a*c]*k + 2*a^2*m) + b*c*(-(b^2*k) + b*Sqrt[b
^2 - 4*a*c]*k - 4*a*b*m + 2*a*Sqrt[b^2 - 4*a*c]*m))*ArcTan[(Sqrt[
2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(7/2)*Sqrt
[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - ((2*c^4*d - c^3*(b*f
 + Sqrt[b^2 - 4*a*c]*f + 2*a*h) + b^3*(b + Sqrt[b^2 - 4*a*c])*m +
 c^2*(b^2*h + b*Sqrt[b^2 - 4*a*c]*h + 3*a*b*k + a*Sqrt[b^2 - 4*a*
c]*k + 2*a^2*m) - b*c*(b^2*k + b*Sqrt[b^2 - 4*a*c]*k + 4*a*b*m +
2*a*Sqrt[b^2 - 4*a*c]*m))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqr
t[b^2 - 4*a*c]]])/(Sqrt[2]*c^(7/2)*Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqr
t[b^2 - 4*a*c]]) + ((2*c^3*e + c^2*(-(b*g) + Sqrt[b^2 - 4*a*c]*g
- 2*a*j) + b^2*(-b + Sqrt[b^2 - 4*a*c])*l + c*(b^2*j - b*Sqrt[b^2
 - 4*a*c]*j + 3*a*b*l - a*Sqrt[b^2 - 4*a*c]*l))*Log[-b + Sqrt[b^2
 - 4*a*c] - 2*c*x^2]])/(4*c^3*Sqrt[b^2 - 4*a*c]) + ((-2*c^3*e + c^
2*(b*g + Sqrt[b^2 - 4*a*c]*g + 2*a*j) + b^2*(b + Sqrt[b^2 - 4*a*c
])*l - c*(b^2*j + b*Sqrt[b^2 - 4*a*c]*j + 3*a*b*l + a*Sqrt[b^2 -
4*a*c]*l))*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(4*c^3*Sqrt[b^2
- 4*a*c])
```

Maple [B] time = 0.067, size = 3835, normalized size = 7.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((m*x^8+l*x^7+k*x^6+j*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/4/c^2/(4*a*c-b^2)*ln(2*c*x^2+(-4*a*c+b^2)^(1/2)+b)*b^3*j+1/4/c^
2/(4*a*c-b^2)*ln(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)*b^3*j-1/4/c^3/(4*
a*c-b^2)*ln(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)*b^4*l-1/4/c^3/(4*a*c-b
^2)*ln(2*c*x^2+(-4*a*c+b^2)^(1/2)+b)*b^4*l-(-4*a*c+b^2)^(1/2)/(4*
a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^
(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*h-(-4*a*c+b^2)^(1/2)/(
4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^
(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*h+1/4*l*x^4/c+1/5*m*x^5
/c+h*x/c-1/2/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)
*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*f*b^2+1/2/(
4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*
2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*f*b^2+2*c/(4*a*c-b^2)*
2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(
-4*a*c+b^2)^(1/2))*c)^(1/2))*f*a-1/2*(-4*a*c+b^2)^(1/2)/(4*a*c-b^
2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((
b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b*f+c*(-4*a*c+b^2)^(1/2)/(4*a*c-b
^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/(
(b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*d-2*c/(4*a*c-b^2)*2^(1/2)/((-b+(
-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)
^(1/2))*c)^(1/2))*f*a-1/2*(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)*2^(1/2)/
((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*
c+b^2)^(1/2))*c)^(1/2))*b*f+1/2*(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)/c*
2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-
b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^2*h+1/(4*a*c-b^2)*ln(2*c*x^2+(-
4*a*c+b^2)^(1/2)+b)*g*a+1/(4*a*c-b^2)*ln(-2*c*x^2+(-4*a*c+b^2)^(1
/2)-b)*g*a+1/2*(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)*e*ln(2*c*x^2+(-4*a*
c+b^2)^(1/2)+b)-1/2*(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)*e*ln(-2*c*x^2+
(-4*a*c+b^2)^(1/2)-b)-2/c^2*(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)*2^(1/2
)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*
a*c+b^2)^(1/2))*c)^(1/2))*a*b^2*m+3/2/c*(-4*a*c+b^2)^(1/2)/(4*a*c
-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/
2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*b*k-2/c^2*(-4*a*c+b^2)^(1
/2)/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c
*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*b^2*m+3/2/c*(-4*a*
c+b^2)^(1/2)/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)
*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*b*k+3/4/c
^2*(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)*ln(2*c*x^2+(-4*a*c+b^2)^(1/2)+b
)*a*b*l+1/2*(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)/c*2^(1/2)/((b+(-4*a*c+
b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c
)^(1/2))*b^2*h-1/2/c^2*(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)*2^(1/2)/((-
b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b
^2)^(1/2))*c)^(1/2))*b^3*k-4/c/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b
^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*
c)^(1/2))*a^2*b*m-3/c^2/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2
))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*
a*b^3*m-5/2/c/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/
2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^2*k*a
+3/c^2/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arct
anh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^3*m*a+4/c/(4
```

```
*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(
1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a^2*b*m+1/2/c^3*(-4*a*c+b^
2)^(1/2)/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arc
tan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^4*m-1/2/c^2*(
-4*a*c+b^2)^(1/2)/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^
(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^3*k-
1/2/c^2/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arct
an(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^4*k-3/4/c^2*(-
4*a*c+b^2)^(1/2)/(4*a*c-b^2)*ln(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)*a*
b*l-1/2/c^3/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)
*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^5*m+1/2
/c/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*
x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^3*h+1/c*(-4*a*c+b^2
)^(1/2)/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arct
an(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a^2*m+1/2/c^3*(-
4*a*c+b^2)^(1/2)/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^
(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^4*
m+5/2/c/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arct
an(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*k*b^2+1/c*(-4*
a*c+b^2)^(1/2)/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1
/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a^2*m+
1/2/c^3/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arct
an(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^5*m+1/2/c^2/(4
*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2
^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^4*k-1/c/(4*a*c-b^2)*l
n(2*c*x^2+(-4*a*c+b^2)^(1/2)+b)*a^2*l-1/c/(4*a*c-b^2)*ln(-2*c*x^2
+(-4*a*c+b^2)^(1/2)-b)*a^2*l+1/4*(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)/c
*ln(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)*b*g-1/4/(4*a*c-b^2)/c*ln(2*c*x
^2+(-4*a*c+b^2)^(1/2)+b)*g*b^2-1/3/c^2*x^3*b*m-1/2/c^2*x^2*b*l-1/
c^2*a*m*x+1/c^3*b^2*m*x-1/c^2*b*k*x-2/(4*a*c-b^2)*2^(1/2)/((b+(-4
*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/
2))*c)^(1/2))*b*h*a+2/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2)
)*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))
*b*h*a-1/2/c/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2
)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^3*h-1/
4*(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)/c*ln(2*c*x^2+(-4*a*c+b^2)^(1/2)+
b)*b*g-1/4/(4*a*c-b^2)/c*ln(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)*g*b^2+
1/3/c*x^3*k+1/2/c*x^2*j+c*(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)*2^(1/2)/
((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*
c+b^2)^(1/2))*c)^(1/2))*d-1/c/(4*a*c-b^2)*ln(2*c*x^2+(-4*a*c+b^2)
^(1/2)+b)*b*j*a-1/c/(4*a*c-b^2)*ln(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)
*b*j*a+5/4/c^2/(4*a*c-b^2)*ln(2*c*x^2+(-4*a*c+b^2)^(1/2)+b)*a*l*b
^2+5/4/c^2/(4*a*c-b^2)*ln(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)*a*l*b^2+
1/2/c*(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)*ln(-2*c*x^2+(-4*a*c+b^2)^(1/
2)-b)*a*j+1/4/c^3*(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)*ln(-2*c*x^2+(-4*
a*c+b^2)^(1/2)-b)*b^3*l-1/4/c^2*(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)*ln
(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)*b^2*j-1/2/c*(-4*a*c+b^2)^(1/2)/(4
*a*c-b^2)*ln(2*c*x^2+(-4*a*c+b^2)^(1/2)+b)*a*j-1/4/c^3*(-4*a*c+b^
2)^(1/2)/(4*a*c-b^2)*ln(2*c*x^2+(-4*a*c+b^2)^(1/2)+b)*b^3*l+1/4/c
^2*(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)*ln(2*c*x^2+(-4*a*c+b^2)^(1/2)+b
)*b^2*j-2/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*ar
ctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a^2*k+2/(4*a*c
-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/
2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a^2*k
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{12\,c^2mx^5+15\,c^2lx^4+20\,\left(c^2k-bcm\right)x^3+30\,\left(c^2j-bcl\right)x^2+60\,\left(c^2h-bck+\left(b^2-ac\right)m\right)x}{60\,c^3}-\frac{-\int\frac{c^3d-ac^2h+abck+\left(c^3g-bc^2j+\left(b^2c-ac^2\right)l\right)x^3+\left(c^3f-bc^2h+\left(b^2c-ac^2\right)k-\left(b^3-2\,abc\right)m\right)x^2-\left(ab^2-a^2c\right)m+\left(c^3e-ac^2j+abcl\right)x}{cx^4+bx^2+a}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((m*x^8 + l*x^7 + k*x^6 + j*x^5 + h*x^4 + g*x^3 + f*x^2 + e*x + d)/(c*x^4 +
```

```
[Out] 1/60*(12*c^2*m*x^5 + 15*c^2*l*x^4 + 20*(c^2*k - b*c*m)*x^3 + 30*(
c^2*j - b*c*l)*x^2 + 60*(c^2*h - b*c*k + (b^2 - a*c)*m)*x)/c^3 -
integrate(-(c^3*d - a*c^2*h + a*b*c*k + (c^3*g - b*c^2*j + (b^2*c
 - a*c^2)*l)*x^3 + (c^3*f - b*c^2*h + (b^2*c - a*c^2)*k - (b^3 -
2*a*b*c)*m)*x^2 - (a*b^2 - a^2*c)*m + (c^3*e - a*c^2*j + a*b*c*l)
*x)/(c*x^4 + b*x^2 + a), x)/c^3
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((m*x^8 + l*x^7 + k*x^6 + j*x^5 + h*x^4 + g*x^3 + f*x^2 + e*x + d)/(c*x^4 +
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((m*x**8+l*x**7+k*x**6+j*x**5+h*x**4+g*x**3+f*x**2+e*x+d)/(c*x**4+b*x
```

```
[Out] Timed out
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((m*x^8 + l*x^7 + k*x^6 + j*x^5 + h*x^4 + g*x^3 + f*x^2 + e*x + d)/(c*x^4 +

[Out] Exception raised: NotImplementedError
```

3.26 $\int \frac{d+ex}{(4-5x^2+x^4)^2}\,dx$

Optimal. Leaf size=94

$$\frac{dx\left(17-5x^2\right)}{72\left(x^4-5x^2+4\right)}+\frac{19}{432}d\tanh^{-1}\left(\frac{x}{2}\right)-\frac{1}{54}d\tanh^{-1}(x)+\frac{1}{27}e\log\left(1-x^2\right)-\frac{1}{27}e\log\left(4-x^2\right)+\frac{e\left(5-2x^2\right)}{18\left(x^4-5x^2+4\right)}$$

```
[Out] (d*x*(17 - 5*x^2))/(72*(4 - 5*x^2 + x^4)) + (e*(5 - 2*x^2))/(18*(
4 - 5*x^2 + x^4)) + (19*d*ArcTanh[x/2])/432 - (d*ArcTanh[x])/54 +
 (e*Log[1 - x^2])/27 - (e*Log[4 - x^2])/27
```

Rubi [A] time = 0.12135, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5

$$\frac{dx\left(17-5x^2\right)}{72\left(x^4-5x^2+4\right)}+\frac{19}{432}d\tanh^{-1}\left(\frac{x}{2}\right)-\frac{1}{54}d\tanh^{-1}(x)+\frac{1}{27}e\log\left(1-x^2\right)-\frac{1}{27}e\log\left(4-x^2\right)+\frac{e\left(5-2x^2\right)}{18\left(x^4-5x^2+4\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x)/(4 - 5*x^2 + x^4)^2,x]
```

```
[Out] (d*x*(17 - 5*x^2))/(72*(4 - 5*x^2 + x^4)) + (e*(5 - 2*x^2))/(18*(
4 - 5*x^2 + x^4)) + (19*d*ArcTanh[x/2])/432 - (d*ArcTanh[x])/54 +
 (e*Log[1 - x^2])/27 - (e*Log[4 - x^2])/27
```

Rubi in Sympy [A] time = 31.8527, size = 71, normalized size = 0.76

$$\frac{19d\operatorname{atanh}\left(\frac{x}{2}\right)}{432}-\frac{d\operatorname{atanh}(x)}{54}+\frac{e\log\left(-x^2+1\right)}{27}-\frac{e\log\left(-x^2+4\right)}{27}+\frac{x\left(-5dx^2+17d-5ex^3+17ex\right)}{72\left(x^4-5x^2+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x+d)/(x**4-5*x**2+4)**2,x)
```

```
[Out] 19*d*atanh(x/2)/432 - d*atanh(x)/54 + e*log(-x**2 + 1)/27 - e*log
(-x**2 + 4)/27 + x*(-5*d*x**2 + 17*d - 5*e*x**3 + 17*e*x)/(72*(x*
```

```
*4 - 5*x**2 + 4))
```

Mathematica [A] time = 0.121014, size = 90, normalized size = 0.96

$$\frac{1}{864}\left(\frac{12\left(dx\left(17-5x^2\right)+e\left(20-8x^2\right)\right)}{x^4-5x^2+4}+8(d+4e)\log(1-x)\right.$$
$$\left.-(19d+32e)\log(2-x)-8(d-4e)\log(x+1)+(19d-32e)\log(x+2)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x)/(4 - 5*x^2 + x^4)^2,x]
```

```
[Out] ((12*(e*(20 - 8*x^2) + d*x*(17 - 5*x^2)))/(4 - 5*x^2 + x^4) + 8*(
d + 4*e)*Log[1 - x] - (19*d + 32*e)*Log[2 - x] - 8*(d - 4*e)*Log[
1 + x] + (19*d - 32*e)*Log[2 + x])/864
```

Maple [A] time = 0.026, size = 122, normalized size = 1.3

$$-\frac{d}{288+144\,x}+\frac{e}{144+72\,x}+\frac{19\,\ln(2+x)\,d}{864}-\frac{\ln(2+x)\,e}{27}-\frac{d}{-36+36\,x}$$
$$-\frac{e}{-36+36\,x}+\frac{\ln(-1+x)\,d}{108}+\frac{\ln(-1+x)\,e}{27}-\frac{\ln(1+x)\,d}{108}+\frac{\ln(1+x)\,e}{27}$$
$$-\frac{d}{36+36\,x}+\frac{e}{36+36\,x}-\frac{19\,\ln(x-2)\,d}{864}-\frac{\ln(x-2)\,e}{27}-\frac{d}{144\,x-288}-\frac{e}{72\,x-144}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x+d)/(x^4-5*x^2+4)^2,x)
```

```
[Out] -1/144/(2+x)*d+1/72/(2+x)*e+19/864*ln(2+x)*d-1/27*ln(2+x)*e-1/36/
(-1+x)*d-1/36/(-1+x)*e+1/108*ln(-1+x)*d+1/27*ln(-1+x)*e-1/108*ln(
1+x)*d+1/27*ln(1+x)*e-1/36/(1+x)*d+1/36/(1+x)*e-19/864*ln(x-2)*d-
1/27*ln(x-2)*e-1/144/(x-2)*d-1/72/(x-2)*e
```

Maxima [A] time = 0.704961, size = 112, normalized size = 1.19

$$\frac{1}{864}\,(19\,d-32\,e)\log(x+2)-\frac{1}{108}\,(d-4\,e)\log(x+1)+\frac{1}{108}\,(d+4\,e)\log(x-1)$$
$$-\frac{1}{864}\,(19\,d+32\,e)\log(x-2)-\frac{5\,dx^3+8\,ex^2-17\,dx-20\,e}{72\,(x^4-5\,x^2+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x + d)/(x^4 - 5*x^2 + 4)^2,x, algorithm="maxima")
```

```
[Out] 1/864*(19*d - 32*e)*log(x + 2) - 1/108*(d - 4*e)*log(x + 1) + 1/1
08*(d + 4*e)*log(x - 1) - 1/864*(19*d + 32*e)*log(x - 2) - 1/72*(
5*d*x^3 + 8*e*x^2 - 17*d*x - 20*e)/(x^4 - 5*x^2 + 4)
```

Fricas [A] time = 0.325222, size = 228, normalized size = 2.43

$$-\frac{60\,dx^3 + 96\,ex^2 - 204\,dx - \left((19\,d - 32\,e)x^4 - 5\,(19\,d - 32\,e)x^2 + 76\,d - 128\,e\right)\log(x+2) + 8\left((d - 4\,e)x^4 - 5\,(d - 4\,e)x^2 +}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x + d)/(x^4 - 5*x^2 + 4)^2,x, algorithm="fricas")
```

```
[Out] -1/864*(60*d*x^3 + 96*e*x^2 - 204*d*x - ((19*d - 32*e)*x^4 - 5*(1
9*d - 32*e)*x^2 + 76*d - 128*e)*log(x + 2) + 8*((d - 4*e)*x^4 - 5
*(d - 4*e)*x^2 + 4*d - 16*e)*log(x + 1) - 8*((d + 4*e)*x^4 - 5*(d
 + 4*e)*x^2 + 4*d + 16*e)*log(x - 1) + ((19*d + 32*e)*x^4 - 5*(19
*d + 32*e)*x^2 + 76*d + 128*e)*log(x - 2) - 240*e)/(x^4 - 5*x^2 +
 4)
```

Sympy [A] time = 8.9459, size = 604, normalized size = 6.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x+d)/(x**4-5*x**2+4)**2,x)
```

```
[Out] -(d - 4*e)*log(x + (-6006260*d**4*e + 2341251*d**4*(d - 4*e) - 18
247680*d**2*e**3 + 24099840*d**2*e**2*(d - 4*e) + 7387904*d**2*e*
(d - 4*e)**2 - 665280*d**2*(d - 4*e)**3 + 587202560*e**5 - 125829
12*e**4*(d - 4*e) - 36700160*e**3*(d - 4*e)**2 + 786432*e**2*(d -
 4*e)**3)/(1675971*d**5 - 66150400*d**3*e**2 + 318767104*d*e**4))
/108 + (d + 4*e)*log(x + (-6006260*d**4*e - 2341251*d**4*(d + 4*e
) - 18247680*d**2*e**3 - 24099840*d**2*e**2*(d + 4*e) + 7387904*d
**2*e*(d + 4*e)**2 + 665280*d**2*(d + 4*e)**3 + 587202560*e**5 +
12582912*e**4*(d + 4*e) - 36700160*e**3*(d + 4*e)**2 - 786432*e**
2*(d + 4*e)**3)/(1675971*d**5 - 66150400*d**3*e**2 + 318767104*d*
e**4))/108 + (19*d - 32*e)*log(x + (-6006260*d**4*e - 2341251*d**
```

```
4*(19*d - 32*e)/8 - 18247680*d**2*e**3 - 3012480*d**2*e**2*(19*d
- 32*e) + 115436*d**2*e*(19*d - 32*e)**2 + 10395*d**2*(19*d - 32*
e)**3/8 + 587202560*e**5 + 1572864*e**4*(19*d - 32*e) - 573440*e*
*3*(19*d - 32*e)**2 - 1536*e**2*(19*d - 32*e)**3)/(1675971*d**5 -
 66150400*d**3*e**2 + 318767104*d*e**4))/864 - (19*d + 32*e)*log(
x + (-6006260*d**4*e + 2341251*d**4*(19*d + 32*e)/8 - 18247680*d*
*2*e**3 + 3012480*d**2*e**2*(19*d + 32*e) + 115436*d**2*e*(19*d +
 32*e)**2 - 10395*d**2*(19*d + 32*e)**3/8 + 587202560*e**5 - 1572
864*e**4*(19*d + 32*e) - 573440*e**3*(19*d + 32*e)**2 + 1536*e**2
*(19*d + 32*e)**3)/(1675971*d**5 - 66150400*d**3*e**2 + 318767104
*d*e**4))/864 - (5*d*x**3 - 17*d*x + 8*e*x**2 - 20*e)/(72*x**4 -
360*x**2 + 288)
```

GIAC/XCAS [A] time = 0.290152, size = 126, normalized size = 1.34

$$\frac{1}{864}\,(19\,d-32\,e)\ln\left(|x+2|\right)-\frac{1}{108}\,(d-4\,e)\ln\left(|x+1|\right)+\frac{1}{108}\,(d+4\,e)\ln\left(|x-1|\right)-\frac{1}{864}\,(19\,d+32\,e)\ln\left(|x-2|\right)-\frac{5\,dx^3+8\,x^2e-17\,dx-20\,e}{72\,(x^4-5\,x^2+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x + d)/(x^4 - 5*x^2 + 4)^2,x, algorithm="giac")
```

```
[Out] 1/864*(19*d - 32*e)*ln(abs(x + 2)) - 1/108*(d - 4*e)*ln(abs(x + 1
)) + 1/108*(d + 4*e)*ln(abs(x - 1)) - 1/864*(19*d + 32*e)*ln(abs(
x - 2)) - 1/72*(5*d*x^3 + 8*x^2*e - 17*d*x - 20*e)/(x^4 - 5*x^2 +
 4)
```

3.27 $\int \frac{d+ex+fx^2}{(4-5x^2+x^4)^2}\,dx$

Optimal. Leaf size=115

$$\frac{x\left(x^2(-(5d+8f))+17d+20f\right)}{72\left(x^4-5x^2+4\right)}+\frac{1}{432}(19d+52f)\tanh^{-1}\left(\frac{x}{2}\right)-\frac{1}{54}(d+7f)\tanh^{-1}(x)+\frac{1}{27}e\log\left(1-x^2\right)-\frac{1}{27}e\log\left(4-x^2\right)+\frac{e\left(5-2x^2\right)}{18\left(x^4-5x^2+4\right)}$$

```
[Out] (e*(5 - 2*x^2))/(18*(4 - 5*x^2 + x^4)) + (x*(17*d + 20*f - (5*d +
 8*f)*x^2))/(72*(4 - 5*x^2 + x^4)) + ((19*d + 52*f)*ArcTanh[x/2])
/432 - ((d + 7*f)*ArcTanh[x])/54 + (e*Log[1 - x^2])/27 - (e*Log[4
 - x^2])/27
```

Rubi [A] time = 0.285621, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$

$$\frac{x\left(x^2(-(5d+8f))+17d+20f\right)}{72\left(x^4-5x^2+4\right)}+\frac{1}{432}(19d+52f)\tanh^{-1}\left(\frac{x}{2}\right)-\frac{1}{54}(d+7f)\tanh^{-1}(x)+\frac{1}{27}e\log\left(1-x^2\right)-\frac{1}{27}e\log\left(4-x^2\right)+\frac{e\left(5-2x^2\right)}{18\left(x^4-5x^2+4\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x + f*x^2)/(4 - 5*x^2 + x^4)^2,x]
```

```
[Out] (e*(5 - 2*x^2))/(18*(4 - 5*x^2 + x^4)) + (x*(17*d + 20*f - (5*d +
 8*f)*x^2))/(72*(4 - 5*x^2 + x^4)) + ((19*d + 52*f)*ArcTanh[x/2])
/432 - ((d + 7*f)*ArcTanh[x])/54 + (e*Log[1 - x^2])/27 - (e*Log[4
 - x^2])/27
```

Rubi in Sympy [A] time = 40.634, size = 88, normalized size = 0.77

$$\frac{e\log\left(-x^2+1\right)}{27}-\frac{e\log\left(-x^2+4\right)}{27}+\frac{x\left(17d-5ex^3+17ex+20f-x^2(5d+8f)\right)}{72\left(x^4-5x^2+4\right)}-\left(\frac{d}{54}+\frac{7f}{54}\right)\operatorname{atanh}(x)+\left(\frac{19d}{432}+\frac{13f}{108}\right)\operatorname{atanh}\left(\frac{x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**2+e*x+d)/(x**4-5*x**2+4)**2,x)
```

```
[Out] e*log(-x**2 + 1)/27 - e*log(-x**2 + 4)/27 + x*(17*d - 5*e*x**3 +
17*e*x + 20*f - x**2*(5*d + 8*f))/(72*(x**4 - 5*x**2 + 4)) - (d/5
4 + 7*f/54)*atanh(x) + (19*d/432 + 13*f/108)*atanh(x/2)
```

Mathematica [A] time = 0.219271, size = 112, normalized size = 0.97

$$\frac{1}{864}\left(\frac{12\left(-5dx^3+17dx+e\left(20-8x^2\right)-8fx^3+20fx\right)}{x^4-5x^2+4}+8\log(1-x)(d+4e+7f)\right.$$
$$\left.-\log(2-x)(19d+32e+52f)-8\log(x+1)(d-4e+7f)+\log(x+2)(19d-32e+52f)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x + f*x^2)/(4 - 5*x^2 + x^4)^2,x]
```

```
[Out] ((12*(17*d*x + 20*f*x - 5*d*x^3 - 8*f*x^3 + e*(20 - 8*x^2)))/(4 -
 5*x^2 + x^4) + 8*(d + 4*e + 7*f)*Log[1 - x] - (19*d + 32*e + 52*
f)*Log[2 - x] - 8*(d - 4*e + 7*f)*Log[1 + x] + (19*d - 32*e + 52*
f)*Log[2 + x])/864
```

Maple [A] time = 0.026, size = 182, normalized size = 1.6

$$-\frac{d}{288+144x}+\frac{e}{144+72x}-\frac{f}{72+36x}+\frac{19\ln(2+x)d}{864}-\frac{\ln(2+x)e}{27}+\frac{13\ln(2+x)f}{216}$$
$$-\frac{d}{-36+36x}-\frac{e}{-36+36x}-\frac{f}{-36+36x}+\frac{\ln(-1+x)d}{108}+\frac{\ln(-1+x)e}{27}+\frac{7\ln(-1+x)f}{108}$$
$$-\frac{\ln(1+x)d}{108}+\frac{\ln(1+x)e}{27}-\frac{7\ln(1+x)f}{108}-\frac{d}{36+36x}+\frac{e}{36+36x}-\frac{f}{36+36x}$$
$$-\frac{19\ln(x-2)d}{864}-\frac{\ln(x-2)e}{27}-\frac{13\ln(x-2)f}{216}-\frac{d}{144x-288}-\frac{e}{72x-144}-\frac{f}{36x-72}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x)
```

```
[Out] -1/144/(2+x)*d+1/72/(2+x)*e-1/36/(2+x)*f+19/864*ln(2+x)*d-1/27*ln
(2+x)*e+13/216*ln(2+x)*f-1/36/(-1+x)*d-1/36/(-1+x)*e-1/36/(-1+x)*
f+1/108*ln(-1+x)*d+1/27*ln(-1+x)*e+7/108*ln(-1+x)*f-1/108*ln(1+x)
*d+1/27*ln(1+x)*e-7/108*ln(1+x)*f-1/36/(1+x)*d+1/36/(1+x)*e-1/36/
(1+x)*f-19/864*ln(x-2)*d-1/27*ln(x-2)*e-13/216*ln(x-2)*f-1/144/(x
-2)*d-1/72/(x-2)*e-1/36/(x-2)*f
```

Maxima [A] time = 0.704709, size = 143, normalized size = 1.24

$$\frac{1}{864}\,(19\,d-32\,e+52\,f)\log\left(x+2\right)-\frac{1}{108}\,(d-4\,e+7\,f)\log\left(x+1\right)+\frac{1}{108}\,(d+4\,e+7\,f)\log\left(x-1\right)-\frac{1}{864}\,(19\,d+32\,e+52\,f)\log\left(x-2\right)-\frac{(5\,d+8\,f)x^3+8\,ex^2-(17\,d+20\,f)x-20\,e}{72\,(x^4-5\,x^2+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2 + e*x + d)/(x^4 - 5*x^2 + 4)^2,x, algorithm="maxima")
```

```
[Out] 1/864*(19*d - 32*e + 52*f)*log(x + 2) - 1/108*(d - 4*e + 7*f)*log
(x + 1) + 1/108*(d + 4*e + 7*f)*log(x - 1) - 1/864*(19*d + 32*e +
 52*f)*log(x - 2) - 1/72*((5*d + 8*f)*x^3 + 8*e*x^2 - (17*d + 20*
f)*x - 20*e)/(x^4 - 5*x^2 + 4)
```

Fricas [A] time = 0.330101, size = 293, normalized size = 2.55

$$-\frac{12\,(5\,d+8\,f)x^3+96\,ex^2-12\,(17\,d+20\,f)x-\left((19\,d-32\,e+52\,f)x^4-5\,(19\,d-32\,e+52\,f)x^2+76\,d-128\,e+208\,f\right)\text{lo}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2 + e*x + d)/(x^4 - 5*x^2 + 4)^2,x, algorithm="fricas")
```

```
[Out] -1/864*(12*(5*d + 8*f)*x^3 + 96*e*x^2 - 12*(17*d + 20*f)*x - ((19
*d - 32*e + 52*f)*x^4 - 5*(19*d - 32*e + 52*f)*x^2 + 76*d - 128*e
 + 208*f)*log(x + 2) + 8*((d - 4*e + 7*f)*x^4 - 5*(d - 4*e + 7*f)
*x^2 + 4*d - 16*e + 28*f)*log(x + 1) - 8*((d + 4*e + 7*f)*x^4 - 5
*(d + 4*e + 7*f)*x^2 + 4*d + 16*e + 28*f)*log(x - 1) + ((19*d + 3
2*e + 52*f)*x^4 - 5*(19*d + 32*e + 52*f)*x^2 + 76*d + 128*e + 208
*f)*log(x - 2) - 240*e)/(x^4 - 5*x^2 + 4)
```

Sympy [A] time = 109.052, size = 2689, normalized size = 23.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x**2+e*x+d)/(x**4-5*x**2+4)**2,x)
```

```
[Out] -(d - 4*e + 7*f)*log(x + (-6006260*d**5*e + 2341251*d**5*(d - 4*e
 + 7*f) - 246016240*d**4*e*f + 31626180*d**4*f*(d - 4*e + 7*f) -
```

```
18247680*d**3*e**3 + 24099840*d**3*e**2*(d - 4*e + 7*f) - 2758371
200*d**3*e*f**2 + 7387904*d**3*e*(d - 4*e + 7*f)**2 + 171122976*d
**3*f**2*(d - 4*e + 7*f) - 665280*d**3*(d - 4*e + 7*f)**3 + 29859
8400*d**2*e**3*f + 369487872*d**2*e**2*f*(d - 4*e + 7*f) - 131922
56000*d**2*e*f**3 + 90885120*d**2*e*f*(d - 4*e + 7*f)**2 + 441486
720*d**2*f**3*(d - 4*e + 7*f) - 5536512*d**2*f*(d - 4*e + 7*f)**3
 + 587202560*d*e**5 - 12582912*d*e**4*(d - 4*e + 7*f) + 135364608
0*d*e**3*f**2 - 36700160*d*e**3*(d - 4*e + 7*f)**2 + 1448755200*d
*e**2*f**2*(d - 4*e + 7*f) + 786432*d*e**2*(d - 4*e + 7*f)**3 - 2
8282393600*d*e*f**4 + 362729472*d*e*f**2*(d - 4*e + 7*f)**2 + 399
575808*d*f**4*(d - 4*e + 7*f) - 10368000*d*f**2*(d - 4*e + 7*f)**
3 + 2751463424*e**5*f + 251658240*e**4*f*(d - 4*e + 7*f) - 530841
600*e**3*f**3 - 171966464*e**3*f*(d - 4*e + 7*f)**2 + 1935212544*
e**2*f**3*(d - 4*e + 7*f) - 15728640*e**2*f*(d - 4*e + 7*f)**3 -
21886889984*e*f**5 + 483737600*e*f**3*(d - 4*e + 7*f)**2 - 212474
880*f**5*(d - 4*e + 7*f) + 4534272*f**3*(d - 4*e + 7*f)**3)/(1675
971*d**6 + 28507545*d**5*f - 66150400*d**4*e**2 + 168075324*d**4*
f**2 - 1091117056*d**3*e**2*f + 384095520*d**3*f**3 + 318767104*d
**2*e**4 - 6528860160*d**2*e**2*f**2 + 162082944*d**2*f**4 + 3103
784960*d*e**4*f - 17414619136*d*e**2*f**3 - 305130240*d*f**5 + 61
06906624*e**4*f**2 - 17414225920*e**2*f**4 + 67931136*f**6))/108
+ (d + 4*e + 7*f)*log(x + (-6006260*d**5*e - 2341251*d**5*(d + 4*
e + 7*f) - 246016240*d**4*e*f - 31626180*d**4*f*(d + 4*e + 7*f) -
 18247680*d**3*e**3 - 24099840*d**3*e**2*(d + 4*e + 7*f) - 275837
1200*d**3*e*f**2 + 7387904*d**3*e*(d + 4*e + 7*f)**2 - 171122976*
d**3*f**2*(d + 4*e + 7*f) + 665280*d**3*(d + 4*e + 7*f)**3 + 2985
98400*d**2*e**3*f - 369487872*d**2*e**2*f*(d + 4*e + 7*f) - 13192
256000*d**2*e*f**3 + 90885120*d**2*e*f*(d + 4*e + 7*f)**2 - 44148
6720*d**2*f**3*(d + 4*e + 7*f) + 5536512*d**2*f*(d + 4*e + 7*f)**
3 + 587202560*d*e**5 + 12582912*d*e**4*(d + 4*e + 7*f) + 13536460
80*d*e**3*f**2 - 36700160*d*e**3*(d + 4*e + 7*f)**2 - 1448755200*
d*e**2*f**2*(d + 4*e + 7*f) - 786432*d*e**2*(d + 4*e + 7*f)**3 -
28282393600*d*e*f**4 + 362729472*d*e*f**2*(d + 4*e + 7*f)**2 - 39
9575808*d*f**4*(d + 4*e + 7*f) + 10368000*d*f**2*(d + 4*e + 7*f)*
*3 + 2751463424*e**5*f - 251658240*e**4*f*(d + 4*e + 7*f) - 53084
1600*e**3*f**3 - 171966464*e**3*f*(d + 4*e + 7*f)**2 - 1935212544
*e**2*f**3*(d + 4*e + 7*f) + 15728640*e**2*f*(d + 4*e + 7*f)**3 -
 21886889984*e*f**5 + 483737600*e*f**3*(d + 4*e + 7*f)**2 + 21247
4880*f**5*(d + 4*e + 7*f) - 4534272*f**3*(d + 4*e + 7*f)**3)/(167
5971*d**6 + 28507545*d**5*f - 66150400*d**4*e**2 + 168075324*d**4
*f**2 - 1091117056*d**3*e**2*f + 384095520*d**3*f**3 + 318767104*
d**2*e**4 - 6528860160*d**2*e**2*f**2 + 162082944*d**2*f**4 + 310
3784960*d*e**4*f - 17414619136*d*e**2*f**3 - 305130240*d*f**5 + 6
106906624*e**4*f**2 - 17414225920*e**2*f**4 + 67931136*f**6))/108
 + (19*d - 32*e + 52*f)*log(x + (-6006260*d**5*e - 2341251*d**5*(
19*d - 32*e + 52*f)/8 - 246016240*d**4*e*f - 7906545*d**4*f*(19*d
 - 32*e + 52*f)/2 - 18247680*d**3*e**3 - 3012480*d**3*e**2*(19*d
- 32*e + 52*f) - 2758371200*d**3*e*f**2 + 115436*d**3*e*(19*d - 3
2*e + 52*f)**2 - 21390372*d**3*f**2*(19*d - 32*e + 52*f) + 10395*
d**3*(19*d - 32*e + 52*f)**3/8 + 298598400*d**2*e**3*f - 46185984
*d**2*e**2*f*(19*d - 32*e + 52*f) - 13192256000*d**2*e*f**3 + 142
0080*d**2*e*f*(19*d - 32*e + 52*f)**2 - 55185840*d**2*f**3*(19*d
- 32*e + 52*f) + 21627*d**2*f*(19*d - 32*e + 52*f)**3/2 + 5872025
60*d*e**5 + 1572864*d*e**4*(19*d - 32*e + 52*f) + 1353646080*d*e*
```

```
*3*f**2 - 573440*d*e**3*(19*d - 32*e + 52*f)**2 - 181094400*d*e**
2*f**2*(19*d - 32*e + 52*f) - 1536*d*e**2*(19*d - 32*e + 52*f)**3
 - 28282393600*d*e*f**4 + 5667648*d*e*f**2*(19*d - 32*e + 52*f)**
2 - 49946976*d*f**4*(19*d - 32*e + 52*f) + 20250*d*f**2*(19*d - 3
2*e + 52*f)**3 + 2751463424*e**5*f - 31457280*e**4*f*(19*d - 32*e
 + 52*f) - 530841600*e**3*f**3 - 2686976*e**3*f*(19*d - 32*e + 52
*f)**2 - 241901568*e**2*f**3*(19*d - 32*e + 52*f) + 30720*e**2*f*
(19*d - 32*e + 52*f)**3 - 21886889984*e*f**5 + 7558400*e*f**3*(19
*d - 32*e + 52*f)**2 + 26559360*f**5*(19*d - 32*e + 52*f) - 8856*
f**3*(19*d - 32*e + 52*f)**3)/(1675971*d**6 + 28507545*d**5*f - 6
6150400*d**4*e**2 + 168075324*d**4*f**2 - 1091117056*d**3*e**2*f
+ 384095520*d**3*f**3 + 318767104*d**2*e**4 - 6528860160*d**2*e**
2*f**2 + 162082944*d**2*f**4 + 3103784960*d*e**4*f - 17414619136*
d*e**2*f**3 - 305130240*d*f**5 + 6106906624*e**4*f**2 - 174142259
20*e**2*f**4 + 67931136*f**6))/864 - (19*d + 32*e + 52*f)*log(x +
 (-6006260*d**5*e + 2341251*d**5*(19*d + 32*e + 52*f)/8 - 2460162
40*d**4*e*f + 7906545*d**4*f*(19*d + 32*e + 52*f)/2 - 18247680*d*
*3*e**3 + 3012480*d**3*e**2*(19*d + 32*e + 52*f) - 2758371200*d**
3*e*f**2 + 115436*d**3*e*(19*d + 32*e + 52*f)**2 + 21390372*d**3*
f**2*(19*d + 32*e + 52*f) - 10395*d**3*(19*d + 32*e + 52*f)**3/8
+ 298598400*d**2*e**3*f + 46185984*d**2*e**2*f*(19*d + 32*e + 52*
f) - 13192256000*d**2*e*f**3 + 1420080*d**2*e*f*(19*d + 32*e + 52
*f)**2 + 55185840*d**2*f**3*(19*d + 32*e + 52*f) - 21627*d**2*f*(
19*d + 32*e + 52*f)**3/2 + 587202560*d*e**5 - 1572864*d*e**4*(19*
d + 32*e + 52*f) + 1353646080*d*e**3*f**2 - 573440*d*e**3*(19*d +
 32*e + 52*f)**2 + 181094400*d*e**2*f**2*(19*d + 32*e + 52*f) + 1
536*d*e**2*(19*d + 32*e + 52*f)**3 - 28282393600*d*e*f**4 + 56676
48*d*e*f**2*(19*d + 32*e + 52*f)**2 + 49946976*d*f**4*(19*d + 32*
e + 52*f) - 20250*d*f**2*(19*d + 32*e + 52*f)**3 + 2751463424*e**
5*f + 31457280*e**4*f*(19*d + 32*e + 52*f) - 530841600*e**3*f**3
- 2686976*e**3*f*(19*d + 32*e + 52*f)**2 + 241901568*e**2*f**3*(1
9*d + 32*e + 52*f) - 30720*e**2*f*(19*d + 32*e + 52*f)**3 - 21886
889984*e*f**5 + 7558400*e*f**3*(19*d + 32*e + 52*f)**2 - 26559360
*f**5*(19*d + 32*e + 52*f) + 8856*f**3*(19*d + 32*e + 52*f)**3)/(
1675971*d**6 + 28507545*d**5*f - 66150400*d**4*e**2 + 168075324*d
**4*f**2 - 1091117056*d**3*e**2*f + 384095520*d**3*f**3 + 3187671
04*d**2*e**4 - 6528860160*d**2*e**2*f**2 + 162082944*d**2*f**4 +
3103784960*d*e**4*f - 17414619136*d*e**2*f**3 - 305130240*d*f**5
+ 6106906624*e**4*f**2 - 17414225920*e**2*f**4 + 67931136*f**6))/
864 - (8*e*x**2 - 20*e + x**3*(5*d + 8*f) + x*(-17*d - 20*f))/(72
*x**4 - 360*x**2 + 288)
```

GIAC/XCAS [A] time = 0.272902, size = 155, normalized size = 1.35

$$\frac{1}{864}(19d+52f-32e)\ln(|x+2|)-\frac{1}{108}(d+7f-4e)\ln(|x+1|)+\frac{1}{108}(d+7f+4e)\ln(|x-1|)-\frac{1}{864}(19d+52f+32e)\ln(|x-2|)-\frac{5dx^3+8fx^3+8x^2e-17dx-20fx-20e}{72(x^4-5x^2+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^2 + e*x + d)/(x^4 - 5*x^2 + 4)^2,x, algorithm="giac")
```

```
[Out] 1/864*(19*d + 52*f - 32*e)*ln(abs(x + 2)) - 1/108*(d + 7*f - 4*e)
*ln(abs(x + 1)) + 1/108*(d + 7*f + 4*e)*ln(abs(x - 1)) - 1/864*(1
9*d + 52*f + 32*e)*ln(abs(x - 2)) - 1/72*(5*d*x^3 + 8*f*x^3 + 8*x
^2*e - 17*d*x - 20*f*x - 20*e)/(x^4 - 5*x^2 + 4)
```

3.28 $\int \frac{d+ex+fx^2+gx^3}{(4-5x^2+x^4)^2}\,dx$

Optimal. Leaf size=138

$$\frac{x\left(x^2(-(5d+8f))+17d+20f\right)}{72\left(x^4-5x^2+4\right)}+\frac{1}{432}(19d+52f)\tanh^{-1}\left(\frac{x}{2}\right)-\frac{1}{54}(d+7f)\tanh^{-1}(x)+\frac{1}{54}(2e+5g)\log\left(1-x^2\right)-\frac{1}{54}(2e+5g)\log\left(4-x^2\right)+\frac{x^2(-(2e+5g))+5e+8g}{18\left(x^4-5x^2+4\right)}$$

```
[Out] (x*(17*d + 20*f - (5*d + 8*f)*x^2))/(72*(4 - 5*x^2 + x^4)) + (5*e
 + 8*g - (2*e + 5*g)*x^2)/(18*(4 - 5*x^2 + x^4)) + ((19*d + 52*f)
*ArcTanh[x/2])/432 - ((d + 7*f)*ArcTanh[x])/54 + ((2*e + 5*g)*Log
[1 - x^2])/54 - ((2*e + 5*g)*Log[4 - x^2])/54
```

Rubi [A] time = 0.351792, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$

$$\frac{x\left(x^2(-(5d+8f))+17d+20f\right)}{72\left(x^4-5x^2+4\right)}+\frac{1}{432}(19d+52f)\tanh^{-1}\left(\frac{x}{2}\right)-\frac{1}{54}(d+7f)\tanh^{-1}(x)+\frac{1}{54}(2e+5g)\log\left(1-x^2\right)-\frac{1}{54}(2e+5g)\log\left(4-x^2\right)+\frac{x^2(-(2e+5g))+5e+8g}{18\left(x^4-5x^2+4\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x + f*x^2 + g*x^3)/(4 - 5*x^2 + x^4)^2,x]
```

```
[Out] (x*(17*d + 20*f - (5*d + 8*f)*x^2))/(72*(4 - 5*x^2 + x^4)) + (5*e
 + 8*g - (2*e + 5*g)*x^2)/(18*(4 - 5*x^2 + x^4)) + ((19*d + 52*f)
*ArcTanh[x/2])/432 - ((d + 7*f)*ArcTanh[x])/54 + ((2*e + 5*g)*Log
[1 - x^2])/54 - ((2*e + 5*g)*Log[4 - x^2])/54
```

Rubi in Sympy [A] time = 45.9873, size = 105, normalized size = 0.76

$$\frac{x\left(17d+20f-x^3\left(5e+8g\right)-x^2\left(5d+8f\right)+x\left(17e+20g\right)\right)}{72\left(x^4-5x^2+4\right)}-\left(\frac{d}{54}+\frac{7f}{54}\right)\operatorname{atanh}(x)+\left(\frac{19d}{432}+\frac{13f}{108}\right)\operatorname{atanh}\left(\frac{x}{2}\right)+\left(\frac{e}{27}+\frac{5g}{54}\right)\log\left(-x^2+1\right)-\left(\frac{e}{27}+\frac{5g}{54}\right)\log\left(-x^2+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((g*x**3+f*x**2+e*x+d)/(x**4-5*x**2+4)**2,x)
```

```
[Out] x*(17*d + 20*f - x**3*(5*e + 8*g) - x**2*(5*d + 8*f) + x*(17*e +
20*g))/(72*(x**4 - 5*x**2 + 4)) - (d/54 + 7*f/54)*atanh(x) + (19*
d/432 + 13*f/108)*atanh(x/2) + (e/27 + 5*g/54)*log(-x**2 + 1) - (
e/27 + 5*g/54)*log(-x**2 + 4)
```

Mathematica [A] time = 0.101133, size = 134, normalized size = 0.97

$$\frac{1}{864}\left(\frac{12\left(-5dx^3+17dx+e\left(20-8x^2\right)-8fx^3+20fx-4g\left(5x^2-8\right)\right)}{x^4-5x^2+4}+8\log(1-x)(d+4e+7f+10g)-\log(2-x)(19d+32e+52f+80g)-8\log(x+1)(d-4e+7f-10g)+\log(x+2)(19d-32e+52f-80g)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x + f*x^2 + g*x^3)/(4 - 5*x^2 + x^4)^2,x]
```

```
[Out] ((12*(17*d*x + 20*f*x - 5*d*x^3 - 8*f*x^3 + e*(20 - 8*x^2) - 4*g*
(-8 + 5*x^2)))/(4 - 5*x^2 + x^4) + 8*(d + 4*e + 7*f + 10*g)*Log[1
 - x] - (19*d + 32*e + 52*f + 80*g)*Log[2 - x] - 8*(d - 4*e + 7*f
 - 10*g)*Log[1 + x] + (19*d - 32*e + 52*f - 80*g)*Log[2 + x])/864
```

Maple [A] time = 0.027, size = 242, normalized size = 1.8

$$\frac{g}{36+36x}-\frac{g}{-36+36x}+\frac{g}{36+18x}-\frac{g}{18x-36}-\frac{f}{36+36x}-\frac{d}{36+36x}+\frac{e}{36+36x}-\frac{d}{144x-288}-\frac{e}{72x-144}-\frac{f}{36x-72}-\frac{f}{-36+36x}-\frac{d}{288+144x}+\frac{e}{144+72x}-\frac{d}{-36+36x}-\frac{e}{-36+36x}-\frac{f}{72+36x}-\frac{\ln(1+x)d}{108}+\frac{\ln(1+x)e}{27}+\frac{\ln(-1+x)d}{108}+\frac{\ln(-1+x)e}{27}+\frac{5\ln(1+x)g}{54}-\frac{5\ln(x-2)g}{54}+\frac{5\ln(-1+x)g}{54}-\frac{5\ln(2+x)g}{54}-\frac{19\ln(x-2)d}{864}-\frac{\ln(x-2)e}{27}-\frac{\ln(2+x)e}{27}-\frac{13\ln(x-2)f}{216}+\frac{19\ln(2+x)d}{864}-\frac{7\ln(1+x)f}{108}+\frac{7\ln(-1+x)f}{108}+\frac{13\ln(2+x)f}{216}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x)
```

```
[Out] 1/36/(1+x)*g-1/36/(-1+x)*g+1/18/(2+x)*g-1/18/(x-2)*g-1/36/(1+x)*f
-1/36/(1+x)*d+1/36/(1+x)*e-1/144/(x-2)*d-1/72/(x-2)*e-1/36/(x-2)*
f-1/36/(-1+x)*f-1/144/(2+x)*d+1/72/(2+x)*e-1/36/(-1+x)*d-1/36/(-1
+x)*e-1/36/(2+x)*f-1/108*ln(1+x)*d+1/27*ln(1+x)*e+1/108*ln(-1+x)*
```

```
d+1/27*ln(-1+x)*e+5/54*ln(1+x)*g-5/54*ln(x-2)*g+5/54*ln(-1+x)*g-5
/54*ln(2+x)*g-19/864*ln(x-2)*d-1/27*ln(x-2)*e-1/27*ln(2+x)*e-13/2
16*ln(x-2)*f+19/864*ln(2+x)*d-7/108*ln(1+x)*f+7/108*ln(-1+x)*f+13
/216*ln(2+x)*f
```

Maxima [A] time = 0.703605, size = 171, normalized size = 1.24

$$\frac{1}{864}\,(19\,d - 32\,e + 52\,f - 80\,g)\log\left(x + 2\right) - \frac{1}{108}\,(d - 4\,e + 7\,f - 10\,g)\log\left(x + 1\right) + \frac{1}{108}\,(d + 4\,e + 7\,f + 10\,g)\log\left(x - 1\right) - \frac{1}{864}\,(19\,d + 32\,e + 52\,f + 80\,g)\log\left(x - 2\right) - \frac{(5\,d + 8\,f)x^3 + 4\,(2\,e + 5\,g)x^2 - (17\,d + 20\,f)x - 20\,e - 32\,g}{72\,(x^4 - 5\,x^2 + 4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3 + f*x^2 + e*x + d)/(x^4 - 5*x^2 + 4)^2,x, algorithm="maxima")
```

```
[Out] 1/864*(19*d - 32*e + 52*f - 80*g)*log(x + 2) - 1/108*(d - 4*e + 7
*f - 10*g)*log(x + 1) + 1/108*(d + 4*e + 7*f + 10*g)*log(x - 1) -
 1/864*(19*d + 32*e + 52*f + 80*g)*log(x - 2) - 1/72*((5*d + 8*f)
*x^3 + 4*(2*e + 5*g)*x^2 - (17*d + 20*f)*x - 20*e - 32*g)/(x^4 -
5*x^2 + 4)
```

Fricas [A] time = 0.561459, size = 354, normalized size = 2.57

$$-\frac{12\,(5\,d + 8\,f)x^3 + 48\,(2\,e + 5\,g)x^2 - 12\,(17\,d + 20\,f)x - \left((19\,d - 32\,e + 52\,f - 80\,g)x^4 - 5\,(19\,d - 32\,e + 52\,f - 80\,g)x^2 + 7\ldots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3 + f*x^2 + e*x + d)/(x^4 - 5*x^2 + 4)^2,x, algorithm="fricas")
```

```
[Out] -1/864*(12*(5*d + 8*f)*x^3 + 48*(2*e + 5*g)*x^2 - 12*(17*d + 20*f
)*x - ((19*d - 32*e + 52*f - 80*g)*x^4 - 5*(19*d - 32*e + 52*f -
80*g)*x^2 + 76*d - 128*e + 208*f - 320*g)*log(x + 2) + 8*((d - 4*
e + 7*f - 10*g)*x^4 - 5*(d - 4*e + 7*f - 10*g)*x^2 + 4*d - 16*e +
 28*f - 40*g)*log(x + 1) - 8*((d + 4*e + 7*f + 10*g)*x^4 - 5*(d +
 4*e + 7*f + 10*g)*x^2 + 4*d + 16*e + 28*f + 40*g)*log(x - 1) + (
(19*d + 32*e + 52*f + 80*g)*x^4 - 5*(19*d + 32*e + 52*f + 80*g)*x
^2 + 76*d + 128*e + 208*f + 320*g)*log(x - 2) - 240*e - 384*g)/(x
^4 - 5*x^2 + 4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((g*x**3+f*x**2+e*x+d)/(x**4-5*x**2+4)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.273555, size = 184, normalized size = 1.33

$$\frac{1}{864}\,(19\,d+52\,f-80\,g-32\,e)\ln\left(|x+2|\right)-\frac{1}{108}\,(d+7\,f-10\,g-4\,e)\ln\left(|x+1|\right)+\frac{1}{108}\,(d+7\,f+10\,g+4\,e)\ln\left(|x-1|\right)-\frac{1}{864}\,(19\,d+52\,f+80\,g+32\,e)\ln\left(|x-2|\right)-\frac{5\,dx^3+8\,fx^3+20\,gx^2+8\,x^2e-17\,dx-20\,fx-32\,g-20\,e}{72\,(x^4-5\,x^2+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3 + f*x^2 + e*x + d)/(x^4 - 5*x^2 + 4)^2,x, algorithm="giac")
```

```
[Out] 1/864*(19*d + 52*f - 80*g - 32*e)*ln(abs(x + 2)) - 1/108*(d + 7*f
 - 10*g - 4*e)*ln(abs(x + 1)) + 1/108*(d + 7*f + 10*g + 4*e)*ln(a
bs(x - 1)) - 1/864*(19*d + 52*f + 80*g + 32*e)*ln(abs(x - 2)) - 1
/72*(5*d*x^3 + 8*f*x^3 + 20*g*x^2 + 8*x^2*e - 17*d*x - 20*f*x - 3
2*g - 20*e)/(x^4 - 5*x^2 + 4)
```

3.29 $\int \frac{d+ex+fx^2+gx^3+hx^4}{(4-5x^2+x^4)^2}\,dx$

Optimal. Leaf size=150

$$\frac{x\left(x^2(-(5d+8f+20h))+17d+20f+32h\right)}{72\left(x^4-5x^2+4\right)}+\frac{1}{432}\tanh^{-1}\left(\frac{x}{2}\right)(19d+52f+112h)$$
$$-\frac{1}{54}\tanh^{-1}(x)(d+7f+13h)+\frac{1}{54}(2e+5g)\log\left(1-x^2\right)$$
$$-\frac{1}{54}(2e+5g)\log\left(4-x^2\right)+\frac{x^2(-(2e+5g))+5e+8g}{18\left(x^4-5x^2+4\right)}$$

```
[Out] (5*e + 8*g - (2*e + 5*g)*x^2)/(18*(4 - 5*x^2 + x^4)) + (x*(17*d +
 20*f + 32*h - (5*d + 8*f + 20*h)*x^2))/(72*(4 - 5*x^2 + x^4)) +
((19*d + 52*f + 112*h)*ArcTanh[x/2])/432 - ((d + 7*f + 13*h)*ArcT
anh[x])/54 + ((2*e + 5*g)*Log[1 - x^2])/54 - ((2*e + 5*g)*Log[4 -
 x^2])/54
```

Rubi [A] time = 0.431328, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242

$$\frac{x\left(x^2(-(5d+8f+20h))+17d+20f+32h\right)}{72\left(x^4-5x^2+4\right)}+\frac{1}{432}\tanh^{-1}\left(\frac{x}{2}\right)(19d+52f+112h)$$
$$-\frac{1}{54}\tanh^{-1}(x)(d+7f+13h)+\frac{1}{54}(2e+5g)\log\left(1-x^2\right)$$
$$-\frac{1}{54}(2e+5g)\log\left(4-x^2\right)+\frac{x^2(-(2e+5g))+5e+8g}{18\left(x^4-5x^2+4\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x + f*x^2 + g*x^3 + h*x^4)/(4 - 5*x^2 + x^4)^2,x]
```

```
[Out] (5*e + 8*g - (2*e + 5*g)*x^2)/(18*(4 - 5*x^2 + x^4)) + (x*(17*d +
 20*f + 32*h - (5*d + 8*f + 20*h)*x^2))/(72*(4 - 5*x^2 + x^4)) +
((19*d + 52*f + 112*h)*ArcTanh[x/2])/432 - ((d + 7*f + 13*h)*ArcT
anh[x])/54 + ((2*e + 5*g)*Log[1 - x^2])/54 - ((2*e + 5*g)*Log[4 -
 x^2])/54
```

Rubi in Sympy [A] time = 58.549, size = 122, normalized size = 0.81

$$\frac{x\left(425d+500f+800h-x^3\left(125e+200g\right)-x^2\left(125d+200f+500h\right)+x\left(425e+500g\right)\right)}{1800\left(x^4-5x^2+4\right)} + \left(\frac{e}{27}+\frac{5g}{54}\right)\log\left(-x^2+1\right)-\left(\frac{e}{27}+\frac{5g}{54}\right)\log\left(-x^2+4\right) - \left(\frac{d}{54}+\frac{7f}{54}+\frac{13h}{54}\right)\operatorname{atanh}(x)+\left(\frac{19d}{432}+\frac{13f}{108}+\frac{7h}{27}\right)\operatorname{atanh}\left(\frac{x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((h*x**4+g*x**3+f*x**2+e*x+d)/(x**4-5*x**2+4)**2,x)
```

```
[Out] x*(425*d + 500*f + 800*h - x**3*(125*e + 200*g) - x**2*(125*d + 2
00*f + 500*h) + x*(425*e + 500*g))/(1800*(x**4 - 5*x**2 + 4)) + (
e/27 + 5*g/54)*log(-x**2 + 1) - (e/27 + 5*g/54)*log(-x**2 + 4) -
(d/54 + 7*f/54 + 13*h/54)*atanh(x) + (19*d/432 + 13*f/108 + 7*h/2
7)*atanh(x/2)
```

Mathematica [A] time = 0.140405, size = 159, normalized size = 1.06

$$\frac{1}{864}\left(-\frac{12\left(x\left(d\left(5x^2-17\right)+4f\left(2x^2-5\right)+4h\left(5x^2-8\right)\right)+4e\left(2x^2-5\right)+4g\left(5x^2-8\right)\right)}{x^4-5x^2+4} + 8\log(1-x)(d+4e+7f+10g+13h)-\log(2-x)(19d+32e+52f+80g+112h) - 8\log(x+1)(d-4e+7f-10g+13h)+\log(x+2)(19d-32e+52f-80g+112h)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x + f*x^2 + g*x^3 + h*x^4)/(4 - 5*x^2 + x^4)^2,x]
```

```
[Out] ((-12*(4*e*(-5 + 2*x^2) + 4*g*(-8 + 5*x^2) + x*(4*f*(-5 + 2*x^2)
+ d*(-17 + 5*x^2) + 4*h*(-8 + 5*x^2))))/(4 - 5*x^2 + x^4) + 8*(d
+ 4*e + 7*f + 10*g + 13*h)*Log[1 - x] - (19*d + 32*e + 52*f + 80*
g + 112*h)*Log[2 - x] - 8*(d - 4*e + 7*f - 10*g + 13*h)*Log[1 + x
] + (19*d - 32*e + 52*f - 80*g + 112*h)*Log[2 + x])/864
```

Maple [B] time = 0.028, size = 302, normalized size = 2.

$$-\frac{h}{9x-18}-\frac{h}{36+36x}-\frac{h}{-36+36x}-\frac{h}{18+9x}+\frac{g}{36+36x}-\frac{g}{-36+36x}+\frac{g}{36+18x}-\frac{g}{18x-36}-\frac{f}{36+36x}-\frac{d}{36+36x}+\frac{e}{36+36x}-\frac{d}{144x-288}-\frac{e}{72x-144}-\frac{f}{36x-72}-\frac{f}{-36+36x}-\frac{d}{288+144x}+\frac{e}{144+72x}-\frac{d}{-36+36x}-\frac{e}{-36+36x}-\frac{f}{72+36x}-\frac{\ln(1+x)d}{108}+\frac{\ln(1+x)e}{27}+\frac{\ln(-1+x)d}{108}+\frac{\ln(-1+x)e}{27}-\frac{7\ln(x-2)h}{54}-\frac{13\ln(1+x)h}{108}+\frac{7\ln(2+x)h}{54}+\frac{13\ln(-1+x)h}{108}+\frac{5\ln(1+x)g}{54}-\frac{5\ln(x-2)g}{54}+\frac{5\ln(-1+x)g}{54}-\frac{5\ln(2+x)g}{54}-\frac{19\ln(x-2)d}{864}-\frac{\ln(x-2)e}{27}-\frac{\ln(2+x)e}{27}-\frac{13\ln(x-2)f}{216}+\frac{19\ln(2+x)d}{864}-\frac{7\ln(1+x)f}{108}+\frac{7\ln(-1+x)f}{108}+\frac{13\ln(2+x)f}{216}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x)
```

```
[Out] -1/9/(x-2)*h-1/36/(1+x)*h-1/36/(-1+x)*h-1/9/(2+x)*h+1/36/(1+x)*g-
1/36/(-1+x)*g+1/18/(2+x)*g-1/18/(x-2)*g-1/36/(1+x)*f-1/36/(1+x)*d
+1/36/(1+x)*e-1/144/(x-2)*d-1/72/(x-2)*e-1/36/(x-2)*f-1/36/(-1+x)
*f-1/144/(2+x)*d+1/72/(2+x)*e-1/36/(-1+x)*d-1/36/(-1+x)*e-1/36/(2
+x)*f-1/108*ln(1+x)*d+1/27*ln(1+x)*e+1/108*ln(-1+x)*d+1/27*ln(-1+
x)*e-7/54*ln(x-2)*h-13/108*ln(1+x)*h+7/54*ln(2+x)*h+13/108*ln(-1+
x)*h+5/54*ln(1+x)*g-5/54*ln(x-2)*g+5/54*ln(-1+x)*g-5/54*ln(2+x)*g
-19/864*ln(x-2)*d-1/27*ln(x-2)*e-1/27*ln(2+x)*e-13/216*ln(x-2)*f+
19/864*ln(2+x)*d-7/108*ln(1+x)*f+7/108*ln(-1+x)*f+13/216*ln(2+x)*
f
```

Maxima [A] time = 0.698831, size = 196, normalized size = 1.31

$$\frac{1}{864}(19d-32e+52f-80g+112h)\log(x+2)-\frac{1}{108}(d-4e+7f-10g+13h)\log(x+1)+\frac{1}{108}(d+4e+7f+10g+13h)\log(x-1)-\frac{1}{864}(19d+32e+52f+80g+112h)\log(x-2)-\frac{(5d+8f+20h)x^3+4(2e+5g)x^2-(17d+20f+32h)x-20e-32g}{72(x^4-5x^2+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((h*x^4 + g*x^3 + f*x^2 + e*x + d)/(x^4 - 5*x^2 + 4)^2,x, algorithm="maxi
```

```
[Out] 1/864*(19*d - 32*e + 52*f - 80*g + 112*h)*log(x + 2) - 1/108*(d -
 4*e + 7*f - 10*g + 13*h)*log(x + 1) + 1/108*(d + 4*e + 7*f + 10*
```

```
g + 13*h)*log(x - 1) - 1/864*(19*d + 32*e + 52*f + 80*g + 112*h)*
log(x - 2) - 1/72*((5*d + 8*f + 20*h)*x^3 + 4*(2*e + 5*g)*x^2 - (
17*d + 20*f + 32*h)*x - 20*e - 32*g)/(x^4 - 5*x^2 + 4)
```

Fricas [A] time = 1.76552, size = 410, normalized size = 2.73

$$-\frac{12\,(5\,d + 8\,f + 20\,h)x^3 + 48\,(2\,e + 5\,g)x^2 - 12\,(17\,d + 20\,f + 32\,h)x - \left((19\,d - 32\,e + 52\,f - 80\,g + 112\,h)x^4 - 5\,(19\,d - 32\,e\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((h*x^4 + g*x^3 + f*x^2 + e*x + d)/(x^4 - 5*x^2 + 4)^2,x, algorithm="frica
```

```
[Out] -1/864*(12*(5*d + 8*f + 20*h)*x^3 + 48*(2*e + 5*g)*x^2 - 12*(17*d
 + 20*f + 32*h)*x - ((19*d - 32*e + 52*f - 80*g + 112*h)*x^4 - 5*
(19*d - 32*e + 52*f - 80*g + 112*h)*x^2 + 76*d - 128*e + 208*f -
320*g + 448*h)*log(x + 2) + 8*((d - 4*e + 7*f - 10*g + 13*h)*x^4
- 5*(d - 4*e + 7*f - 10*g + 13*h)*x^2 + 4*d - 16*e + 28*f - 40*g
+ 52*h)*log(x + 1) - 8*((d + 4*e + 7*f + 10*g + 13*h)*x^4 - 5*(d
+ 4*e + 7*f + 10*g + 13*h)*x^2 + 4*d + 16*e + 28*f + 40*g + 52*h)
*log(x - 1) + ((19*d + 32*e + 52*f + 80*g + 112*h)*x^4 - 5*(19*d
+ 32*e + 52*f + 80*g + 112*h)*x^2 + 76*d + 128*e + 208*f + 320*g
+ 448*h)*log(x - 2) - 240*e - 384*g)/(x^4 - 5*x^2 + 4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((h*x**4+g*x**3+f*x**2+e*x+d)/(x**4-5*x**2+4)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.277695, size = 213, normalized size = 1.42

$$\frac{1}{864}\,(19\,d + 52\,f - 80\,g + 112\,h - 32\,e)\ln\left(|x + 2|\right) - \frac{1}{108}\,(d + 7\,f - 10\,g + 13\,h - 4\,e)\ln\left(|x + 1|\right) + \frac{1}{108}\,(d + 7\,f + 10\,g + 13\,h + 4\,e)\ln\left(|x - 1|\right) - \frac{1}{864}\,(19\,d + 52\,f + 80\,g + 112\,h + 32\,e)\ln\left(|x - 2|\right) - \frac{5\,dx^3 + 8\,fx^3 + 20\,hx^3 + 20\,gx^2 + 8\,x^2e - 17\,dx - 20\,fx - 32\,hx - 32\,g - 20\,e}{72\,(x^4 - 5\,x^2 + 4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((h*x^4 + g*x^3 + f*x^2 + e*x + d)/(x^4 - 5*x^2 + 4)^2,x, algorithm="giac
```

```
[Out] 1/864*(19*d + 52*f - 80*g + 112*h - 32*e)*ln(abs(x + 2)) - 1/108*
(d + 7*f - 10*g + 13*h - 4*e)*ln(abs(x + 1)) + 1/108*(d + 7*f + 1
0*g + 13*h + 4*e)*ln(abs(x - 1)) - 1/864*(19*d + 52*f + 80*g + 11
2*h + 32*e)*ln(abs(x - 2)) - 1/72*(5*d*x^3 + 8*f*x^3 + 20*h*x^3 +
 20*g*x^2 + 8*x^2*e - 17*d*x - 20*f*x - 32*h*x - 32*g - 20*e)/(x^
4 - 5*x^2 + 4)
```

3.30 $\int \frac{d+ex+fx^2+gx^3+hx^4+ix^5}{(4-5x^2+x^4)^2}\,dx$

Optimal. Leaf size=162

$$\frac{x\left(x^2(-(5d+8f+20h))+17d+20f+32h\right)}{72\left(x^4-5x^2+4\right)}+\frac{1}{432}\tanh^{-1}\left(\frac{x}{2}\right)(19d+52f+112h)$$
$$-\frac{1}{54}\tanh^{-1}(x)(d+7f+13h)+\frac{1}{54}\log\left(1-x^2\right)(2e+5g+8i)$$
$$-\frac{1}{54}\log\left(4-x^2\right)(2e+5g+8i)+\frac{x^2(-(2e+5g+17i))+5e+8g+20i}{18\left(x^4-5x^2+4\right)}$$

```
[Out] (x*(17*d + 20*f + 32*h - (5*d + 8*f + 20*h)*x^2))/(72*(4 - 5*x^2
+ x^4)) + (5*e + 8*g + 20*i - (2*e + 5*g + 17*i)*x^2)/(18*(4 - 5*
x^2 + x^4)) + ((19*d + 52*f + 112*h)*ArcTanh[x/2])/432 - ((d + 7*
f + 13*h)*ArcTanh[x])/54 + ((2*e + 5*g + 8*i)*Log[1 - x^2])/54 -
((2*e + 5*g + 8*i)*Log[4 - x^2])/54
```

Rubi [A] time = 0.470195, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.237

$$\frac{x\left(x^2(-(5d+8f+20h))+17d+20f+32h\right)}{72\left(x^4-5x^2+4\right)}+\frac{1}{432}\tanh^{-1}\left(\frac{x}{2}\right)(19d+52f+112h)$$
$$-\frac{1}{54}\tanh^{-1}(x)(d+7f+13h)+\frac{1}{54}\log\left(1-x^2\right)(2e+5g+8i)$$
$$-\frac{1}{54}\log\left(4-x^2\right)(2e+5g+8i)+\frac{x^2(-(2e+5g+17i))+5e+8g+20i}{18\left(x^4-5x^2+4\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5)/(4 - 5*x^2 + x^4)^2,x]
```


```
[Out] (x*(17*d + 20*f + 32*h - (5*d + 8*f + 20*h)*x^2))/(72*(4 - 5*x^2
+ x^4)) + (5*e + 8*g + 20*i - (2*e + 5*g + 17*i)*x^2)/(18*(4 - 5*
x^2 + x^4)) + ((19*d + 52*f + 112*h)*ArcTanh[x/2])/432 - ((d + 7*
f + 13*h)*ArcTanh[x])/54 + ((2*e + 5*g + 8*i)*Log[1 - x^2])/54 -
((2*e + 5*g + 8*i)*Log[4 - x^2])/54
```

Rubi in Sympy [A] time = 69.3777, size = 139, normalized size = 0.86

$$\frac{x\left(2125d+2500f+4000h-x^{3}\left(625e+1000g+2500\right)-x^{2}\left(625d+1000f+2500h\right)+x\left(2125e+2500g+4000\right)\right)}{9000\left(x^{4}-5x^{2}+4\right)}-\left(\frac{d}{54}+\frac{7f}{54}+\frac{13h}{54}\right)\operatorname{atanh}\left(x\right)+\left(\frac{19d}{432}+\frac{13f}{108}+\frac{7h}{27}\right)\operatorname{atanh}\left(\frac{x}{2}\right)+\left(\frac{e}{27}+\frac{5g}{54}-\frac{i}{6}+\frac{17}{54}\right)\log\left(-x^{2}+1\right)-\left(\frac{e}{27}+\frac{5g}{54}-\frac{i}{6}+\frac{17}{54}\right)\log\left(-x^{2}+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((i*x**5+h*x**4+g*x**3+f*x**2+e*x+d)/(x**4-5*x**2+4)**2,x)
```

```
[Out] x*(2125*d + 2500*f + 4000*h - x**3*(625*e + 1000*g + 2500) - x**2
*(625*d + 1000*f + 2500*h) + x*(2125*e + 2500*g + 4000))/(9000*(x
**4 - 5*x**2 + 4)) - (d/54 + 7*f/54 + 13*h/54)*atanh(x) + (19*d/4
32 + 13*f/108 + 7*h/27)*atanh(x/2) + (e/27 + 5*g/54 - i/6 + 17/54
)*log(-x**2 + 1) - (e/27 + 5*g/54 - i/6 + 17/54)*log(-x**2 + 4)
```

Mathematica [A] time = 0.188378, size = 185, normalized size = 1.14

$$\frac{-5dx^{3}+17dx-8ex^{2}+20e-8fx^{3}+20fx-20gx^{2}+32g-20hx^{3}+32hx-68ix^{2}+80i}{72\left(x^{4}-5x^{2}+4\right)}+\frac{1}{108}\log(1-x)(d+4e+7f+10g+13h+16i)+\frac{1}{864}\log(2-x)(-19d-32e-52f-80g-112h-128i)+\frac{1}{108}\log(x+1)(-d+4e-7f+10g-13h+16i)+\frac{1}{864}\log(x+2)(19d-32e+52f-80g+112h-128i)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5)/(4 - 5*x^2 + x^4)^2,x]
```

```
[Out] (20*e + 32*g + 80*i + 17*d*x + 20*f*x + 32*h*x - 8*e*x^2 - 20*g*x
^2 - 68*i*x^2 - 5*d*x^3 - 8*f*x^3 - 20*h*x^3)/(72*(4 - 5*x^2 + x^
4)) + ((d + 4*e + 7*f + 10*g + 13*h + 16*i)*Log[1 - x])/108 + ((-
19*d - 32*e - 52*f - 80*g - 112*h - 128*i)*Log[2 - x])/864 + ((-d
 + 4*e - 7*f + 10*g - 13*h + 16*i)*Log[1 + x])/108 + ((19*d - 32*
e + 52*f - 80*g + 112*h - 128*i)*Log[2 + x])/864
```

Maple [B] time = 0.028, size = 362, normalized size = 2.2

$$-\frac{2i}{9x-18}+\frac{i}{36+36x}-\frac{i}{-36+36x}+\frac{2i}{18+9x}-\frac{h}{9x-18}-\frac{h}{36+36x}-\frac{h}{-36+36x}-\frac{h}{18+9x}+\frac{g}{36+36x}-\frac{g}{-36+36x}+\frac{g}{36+18x}-\frac{g}{18x-36}-\frac{f}{36+36x}-\frac{d}{36+36x}+\frac{e}{36+36x}-\frac{d}{144x-288}-\frac{e}{72x-144}-\frac{f}{36x-72}-\frac{f}{-36+36x}-\frac{d}{288+144x}+\frac{e}{144+72x}-\frac{d}{-36+36x}-\frac{e}{-36+36x}-\frac{f}{72+36x}-\frac{\ln(1+x)\,d}{108}+\frac{\ln(1+x)\,e}{27}+\frac{\ln(-1+x)\,d}{108}+\frac{\ln(-1+x)\,e}{27}-\frac{4\ln(x-2)\,i}{27}+\frac{4\ln(1+x)\,i}{27}+\frac{4\ln(-1+x)\,i}{27}-\frac{4\ln(2+x)\,i}{27}-\frac{7\ln(x-2)\,h}{54}-\frac{13\ln(1+x)\,h}{108}+\frac{7\ln(2+x)\,h}{54}+\frac{13\ln(-1+x)\,h}{108}+\frac{5\ln(1+x)\,g}{54}-\frac{5\ln(x-2)\,g}{54}+\frac{5\ln(-1+x)\,g}{54}-\frac{5\ln(2+x)\,g}{54}-\frac{19\ln(x-2)\,d}{864}-\frac{\ln(x-2)\,e}{27}-\frac{\ln(2+x)\,e}{27}-\frac{13\ln(x-2)\,f}{216}+\frac{19\ln(2+x)\,d}{864}-\frac{7\ln(1+x)\,f}{108}+\frac{7\ln(-1+x)\,f}{108}+\frac{13\ln(2+x)\,f}{216}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int((i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x)
```

```
[Out] -2/9/(x-2)*i+1/36/(1+x)*i-1/36/(-1+x)*i+2/9/(2+x)*i-1/9/(x-2)*h-1
/36/(1+x)*h-1/36/(-1+x)*h-1/9/(2+x)*h+1/36/(1+x)*g-1/36/(-1+x)*g+
1/18/(2+x)*g-1/18/(x-2)*g-1/36/(1+x)*f-1/36/(1+x)*d+1/36/(1+x)*e-
1/144/(x-2)*d-1/72/(x-2)*e-1/36/(x-2)*f-1/36/(-1+x)*f-1/144/(2+x)
*d+1/72/(2+x)*e-1/36/(-1+x)*d-1/36/(-1+x)*e-1/36/(2+x)*f-1/108*ln
(1+x)*d+1/27*ln(1+x)*e+1/108*ln(-1+x)*d+1/27*ln(-1+x)*e-4/27*ln(x
-2)*i+4/27*ln(1+x)*i+4/27*ln(-1+x)*i-4/27*ln(2+x)*i-7/54*ln(x-2)*
h-13/108*ln(1+x)*h+7/54*ln(2+x)*h+13/108*ln(-1+x)*h+5/54*ln(1+x)*
g-5/54*ln(x-2)*g+5/54*ln(-1+x)*g-5/54*ln(2+x)*g-19/864*ln(x-2)*d-
1/27*ln(x-2)*e-1/27*ln(2+x)*e-13/216*ln(x-2)*f+19/864*ln(2+x)*d-7
/108*ln(1+x)*f+7/108*ln(-1+x)*f+13/216*ln(2+x)*f
```

Maxima [A] time = 0.710376, size = 220, normalized size = 1.36

$$\frac{1}{864}\,(19\,d-32\,e+52\,f-80\,g+112\,h-128\,i)\log(x+2)-\frac{1}{108}\,(d-4\,e+7\,f-10\,g+13\,h-16\,i)\log(x+1)+\frac{1}{108}\,(d+4\,e+7\,f+10\,g+13\,h+16\,i)\log(x-1)-\frac{1}{864}\,(19\,d+32\,e+52\,f+80\,g+112\,h+128\,i)\log(x-2)-\frac{(5\,d+8\,f+20\,h)x^3+4\,(2\,e+5\,g+17\,i)x^2-(17\,d+20\,f+32\,h)x-20\,e-32\,g-80\,i}{72\,(x^4-5\,x^2+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^5 + h*x^4 + g*x^3 + f*x^2 + e*x + d)/(x^4 - 5*x^2 + 4)^2,x, algorith
```

```
[Out] 1/864*(19*d - 32*e + 52*f - 80*g + 112*h - 128*i)*log(x + 2) - 1/
108*(d - 4*e + 7*f - 10*g + 13*h - 16*i)*log(x + 1) + 1/108*(d +
4*e + 7*f + 10*g + 13*h + 16*i)*log(x - 1) - 1/864*(19*d + 32*e +
 52*f + 80*g + 112*h + 128*i)*log(x - 2) - 1/72*((5*d + 8*f + 20*
h)*x^3 + 4*(2*e + 5*g + 17*i)*x^2 - (17*d + 20*f + 32*h)*x - 20*e
 - 32*g - 80*i)/(x^4 - 5*x^2 + 4)
```

Fricas [A] time = 8.51822, size = 467, normalized size = 2.88

$$-\frac{12\,(5\,d+8\,f+20\,h)x^3+48\,(2\,e+5\,g+17\,i)x^2-12\,(17\,d+20\,f+32\,h)x-\big((19\,d-32\,e+52\,f-80\,g+112\,h-128\,i)x^4-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^5 + h*x^4 + g*x^3 + f*x^2 + e*x + d)/(x^4 - 5*x^2 + 4)^2,x, algorith
```

```
[Out] -1/864*(12*(5*d + 8*f + 20*h)*x^3 + 48*(2*e + 5*g + 17*i)*x^2 - 1
2*(17*d + 20*f + 32*h)*x - ((19*d - 32*e + 52*f - 80*g + 112*h -
128*i)*x^4 - 5*(19*d - 32*e + 52*f - 80*g + 112*h - 128*i)*x^2 +
76*d - 128*e + 208*f - 320*g + 448*h - 512*i)*log(x + 2) + 8*((d
- 4*e + 7*f - 10*g + 13*h - 16*i)*x^4 - 5*(d - 4*e + 7*f - 10*g +
 13*h - 16*i)*x^2 + 4*d - 16*e + 28*f - 40*g + 52*h - 64*i)*log(x
 + 1) - 8*((d + 4*e + 7*f + 10*g + 13*h + 16*i)*x^4 - 5*(d + 4*e
+ 7*f + 10*g + 13*h + 16*i)*x^2 + 4*d + 16*e + 28*f + 40*g + 52*h
 + 64*i)*log(x - 1) + ((19*d + 32*e + 52*f + 80*g + 112*h + 128*i
)*x^4 - 5*(19*d + 32*e + 52*f + 80*g + 112*h + 128*i)*x^2 + 76*d
+ 128*e + 208*f + 320*g + 448*h + 512*i)*log(x - 2) - 240*e - 384
*g - 960*i)/(x^4 - 5*x^2 + 4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x**5+h*x**4+g*x**3+f*x**2+e*x+d)/(x**4-5*x**2+4)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.275813, size = 242, normalized size = 1.49

$$\frac{1}{864}\left(19\,d + 52\,f - 80\,g + 112\,h - 128\,i - 32\,e\right)\ln\left(|x + 2|\right) - \frac{1}{108}\left(d + 7\,f - 10\,g + 13\,h - 16\,i - 4\,e\right)\ln\left(|x + 1|\right) + \frac{1}{108}\left(d + 7\,f + 10\,g + 13\,h + 16\,i + 4\,e\right)\ln\left(|x - 1|\right) - \frac{1}{864}\left(19\,d + 52\,f + 80\,g + 112\,h + 128\,i + 32\,e\right)\ln\left(|x - 2|\right) - \frac{5\,dx^3 + 8\,fx^3 + 20\,hx^3 + 20\,gx^2 + 68\,ix^2 + 8\,x^2e - 17\,dx - 20\,fx - 32\,hx - 32\,g - 80\,i - 20\,e}{72\,(x^4 - 5\,x^2 + 4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^5 + h*x^4 + g*x^3 + f*x^2 + e*x + d)/(x^4 - 5*x^2 + 4)^2,x, algorith
```

```
[Out] 1/864*(19*d + 52*f - 80*g + 112*h - 128*i - 32*e)*ln(abs(x + 2))
- 1/108*(d + 7*f - 10*g + 13*h - 16*i - 4*e)*ln(abs(x + 1)) + 1/1
08*(d + 7*f + 10*g + 13*h + 16*i + 4*e)*ln(abs(x - 1)) - 1/864*(1
9*d + 52*f + 80*g + 112*h + 128*i + 32*e)*ln(abs(x - 2)) - 1/72*(
5*d*x^3 + 8*f*x^3 + 20*h*x^3 + 20*g*x^2 + 68*i*x^2 + 8*x^2*e - 17
*d*x - 20*f*x - 32*h*x - 32*g - 80*i - 20*e)/(x^4 - 5*x^2 + 4)
```

3.31 $\int \frac{d+ex}{(1+x^2+x^4)^2}\,dx$

Optimal. Leaf size=140

$$-\frac{1}{4}d\log\left(x^2-x+1\right)+\frac{1}{4}d\log\left(x^2+x+1\right)+\frac{dx\left(1-x^2\right)}{6\left(x^4+x^2+1\right)}-\frac{d\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{d\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{2e\tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{e\left(2x^2+1\right)}{6\left(x^4+x^2+1\right)}$$

```
[Out] (d*x*(1 - x^2))/(6*(1 + x^2 + x^4)) + (e*(1 + 2*x^2))/(6*(1 + x^2
 + x^4)) - (d*ArcTan[(1 - 2*x)/Sqrt[3]])/(3*Sqrt[3]) + (d*ArcTan[
(1 + 2*x)/Sqrt[3]])/(3*Sqrt[3]) + (2*e*ArcTan[(1 + 2*x^2)/Sqrt[3]
])/(3*Sqrt[3]) - (d*Log[1 - x + x^2])/4 + (d*Log[1 + x + x^2])/4
```

Rubi [A] time = 0.218017, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 10, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625

$$-\frac{1}{4}d\log\left(x^2-x+1\right)+\frac{1}{4}d\log\left(x^2+x+1\right)+\frac{dx\left(1-x^2\right)}{6\left(x^4+x^2+1\right)}-\frac{d\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{d\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{2e\tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{e\left(2x^2+1\right)}{6\left(x^4+x^2+1\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x)/(1 + x^2 + x^4)^2,x]
```

```
[Out] (d*x*(1 - x^2))/(6*(1 + x^2 + x^4)) + (e*(1 + 2*x^2))/(6*(1 + x^2
 + x^4)) - (d*ArcTan[(1 - 2*x)/Sqrt[3]])/(3*Sqrt[3]) + (d*ArcTan[
(1 + 2*x)/Sqrt[3]])/(3*Sqrt[3]) + (2*e*ArcTan[(1 + 2*x^2)/Sqrt[3]
])/(3*Sqrt[3]) - (d*Log[1 - x + x^2])/4 + (d*Log[1 + x + x^2])/4
```

Rubi in Sympy [A] time = 39.6818, size = 126, normalized size = 0.9

$$-\frac{d\log\left(x^2-x+1\right)}{4}+\frac{d\log\left(x^2+x+1\right)}{4}+\frac{\sqrt{3}d\,\mathrm{atan}\left(\sqrt{3}\left(\frac{2x}{3}-\frac{1}{3}\right)\right)}{9}+\frac{\sqrt{3}d\,\mathrm{atan}\left(\sqrt{3}\left(\frac{2x}{3}+\frac{1}{3}\right)\right)}{9}+\frac{2\sqrt{3}e\,\mathrm{atan}\left(\sqrt{3}\left(\frac{2x^2}{3}+\frac{1}{3}\right)\right)}{9}+\frac{x\left(-dx^2+d-ex^3+ex\right)}{6\left(x^4+x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x+d)/(x**4+x**2+1)**2,x)
```

```
[Out] -d*log(x**2 - x + 1)/4 + d*log(x**2 + x + 1)/4 + sqrt(3)*d*atan(s
qrt(3)*(2*x/3 - 1/3))/9 + sqrt(3)*d*atan(sqrt(3)*(2*x/3 + 1/3))/9
 + 2*sqrt(3)*e*atan(sqrt(3)*(2*x**2/3 + 1/3))/9 + x*(-d*x**2 + d
- e*x**3 + e*x)/(6*(x**4 + x**2 + 1))
```

Mathematica [C]   time = 0.986055, size = 146, normalized size = 1.04

$$\frac{d\left(x-x^3\right)+2ex^2+e}{6\left(x^4+x^2+1\right)}-\frac{\left(\sqrt{3}-11i\right)d\tan^{-1}\left(\frac{1}{2}\left(\sqrt{3}-i\right)x\right)}{6\sqrt{6+6i\sqrt{3}}}-\frac{\left(\sqrt{3}+11i\right)d\tan^{-1}\left(\frac{1}{2}\left(\sqrt{3}+i\right)x\right)}{6\sqrt{6-6i\sqrt{3}}}-\frac{2e\tan^{-1}\left(\frac{\sqrt{3}}{2x^2+1}\right)}{3\sqrt{3}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(d + e*x)/(1 + x^2 + x^4)^2,x]
```

```
[Out] (e + 2*e*x^2 + d*(x - x^3))/(6*(1 + x^2 + x^4)) - ((-11*I + Sqrt[
3])*d*ArcTan[((-I + Sqrt[3])*x)/2])/(6*Sqrt[6 + (6*I)*Sqrt[3]]) -
 ((11*I + Sqrt[3])*d*ArcTan[((I + Sqrt[3])*x)/2])/(6*Sqrt[6 - (6*
I)*Sqrt[3]]) - (2*e*ArcTan[Sqrt[3]/(1 + 2*x^2)])/(3*Sqrt[3])
```

Maple [A]   time = 0.016, size = 146, normalized size = 1.

$$\frac{1}{4x^2+4x+4}\left(\left(-\frac{d}{3}-\frac{e}{3}\right)x-\frac{2d}{3}+\frac{e}{3}\right)+\frac{d\ln\left(x^2+x+1\right)}{4}+\frac{d\sqrt{3}}{9}\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)-\frac{2\sqrt{3}e}{9}\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)-\frac{1}{4x^2-4x+4}\left(\left(\frac{d}{3}-\frac{e}{3}\right)x-\frac{2d}{3}-\frac{e}{3}\right)-\frac{d\ln\left(x^2-x+1\right)}{4}+\frac{d\sqrt{3}}{9}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)+\frac{2\sqrt{3}e}{9}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x+d)/(x^4+x^2+1)^2,x)
```

```
[Out] 1/4*((-1/3*d-1/3*e)*x-2/3*d+1/3*e)/(x^2+x+1)+1/4*d*ln(x^2+x+1)+1/
9*d*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)-2/9*3^(1/2)*arctan(1/3*(1
+2*x)*3^(1/2))*e-1/4*((1/3*d-1/3*e)*x-2/3*d-1/3*e)/(x^2-x+1)-1/4*
d*ln(x^2-x+1)+1/9*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))*d+2/9*3^(1/
```

```
2)*arctan(1/3*(2*x-1)*3^(1/2))*e
```

Maxima [A] time = 0.777721, size = 130, normalized size = 0.93

$$\frac{1}{9}\sqrt{3}(d-2e)\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{9}\sqrt{3}(d+2e)\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{4}d\log\left(x^2+x+1\right)-\frac{1}{4}d\log\left(x^2-x+1\right)-\frac{dx^3-2ex^2-dx-e}{6\left(x^4+x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x + d)/(x^4 + x^2 + 1)^2,x, algorithm="maxima")
```

```
[Out] 1/9*sqrt(3)*(d - 2*e)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/9*sqrt(3)
*(d + 2*e)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/4*d*log(x^2 + x + 1)
 - 1/4*d*log(x^2 - x + 1) - 1/6*(d*x^3 - 2*e*x^2 - d*x - e)/(x^4
+ x^2 + 1)
```

Fricas [A] time = 0.273321, size = 219, normalized size = 1.56

$$\frac{\sqrt{3}\left(3\sqrt{3}\left(dx^4+dx^2+d\right)\log\left(x^2+x+1\right)-3\sqrt{3}\left(dx^4+dx^2+d\right)\log\left(x^2-x+1\right)+4\left((d-2e)x^4+(d-2e)x^2+d-2e\right)\text{ar}\right.}{36\left(x^4+x^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x + d)/(x^4 + x^2 + 1)^2,x, algorithm="fricas")
```

```
[Out] 1/36*sqrt(3)*(3*sqrt(3)*(d*x^4 + d*x^2 + d)*log(x^2 + x + 1) - 3*
sqrt(3)*(d*x^4 + d*x^2 + d)*log(x^2 - x + 1) + 4*((d - 2*e)*x^4 +
 (d - 2*e)*x^2 + d - 2*e)*arctan(1/3*sqrt(3)*(2*x + 1)) + 4*((d +
 2*e)*x^4 + (d + 2*e)*x^2 + d + 2*e)*arctan(1/3*sqrt(3)*(2*x - 1)
) - 2*sqrt(3)*(d*x^3 - 2*e*x^2 - d*x - e))/(x^4 + x^2 + 1)
```

Sympy [A] time = 8.57345, size = 952, normalized size = 6.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x+d)/(x**4+x**2+1)**2,x)
```

```
[Out] (-d/4 - sqrt(3)*I*(d + 2*e)/18)*log(x + (-10309*d**4*e + 1026*d**
4*(-d/4 - sqrt(3)*I*(d + 2*e)/18) - 7200*d**2*e**3 - 31536*d**2*e
**2*(-d/4 - sqrt(3)*I*(d + 2*e)/18) + 108432*d**2*e*(-d/4 - sqrt(
3)*I*(d + 2*e)/18)**2 + 163296*d**2*(-d/4 - sqrt(3)*I*(d + 2*e)/1
8)**3 + 1792*e**5 + 11520*e**4*(-d/4 - sqrt(3)*I*(d + 2*e)/18) +
48384*e**3*(-d/4 - sqrt(3)*I*(d + 2*e)/18)**2 + 311040*e**2*(-d/4
 - sqrt(3)*I*(d + 2*e)/18)**3)/(3348*d**5 - 11408*d**3*e**2 - 793
6*d*e**4)) + (-d/4 + sqrt(3)*I*(d + 2*e)/18)*log(x + (-10309*d**4
*e + 1026*d**4*(-d/4 + sqrt(3)*I*(d + 2*e)/18) - 7200*d**2*e**3 -
 31536*d**2*e**2*(-d/4 + sqrt(3)*I*(d + 2*e)/18) + 108432*d**2*e*
(-d/4 + sqrt(3)*I*(d + 2*e)/18)**2 + 163296*d**2*(-d/4 + sqrt(3)*
I*(d + 2*e)/18)**3 + 1792*e**5 + 11520*e**4*(-d/4 + sqrt(3)*I*(d
+ 2*e)/18) + 48384*e**3*(-d/4 + sqrt(3)*I*(d + 2*e)/18)**2 + 3110
40*e**2*(-d/4 + sqrt(3)*I*(d + 2*e)/18)**3)/(3348*d**5 - 11408*d*
*3*e**2 - 7936*d*e**4)) + (d/4 - sqrt(3)*I*(d - 2*e)/18)*log(x +
(-10309*d**4*e + 1026*d**4*(d/4 - sqrt(3)*I*(d - 2*e)/18) - 7200*
d**2*e**3 - 31536*d**2*e**2*(d/4 - sqrt(3)*I*(d - 2*e)/18) + 1084
32*d**2*e*(d/4 - sqrt(3)*I*(d - 2*e)/18)**2 + 163296*d**2*(d/4 -
sqrt(3)*I*(d - 2*e)/18)**3 + 1792*e**5 + 11520*e**4*(d/4 - sqrt(3
)*I*(d - 2*e)/18) + 48384*e**3*(d/4 - sqrt(3)*I*(d - 2*e)/18)**2
+ 311040*e**2*(d/4 - sqrt(3)*I*(d - 2*e)/18)**3)/(3348*d**5 - 114
08*d**3*e**2 - 7936*d*e**4)) + (d/4 + sqrt(3)*I*(d - 2*e)/18)*log
(x + (-10309*d**4*e + 1026*d**4*(d/4 + sqrt(3)*I*(d - 2*e)/18) -
7200*d**2*e**3 - 31536*d**2*e**2*(d/4 + sqrt(3)*I*(d - 2*e)/18) +
 108432*d**2*e*(d/4 + sqrt(3)*I*(d - 2*e)/18)**2 + 163296*d**2*(d
/4 + sqrt(3)*I*(d - 2*e)/18)**3 + 1792*e**5 + 11520*e**4*(d/4 + s
qrt(3)*I*(d - 2*e)/18) + 48384*e**3*(d/4 + sqrt(3)*I*(d - 2*e)/18
)**2 + 311040*e**2*(d/4 + sqrt(3)*I*(d - 2*e)/18)**3)/(3348*d**5
- 11408*d**3*e**2 - 7936*d*e**4)) - (d*x**3 - d*x - 2*e*x**2 - e)
/(6*x**4 + 6*x**2 + 6)
```

GIAC/XCAS [A] time = 0.273584, size = 135, normalized size = 0.96

$$\frac{1}{9}\sqrt{3}(d-2e)\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{9}\sqrt{3}(d+2e)\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{4}d\ln\left(x^2+x+1\right)-\frac{1}{4}d\ln\left(x^2-x+1\right)-\frac{dx^3-2x^2e-dx-e}{6\left(x^4+x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x + d)/(x^4 + x^2 + 1)^2,x, algorithm="giac")
```

```
[Out] 1/9*sqrt(3)*(d - 2*e)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/9*sqrt(3)
*(d + 2*e)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/4*d*ln(x^2 + x + 1)
- 1/4*d*ln(x^2 - x + 1) - 1/6*(d*x^3 - 2*x^2*e - d*x - e)/(x^4 +
x^2 + 1)
```

3.32 $\int \frac{d+ex+fx^2}{(1+x^2+x^4)^2}\,dx$

Optimal. Leaf size=165

$$-\frac{1}{8}(2d-f)\log\left(x^2-x+1\right)+\frac{1}{8}(2d-f)\log\left(x^2+x+1\right)+\frac{x\left(x^2(-(d-2f))+d+f\right)}{6\left(x^4+x^2+1\right)}-\frac{(4d+f)\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{12\sqrt{3}}+\frac{(4d+f)\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{12\sqrt{3}}+\frac{2e\tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{e\left(2x^2+1\right)}{6\left(x^4+x^2+1\right)}$$

```
[Out] (e*(1 + 2*x^2))/(6*(1 + x^2 + x^4)) + (x*(d + f - (d - 2*f)*x^2))
/(6*(1 + x^2 + x^4)) - ((4*d + f)*ArcTan[(1 - 2*x)/Sqrt[3]])/(12*
Sqrt[3]) + ((4*d + f)*ArcTan[(1 + 2*x)/Sqrt[3]])/(12*Sqrt[3]) + (
2*e*ArcTan[(1 + 2*x^2)/Sqrt[3]])/(3*Sqrt[3]) - ((2*d - f)*Log[1 -
 x + x^2])/8 + ((2*d - f)*Log[1 + x + x^2])/8
```

Rubi [A] time = 0.296925, antiderivative size = 165, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476

$$-\frac{1}{8}(2d-f)\log\left(x^2-x+1\right)+\frac{1}{8}(2d-f)\log\left(x^2+x+1\right)+\frac{x\left(x^2(-(d-2f))+d+f\right)}{6\left(x^4+x^2+1\right)}-\frac{(4d+f)\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{12\sqrt{3}}+\frac{(4d+f)\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{12\sqrt{3}}+\frac{2e\tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{e\left(2x^2+1\right)}{6\left(x^4+x^2+1\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x + f*x^2)/(1 + x^2 + x^4)^2,x]
```

```
[Out] (e*(1 + 2*x^2))/(6*(1 + x^2 + x^4)) + (x*(d + f - (d - 2*f)*x^2))
/(6*(1 + x^2 + x^4)) - ((4*d + f)*ArcTan[(1 - 2*x)/Sqrt[3]])/(12*
Sqrt[3]) + ((4*d + f)*ArcTan[(1 + 2*x)/Sqrt[3]])/(12*Sqrt[3]) + (
2*e*ArcTan[(1 + 2*x^2)/Sqrt[3]])/(3*Sqrt[3]) - ((2*d - f)*Log[1 -
 x + x^2])/8 + ((2*d - f)*Log[1 + x + x^2])/8
```

Rubi in Sympy [A] time = 52.5985, size = 144, normalized size = 0.87

$$\frac{2\sqrt{3}e\,\mathrm{atan}\left(\sqrt{3}\left(\frac{2x^2}{3}+\frac{1}{3}\right)\right)}{9}+\frac{x\left(d-ex^3+ex+f-x^2(d-2f)\right)}{6\left(x^4+x^2+1\right)}-\left(\frac{d}{4}-\frac{f}{8}\right)\log\left(x^2-x+1\right)+\left(\frac{d}{4}-\frac{f}{8}\right)\log\left(x^2+x+1\right)+\frac{\sqrt{3}(4d+f)\,\mathrm{atan}\left(\sqrt{3}\left(\frac{2x}{3}-\frac{1}{3}\right)\right)}{36}+\frac{\sqrt{3}(4d+f)\,\mathrm{atan}\left(\sqrt{3}\left(\frac{2x}{3}+\frac{1}{3}\right)\right)}{36}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**2+e*x+d)/(x**4+x**2+1)**2,x)
```

```
[Out] 2*sqrt(3)*e*atan(sqrt(3)*(2*x**2/3 + 1/3))/9 + x*(d - e*x**3 + e*
x + f - x**2*(d - 2*f))/(6*(x**4 + x**2 + 1)) - (d/4 - f/8)*log(x
**2 - x + 1) + (d/4 - f/8)*log(x**2 + x + 1) + sqrt(3)*(4*d + f)*
atan(sqrt(3)*(2*x/3 - 1/3))/36 + sqrt(3)*(4*d + f)*atan(sqrt(3)*(
2*x/3 + 1/3))/36
```

Mathematica [C] time = 0.828077, size = 186, normalized size = 1.13

$$\frac{1}{36}\left(\frac{6\left(x\left(-dx^2+d+2fx^2+f\right)+2ex^2+e\right)}{x^4+x^2+1}-\frac{\left(\left(\sqrt{3}-11i\right)d-2\left(\sqrt{3}-2i\right)f\right)\tan^{-1}\left(\frac{1}{2}\left(\sqrt{3}-i\right)x\right)}{\sqrt{\frac{1}{6}\left(1+i\sqrt{3}\right)}}-\frac{\left(\left(\sqrt{3}+11i\right)d-2\left(\sqrt{3}+2i\right)f\right)\tan^{-1}\left(\frac{1}{2}\left(\sqrt{3}+i\right)x\right)}{\sqrt{\frac{1}{6}\left(1-i\sqrt{3}\right)}}-8\sqrt{3}e\tan^{-1}\left(\frac{\sqrt{3}}{2x^2+1}\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(d + e*x + f*x^2)/(1 + x^2 + x^4)^2,x]
```

```
[Out] ((6*(e + 2*e*x^2 + x*(d + f - d*x^2 + 2*f*x^2)))/(1 + x^2 + x^4)
- (((-11*I + Sqrt[3])*d - 2*(-2*I + Sqrt[3])*f)*ArcTan[((-I + Sqr
t[3])*x)/2])/Sqrt[(1 + I*Sqrt[3])/6] - (((11*I + Sqrt[3])*d - 2*(
2*I + Sqrt[3])*f)*ArcTan[((I + Sqrt[3])*x)/2])/Sqrt[(1 - I*Sqrt[3
])/6] - 8*Sqrt[3]*e*ArcTan[Sqrt[3]/(1 + 2*x^2)])/36
```

Maple [A] time = 0.016, size = 214, normalized size = 1.3

$$\frac{1}{4x^2+4x+4}\left(\left(-\frac{d}{3}-\frac{e}{3}+\frac{2f}{3}\right)x-\frac{2d}{3}+\frac{e}{3}+\frac{f}{3}\right)+\frac{d\ln\left(x^2+x+1\right)}{4}-\frac{\ln\left(x^2+x+1\right)f}{8}+\frac{d\sqrt{3}}{9}\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)-\frac{2\sqrt{3}e}{9}\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)+\frac{\sqrt{3}f}{36}\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)-\frac{1}{4x^2-4x+4}\left(\left(\frac{d}{3}-\frac{e}{3}-\frac{2f}{3}\right)x-\frac{2d}{3}-\frac{e}{3}+\frac{f}{3}\right)-\frac{d\ln\left(x^2-x+1\right)}{4}+\frac{\ln\left(x^2-x+1\right)f}{8}+\frac{d\sqrt{3}}{9}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)+\frac{2\sqrt{3}e}{9}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)+\frac{\sqrt{3}f}{36}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x^2+e*x+d)/(x^4+x^2+1)^2,x)
```

```
[Out] 1/4*((-1/3*d-1/3*e+2/3*f)*x-2/3*d+1/3*e+1/3*f)/(x^2+x+1)+1/4*d*ln
(x^2+x+1)-1/8*ln(x^2+x+1)*f+1/9*d*arctan(1/3*(1+2*x)*3^(1/2))*3^(
1/2)-2/9*3^(1/2)*arctan(1/3*(1+2*x)*3^(1/2))*e+1/36*3^(1/2)*arcta
n(1/3*(1+2*x)*3^(1/2))*f-1/4*((1/3*d-1/3*e-2/3*f)*x-2/3*d-1/3*e+1
/3*f)/(x^2-x+1)-1/4*d*ln(x^2-x+1)+1/8*ln(x^2-x+1)*f+1/9*3^(1/2)*a
rctan(1/3*(2*x-1)*3^(1/2))*d+2/9*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/
2))*e+1/36*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))*f
```

Maxima [A] time = 0.781035, size = 162, normalized size = 0.98

$$\frac{1}{36}\sqrt{3}(4d-8e+f)\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{36}\sqrt{3}(4d+8e+f)\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{8}(2d-f)\log\left(x^2+x+1\right)-\frac{1}{8}(2d-f)\log\left(x^2-x+1\right)-\frac{(d-2f)x^3-2ex^2-(d+f)x-e}{6(x^4+x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^2 + e*x + d)/(x^4 + x^2 + 1)^2,x, algorithm="maxima")
```

```
[Out] 1/36*sqrt(3)*(4*d - 8*e + f)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/36
*sqrt(3)*(4*d + 8*e + f)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/8*(2*d
 - f)*log(x^2 + x + 1) - 1/8*(2*d - f)*log(x^2 - x + 1) - 1/6*((d
 - 2*f)*x^3 - 2*e*x^2 - (d + f)*x - e)/(x^4 + x^2 + 1)
```

Fricas [A] time = 0.311506, size = 297, normalized size = 1.8

$$\sqrt{3}\left(3\sqrt{3}\left((2d-f)x^4+(2d-f)x^2+2d-f\right)\log\left(x^2+x+1\right)-3\sqrt{3}\left((2d-f)x^4+(2d-f)x^2+2d-f\right)\log\left(x^2-x+1\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^2 + e*x + d)/(x^4 + x^2 + 1)^2,x, algorithm="fricas")
```

```
[Out] 1/72*sqrt(3)*(3*sqrt(3)*((2*d - f)*x^4 + (2*d - f)*x^2 + 2*d - f)
*log(x^2 + x + 1) - 3*sqrt(3)*((2*d - f)*x^4 + (2*d - f)*x^2 + 2*
d - f)*log(x^2 - x + 1) + 2*((4*d - 8*e + f)*x^4 + (4*d - 8*e + f
)*x^2 + 4*d - 8*e + f)*arctan(1/3*sqrt(3)*(2*x + 1)) + 2*((4*d +
8*e + f)*x^4 + (4*d + 8*e + f)*x^2 + 4*d + 8*e + f)*arctan(1/3*sq
rt(3)*(2*x - 1)) - 4*sqrt(3)*((d - 2*f)*x^3 - 2*e*x^2 - (d + f)*x
 - e))/(x^4 + x^2 + 1)
```

Sympy [A] time = 91.8248, size = 4107, normalized size = 24.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x**2+e*x+d)/(x**4+x**2+1)**2,x)
```

```
[Out] (-d/4 + f/8 - sqrt(3)*I*(4*d + 8*e + f)/72)*log(x + (-164944*d**5
*e + 16416*d**5*(-d/4 + f/8 - sqrt(3)*I*(4*d + 8*e + f)/72) + 336
520*d**4*e*f + 200664*d**4*f*(-d/4 + f/8 - sqrt(3)*I*(4*d + 8*e +
 f)/72) - 115200*d**3*e**3 - 504576*d**3*e**2*(-d/4 + f/8 - sqrt(
3)*I*(4*d + 8*e + f)/72) - 272380*d**3*e*f**2 + 1734912*d**3*e*(-
d/4 + f/8 - sqrt(3)*I*(4*d + 8*e + f)/72)**2 - 229500*d**3*f**2*(
-d/4 + f/8 - sqrt(3)*I*(4*d + 8*e + f)/72) + 2612736*d**3*(-d/4 +
 f/8 - sqrt(3)*I*(4*d + 8*e + f)/72)**3 + 51840*d**2*e**3*f + 881
280*d**2*e**2*f*(-d/4 + f/8 - sqrt(3)*I*(4*d + 8*e + f)/72) + 119
420*d**2*e*f**3 - 2477952*d**2*e*f*(-d/4 + f/8 - sqrt(3)*I*(4*d +
 8*e + f)/72)**2 + 50436*d**2*f**3*(-d/4 + f/8 - sqrt(3)*I*(4*d +
 8*e + f)/72) - 2519424*d**2*f*(-d/4 + f/8 - sqrt(3)*I*(4*d + 8*e
 + f)/72)**3 + 28672*d*e**5 + 184320*d*e**4*(-d/4 + f/8 - sqrt(3)
*I*(4*d + 8*e + f)/72) + 8640*d*e**3*f**2 + 774144*d*e**3*(-d/4 +
 f/8 - sqrt(3)*I*(4*d + 8*e + f)/72)**2 - 409536*d*e**2*f**2*(-d/
4 + f/8 - sqrt(3)*I*(4*d + 8*e + f)/72) + 4976640*d*e**2*(-d/4 +
f/8 - sqrt(3)*I*(4*d + 8*e + f)/72)**3 - 31040*d*e*f**4 + 1270080
*d*e*f**2*(-d/4 + f/8 - sqrt(3)*I*(4*d + 8*e + f)/72)**2 + 14040*
d*f**4*(-d/4 + f/8 - sqrt(3)*I*(4*d + 8*e + f)/72) + 139968*d*f**
2*(-d/4 + f/8 - sqrt(3)*I*(4*d + 8*e + f)/72)**3 - 20480*e**5*f -
 36864*e**4*f*(-d/4 + f/8 - sqrt(3)*I*(4*d + 8*e + f)/72) - 2880*
e**3*f**3 - 552960*e**3*f*(-d/4 + f/8 - sqrt(3)*I*(4*d + 8*e + f)
/72)**2 + 70848*e**2*f**3*(-d/4 + f/8 - sqrt(3)*I*(4*d + 8*e + f)
/72) - 995328*e**2*f*(-d/4 + f/8 - sqrt(3)*I*(4*d + 8*e + f)/72)*
*3 + 3956*e*f**5 - 209088*e*f**3*(-d/4 + f/8 - sqrt(3)*I*(4*d + 8
*e + f)/72)**2 - 3996*f**5*(-d/4 + f/8 - sqrt(3)*I*(4*d + 8*e + f
)/72) + 233280*f**3*(-d/4 + f/8 - sqrt(3)*I*(4*d + 8*e + f)/72)**
3)/(53568*d**6 - 69984*d**5*f - 182528*d**4*e**2 + 23652*d**4*f**
```

```
2 + 377344*d**3*e**2*f + 5400*d**3*f**3 - 126976*d**2*e**4 - 2784
00*d**2*e**2*f**2 - 4131*d**2*f**4 + 102400*d*e**4*f + 93568*d*e*
*2*f**3 + 81*d*f**5 - 28672*e**4*f**2 - 11648*e**2*f**4 + 189*f**
6)) + (-d/4 + f/8 + sqrt(3)*I*(4*d + 8*e + f)/72)*log(x + (-16494
4*d**5*e + 16416*d**5*(-d/4 + f/8 + sqrt(3)*I*(4*d + 8*e + f)/72)
 + 336520*d**4*e*f + 200664*d**4*f*(-d/4 + f/8 + sqrt(3)*I*(4*d +
 8*e + f)/72) - 115200*d**3*e**3 - 504576*d**3*e**2*(-d/4 + f/8 +
 sqrt(3)*I*(4*d + 8*e + f)/72) - 272380*d**3*e*f**2 + 1734912*d**
3*e*(-d/4 + f/8 + sqrt(3)*I*(4*d + 8*e + f)/72)**2 - 229500*d**3*
f**2*(-d/4 + f/8 + sqrt(3)*I*(4*d + 8*e + f)/72) + 2612736*d**3*(
-d/4 + f/8 + sqrt(3)*I*(4*d + 8*e + f)/72)**3 + 51840*d**2*e**3*f
 + 881280*d**2*e**2*f*(-d/4 + f/8 + sqrt(3)*I*(4*d + 8*e + f)/72)
 + 119420*d**2*e*f**3 - 2477952*d**2*e*f*(-d/4 + f/8 + sqrt(3)*I*
(4*d + 8*e + f)/72)**2 + 50436*d**2*f**3*(-d/4 + f/8 + sqrt(3)*I*
(4*d + 8*e + f)/72) - 2519424*d**2*f*(-d/4 + f/8 + sqrt(3)*I*(4*d
 + 8*e + f)/72)**3 + 28672*d*e**5 + 184320*d*e**4*(-d/4 + f/8 + s
qrt(3)*I*(4*d + 8*e + f)/72) + 8640*d*e**3*f**2 + 774144*d*e**3*(
-d/4 + f/8 + sqrt(3)*I*(4*d + 8*e + f)/72)**2 - 409536*d*e**2*f**
2*(-d/4 + f/8 + sqrt(3)*I*(4*d + 8*e + f)/72) + 4976640*d*e**2*(-
d/4 + f/8 + sqrt(3)*I*(4*d + 8*e + f)/72)**3 - 31040*d*e*f**4 + 1
270080*d*e*f**2*(-d/4 + f/8 + sqrt(3)*I*(4*d + 8*e + f)/72)**2 +
14040*d*f**4*(-d/4 + f/8 + sqrt(3)*I*(4*d + 8*e + f)/72) + 139968
*d*f**2*(-d/4 + f/8 + sqrt(3)*I*(4*d + 8*e + f)/72)**3 - 20480*e*
*5*f - 36864*e**4*f*(-d/4 + f/8 + sqrt(3)*I*(4*d + 8*e + f)/72) -
 2880*e**3*f**3 - 552960*e**3*f*(-d/4 + f/8 + sqrt(3)*I*(4*d + 8*
e + f)/72)**2 + 70848*e**2*f**3*(-d/4 + f/8 + sqrt(3)*I*(4*d + 8*
e + f)/72) - 995328*e**2*f*(-d/4 + f/8 + sqrt(3)*I*(4*d + 8*e + f
)/72)**3 + 3956*e*f**5 - 209088*e*f**3*(-d/4 + f/8 + sqrt(3)*I*(4
*d + 8*e + f)/72)**2 - 3996*f**5*(-d/4 + f/8 + sqrt(3)*I*(4*d + 8
*e + f)/72) + 233280*f**3*(-d/4 + f/8 + sqrt(3)*I*(4*d + 8*e + f)
/72)**3)/(53568*d**6 - 69984*d**5*f - 182528*d**4*e**2 + 23652*d*
*4*f**2 + 377344*d**3*e**2*f + 5400*d**3*f**3 - 126976*d**2*e**4
- 278400*d**2*e**2*f**2 - 4131*d**2*f**4 + 102400*d*e**4*f + 9356
8*d*e**2*f**3 + 81*d*f**5 - 28672*e**4*f**2 - 11648*e**2*f**4 + 1
89*f**6)) + (d/4 - f/8 - sqrt(3)*I*(4*d - 8*e + f)/72)*log(x + (-
164944*d**5*e + 16416*d**5*(d/4 - f/8 - sqrt(3)*I*(4*d - 8*e + f)
/72) + 336520*d**4*e*f + 200664*d**4*f*(d/4 - f/8 - sqrt(3)*I*(4*
d - 8*e + f)/72) - 115200*d**3*e**3 - 504576*d**3*e**2*(d/4 - f/8
 - sqrt(3)*I*(4*d - 8*e + f)/72) - 272380*d**3*e*f**2 + 1734912*d
**3*e*(d/4 - f/8 - sqrt(3)*I*(4*d - 8*e + f)/72)**2 - 229500*d**3
*f**2*(d/4 - f/8 - sqrt(3)*I*(4*d - 8*e + f)/72) + 2612736*d**3*(
d/4 - f/8 - sqrt(3)*I*(4*d - 8*e + f)/72)**3 + 51840*d**2*e**3*f
+ 881280*d**2*e**2*f*(d/4 - f/8 - sqrt(3)*I*(4*d - 8*e + f)/72) +
 119420*d**2*e*f**3 - 2477952*d**2*e*f*(d/4 - f/8 - sqrt(3)*I*(4*
d - 8*e + f)/72)**2 + 50436*d**2*f**3*(d/4 - f/8 - sqrt(3)*I*(4*d
 - 8*e + f)/72) - 2519424*d**2*f*(d/4 - f/8 - sqrt(3)*I*(4*d - 8*
e + f)/72)**3 + 28672*d*e**5 + 184320*d*e**4*(d/4 - f/8 - sqrt(3)
*I*(4*d - 8*e + f)/72) + 8640*d*e**3*f**2 + 774144*d*e**3*(d/4 -
f/8 - sqrt(3)*I*(4*d - 8*e + f)/72)**2 - 409536*d*e**2*f**2*(d/4
- f/8 - sqrt(3)*I*(4*d - 8*e + f)/72) + 4976640*d*e**2*(d/4 - f/8
 - sqrt(3)*I*(4*d - 8*e + f)/72)**3 - 31040*d*e*f**4 + 1270080*d*
e*f**2*(d/4 - f/8 - sqrt(3)*I*(4*d - 8*e + f)/72)**2 + 14040*d*f*
*4*(d/4 - f/8 - sqrt(3)*I*(4*d - 8*e + f)/72) + 139968*d*f**2*(d/
4 - f/8 - sqrt(3)*I*(4*d - 8*e + f)/72)**3 - 20480*e**5*f - 36864
```

```
*e**4*f*(d/4 - f/8 - sqrt(3)*I*(4*d - 8*e + f)/72) - 2880*e**3*f*
*3 - 552960*e**3*f*(d/4 - f/8 - sqrt(3)*I*(4*d - 8*e + f)/72)**2
+ 70848*e**2*f**3*(d/4 - f/8 - sqrt(3)*I*(4*d - 8*e + f)/72) - 99
5328*e**2*f*(d/4 - f/8 - sqrt(3)*I*(4*d - 8*e + f)/72)**3 + 3956*
e*f**5 - 209088*e*f**3*(d/4 - f/8 - sqrt(3)*I*(4*d - 8*e + f)/72)
**2 - 3996*f**5*(d/4 - f/8 - sqrt(3)*I*(4*d - 8*e + f)/72) + 2332
80*f**3*(d/4 - f/8 - sqrt(3)*I*(4*d - 8*e + f)/72)**3)/(53568*d**
6 - 69984*d**5*f - 182528*d**4*e**2 + 23652*d**4*f**2 + 377344*d*
*3*e**2*f + 5400*d**3*f**3 - 126976*d**2*e**4 - 278400*d**2*e**2*
f**2 - 4131*d**2*f**4 + 102400*d*e**4*f + 93568*d*e**2*f**3 + 81*
d*f**5 - 28672*e**4*f**2 - 11648*e**2*f**4 + 189*f**6)) + (d/4 -
f/8 + sqrt(3)*I*(4*d - 8*e + f)/72)*log(x + (-164944*d**5*e + 164
16*d**5*(d/4 - f/8 + sqrt(3)*I*(4*d - 8*e + f)/72) + 336520*d**4*
e*f + 200664*d**4*f*(d/4 - f/8 + sqrt(3)*I*(4*d - 8*e + f)/72) -
115200*d**3*e**3 - 504576*d**3*e**2*(d/4 - f/8 + sqrt(3)*I*(4*d -
 8*e + f)/72) - 272380*d**3*e*f**2 + 1734912*d**3*e*(d/4 - f/8 +
sqrt(3)*I*(4*d - 8*e + f)/72)**2 - 229500*d**3*f**2*(d/4 - f/8 +
sqrt(3)*I*(4*d - 8*e + f)/72) + 2612736*d**3*(d/4 - f/8 + sqrt(3)
*I*(4*d - 8*e + f)/72)**3 + 51840*d**2*e**3*f + 881280*d**2*e**2*
f*(d/4 - f/8 + sqrt(3)*I*(4*d - 8*e + f)/72) + 119420*d**2*e*f**3
 - 2477952*d**2*e*f*(d/4 - f/8 + sqrt(3)*I*(4*d - 8*e + f)/72)**2
 + 50436*d**2*f**3*(d/4 - f/8 + sqrt(3)*I*(4*d - 8*e + f)/72) - 2
519424*d**2*f*(d/4 - f/8 + sqrt(3)*I*(4*d - 8*e + f)/72)**3 + 286
72*d*e**5 + 184320*d*e**4*(d/4 - f/8 + sqrt(3)*I*(4*d - 8*e + f)/
72) + 8640*d*e**3*f**2 + 774144*d*e**3*(d/4 - f/8 + sqrt(3)*I*(4*
d - 8*e + f)/72)**2 - 409536*d*e**2*f**2*(d/4 - f/8 + sqrt(3)*I*(
4*d - 8*e + f)/72) + 4976640*d*e**2*(d/4 - f/8 + sqrt(3)*I*(4*d -
 8*e + f)/72)**3 - 31040*d*e*f**4 + 1270080*d*e*f**2*(d/4 - f/8 +
 sqrt(3)*I*(4*d - 8*e + f)/72)**2 + 14040*d*f**4*(d/4 - f/8 + sqr
t(3)*I*(4*d - 8*e + f)/72) + 139968*d*f**2*(d/4 - f/8 + sqrt(3)*I
*(4*d - 8*e + f)/72)**3 - 20480*e**5*f - 36864*e**4*f*(d/4 - f/8
+ sqrt(3)*I*(4*d - 8*e + f)/72) - 2880*e**3*f**3 - 552960*e**3*f*
(d/4 - f/8 + sqrt(3)*I*(4*d - 8*e + f)/72)**2 + 70848*e**2*f**3*(
d/4 - f/8 + sqrt(3)*I*(4*d - 8*e + f)/72) - 995328*e**2*f*(d/4 -
f/8 + sqrt(3)*I*(4*d - 8*e + f)/72)**3 + 3956*e*f**5 - 209088*e*f
**3*(d/4 - f/8 + sqrt(3)*I*(4*d - 8*e + f)/72)**2 - 3996*f**5*(d/
4 - f/8 + sqrt(3)*I*(4*d - 8*e + f)/72) + 233280*f**3*(d/4 - f/8
+ sqrt(3)*I*(4*d - 8*e + f)/72)**3)/(53568*d**6 - 69984*d**5*f -
182528*d**4*e**2 + 23652*d**4*f**2 + 377344*d**3*e**2*f + 5400*d*
*3*f**3 - 126976*d**2*e**4 - 278400*d**2*e**2*f**2 - 4131*d**2*f*
*4 + 102400*d*e**4*f + 93568*d*e**2*f**3 + 81*d*f**5 - 28672*e**4
*f**2 - 11648*e**2*f**4 + 189*f**6)) - (-2*e*x**2 - e + x**3*(d -
 2*f) + x*(-d - f))/(6*x**4 + 6*x**2 + 6)
```

GIAC/XCAS [A] time = 0.280608, size = 173, normalized size = 1.05

$$\frac{1}{36}\sqrt{3}(4\,d+f-8\,e)\arctan\left(\frac{1}{3}\sqrt{3}(2\,x+1)\right)+\frac{1}{36}\sqrt{3}(4\,d+f+8\,e)\arctan\left(\frac{1}{3}\sqrt{3}(2\,x-1)\right)+\frac{1}{8}(2\,d-f)\ln\left(x^2+x+1\right)-\frac{1}{8}(2\,d-f)\ln\left(x^2-x+1\right)-\frac{dx^3-2\,fx^3-2\,x^2e-dx-fx-e}{6\,(x^4+x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^2 + e*x + d)/(x^4 + x^2 + 1)^2,x, algorithm="giac")
```

```
[Out] 1/36*sqrt(3)*(4*d + f - 8*e)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/36
*sqrt(3)*(4*d + f + 8*e)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/8*(2*d
 - f)*ln(x^2 + x + 1) - 1/8*(2*d - f)*ln(x^2 - x + 1) - 1/6*(d*x^
3 - 2*f*x^3 - 2*x^2*e - d*x - f*x - e)/(x^4 + x^2 + 1)
```

3.33 $\int \frac{d+ex+fx^2+gx^3}{(1+x^2+x^4)^2} dx$

Optimal. Leaf size=179

$$-\frac{1}{8}(2d-f)\log\left(x^2-x+1\right)+\frac{1}{8}(2d-f)\log\left(x^2+x+1\right)+\frac{x\left(x^2(-(d-2f))+d+f\right)}{6\left(x^4+x^2+1\right)}-\frac{(4d+f)\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{12\sqrt{3}}+\frac{(4d+f)\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{12\sqrt{3}}+\frac{(2e-g)\tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{x^2(2e-g)+e-2g}{6\left(x^4+x^2+1\right)}$$

```
[Out] (x*(d + f - (d - 2*f)*x^2))/(6*(1 + x^2 + x^4)) + (e - 2*g + (2*e
 - g)*x^2)/(6*(1 + x^2 + x^4)) - ((4*d + f)*ArcTan[(1 - 2*x)/Sqrt
[3]])/(12*Sqrt[3]) + ((4*d + f)*ArcTan[(1 + 2*x)/Sqrt[3]])/(12*Sq
rt[3]) + ((2*e - g)*ArcTan[(1 + 2*x^2)/Sqrt[3]])/(3*Sqrt[3]) - ((
2*d - f)*Log[1 - x + x^2])/8 + ((2*d - f)*Log[1 + x + x^2])/8
```

Rubi [A] time = 0.347542, antiderivative size = 179, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346

$$-\frac{1}{8}(2d-f)\log\left(x^2-x+1\right)+\frac{1}{8}(2d-f)\log\left(x^2+x+1\right)+\frac{x\left(x^2(-(d-2f))+d+f\right)}{6\left(x^4+x^2+1\right)}-\frac{(4d+f)\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{12\sqrt{3}}+\frac{(4d+f)\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{12\sqrt{3}}+\frac{(2e-g)\tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{x^2(2e-g)+e-2g}{6\left(x^4+x^2+1\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x + f*x^2 + g*x^3)/(1 + x^2 + x^4)^2,x]
```

```
[Out] (x*(d + f - (d - 2*f)*x^2))/(6*(1 + x^2 + x^4)) + (e - 2*g + (2*e
 - g)*x^2)/(6*(1 + x^2 + x^4)) - ((4*d + f)*ArcTan[(1 - 2*x)/Sqrt
[3]])/(12*Sqrt[3]) + ((4*d + f)*ArcTan[(1 + 2*x)/Sqrt[3]])/(12*Sq
rt[3]) + ((2*e - g)*ArcTan[(1 + 2*x^2)/Sqrt[3]])/(3*Sqrt[3]) - ((
2*d - f)*Log[1 - x + x^2])/8 + ((2*d - f)*Log[1 + x + x^2])/8
```

Rubi in Sympy [A] time = 59.4685, size = 151, normalized size = 0.84

$$\frac{x\left(d+f-x^3(e-2g)-x^2(d-2f)+x(e+g)\right)}{6(x^4+x^2+1)}-\left(\frac{d}{4}-\frac{f}{8}\right)\log\left(x^2-x+1\right)+\left(\frac{d}{4}-\frac{f}{8}\right)\log\left(x^2+x+1\right)+\frac{\sqrt{3}(4d+f)\,\mathrm{atan}\left(\sqrt{3}\left(\frac{2x}{3}-\frac{1}{3}\right)\right)}{36}+\frac{\sqrt{3}(4d+f)\,\mathrm{atan}\left(\sqrt{3}\left(\frac{2x}{3}+\frac{1}{3}\right)\right)}{36}+\frac{\sqrt{3}(2e-g)\,\mathrm{atan}\left(\sqrt{3}\left(\frac{2x^2}{3}+\frac{1}{3}\right)\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((g*x**3+f*x**2+e*x+d)/(x**4+x**2+1)**2,x)
```

```
[Out] x*(d + f - x**3*(e - 2*g) - x**2*(d - 2*f) + x*(e + g))/(6*(x**4
+ x**2 + 1)) - (d/4 - f/8)*log(x**2 - x + 1) + (d/4 - f/8)*log(x*
*2 + x + 1) + sqrt(3)*(4*d + f)*atan(sqrt(3)*(2*x/3 - 1/3))/36 +
sqrt(3)*(4*d + f)*atan(sqrt(3)*(2*x/3 + 1/3))/36 + sqrt(3)*(2*e -
 g)*atan(sqrt(3)*(2*x**2/3 + 1/3))/9
```

Mathematica [C] time = 0.979685, size = 200, normalized size = 1.12

$$\frac{1}{36}\left(\frac{6\left(x\left(-dx^2+d+2fx^2+f\right)+2ex^2+e-g\left(x^2+2\right)\right)}{x^4+x^2+1}-\frac{\left(\left(\sqrt{3}-11i\right)d-2\left(\sqrt{3}-2i\right)f\right)\tan^{-1}\left(\frac{1}{2}\left(\sqrt{3}-i\right)x\right)}{\sqrt{\frac{1}{6}\left(1+i\sqrt{3}\right)}}-\frac{\left(\left(\sqrt{3}+11i\right)d-2\left(\sqrt{3}+2i\right)f\right)\tan^{-1}\left(\frac{1}{2}\left(\sqrt{3}+i\right)x\right)}{\sqrt{\frac{1}{6}\left(1-i\sqrt{3}\right)}}-4\sqrt{3}(2e-g)\tan^{-1}\left(\frac{\sqrt{3}}{2x^2+1}\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(d + e*x + f*x^2 + g*x^3)/(1 + x^2 + x^4)^2,x]
```

```
[Out] ((6*(e + 2*e*x^2 - g*(2 + x^2) + x*(d + f - d*x^2 + 2*f*x^2)))/(1
 + x^2 + x^4) - (((-11*I + Sqrt[3])*d - 2*(-2*I + Sqrt[3])*f)*Arc
Tan[((-I + Sqrt[3])*x)/2])/Sqrt[(1 + I*Sqrt[3])/6] - (((11*I + Sq
rt[3])*d - 2*(2*I + Sqrt[3])*f)*ArcTan[((I + Sqrt[3])*x)/2])/Sqrt
```

```
[(1 - I*Sqrt[3])/6] - 4*Sqrt[3]*(2*e - g)*ArcTan[Sqrt[3]/(1 + 2*x
^2)])/36
```

Maple [A] time = 0.016, size = 260, normalized size = 1.5

$$\frac{1}{4x^2+4x+4}\left(\left(-\frac{d}{3}-\frac{e}{3}-\frac{g}{3}+\frac{2f}{3}\right)x-\frac{2d}{3}+\frac{e}{3}-\frac{2g}{3}+\frac{f}{3}\right)+\frac{d\ln\left(x^2+x+1\right)}{4}-\frac{\ln\left(x^2+x+1\right)f}{8}+\frac{d\sqrt{3}}{9}\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)-\frac{2\sqrt{3}e}{9}\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)+\frac{\sqrt{3}f}{36}\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)+\frac{\sqrt{3}g}{9}\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)-\frac{1}{4x^2-4x+4}\left(\left(\frac{d}{3}-\frac{e}{3}-\frac{g}{3}-\frac{2f}{3}\right)x-\frac{2d}{3}-\frac{e}{3}+\frac{2g}{3}+\frac{f}{3}\right)-\frac{d\ln\left(x^2-x+1\right)}{4}+\frac{\ln\left(x^2-x+1\right)f}{8}+\frac{d\sqrt{3}}{9}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)+\frac{2\sqrt{3}e}{9}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)+\frac{\sqrt{3}f}{36}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)-\frac{\sqrt{3}g}{9}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int((g*x^3+f*x^2+e*x+d)/(x^4+x^2+1)^2,x)
```

```
[Out] 1/4*((-1/3*d-1/3*e-1/3*g+2/3*f)*x-2/3*d+1/3*e-2/3*g+1/3*f)/(x^2+x
+1)+1/4*d*ln(x^2+x+1)-1/8*ln(x^2+x+1)*f+1/9*d*arctan(1/3*(1+2*x)*
3^(1/2))*3^(1/2)-2/9*3^(1/2)*arctan(1/3*(1+2*x)*3^(1/2))*e+1/36*3
^(1/2)*arctan(1/3*(1+2*x)*3^(1/2))*f+1/9*3^(1/2)*arctan(1/3*(1+2*
x)*3^(1/2))*g-1/4*((1/3*d-1/3*e-1/3*g-2/3*f)*x-2/3*d-1/3*e+2/3*g+
1/3*f)/(x^2-x+1)-1/4*d*ln(x^2-x+1)+1/8*ln(x^2-x+1)*f+1/9*3^(1/2)*
arctan(1/3*(2*x-1)*3^(1/2))*d+2/9*3^(1/2)*arctan(1/3*(2*x-1)*3^(1
/2))*e+1/36*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))*f-1/9*3^(1/2)*arc
tan(1/3*(2*x-1)*3^(1/2))*g
```

Maxima [A] time = 0.780339, size = 182, normalized size = 1.02

$$\frac{1}{36}\sqrt{3}(4d-8e+f+4g)\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{36}\sqrt{3}(4d+8e+f-4g)\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{8}(2d-f)\log\left(x^2+x+1\right)-\frac{1}{8}(2d-f)\log\left(x^2-x+1\right)-\frac{(d-2f)x^3-(2e-g)x^2-(d+f)x-e+2g}{6\left(x^4+x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((g*x^3 + f*x^2 + e*x + d)/(x^4 + x^2 + 1)^2,x, algorithm="maxima")
```

```
[Out] 1/36*sqrt(3)*(4*d - 8*e + f + 4*g)*arctan(1/3*sqrt(3)*(2*x + 1))
+ 1/36*sqrt(3)*(4*d + 8*e + f - 4*g)*arctan(1/3*sqrt(3)*(2*x - 1)
) + 1/8*(2*d - f)*log(x^2 + x + 1) - 1/8*(2*d - f)*log(x^2 - x +
1) - 1/6*((d - 2*f)*x^3 - (2*e - g)*x^2 - (d + f)*x - e + 2*g)/(x
^4 + x^2 + 1)
```

Fricas [A] time = 0.475966, size = 333, normalized size = 1.86

$$\frac{\sqrt{3}\left(3\,\sqrt{3}\left((2\,d-f)x^4+(2\,d-f)x^2+2\,d-f\right)\,\log\left(x^2+x+1\right)-3\,\sqrt{3}\left((2\,d-f)x^4+(2\,d-f)x^2+2\,d-f\right)\,\log\left(x^2-x+1\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((g*x^3 + f*x^2 + e*x + d)/(x^4 + x^2 + 1)^2,x, algorithm="fricas")
```

```
[Out] 1/72*sqrt(3)*(3*sqrt(3)*((2*d - f)*x^4 + (2*d - f)*x^2 + 2*d - f)
*log(x^2 + x + 1) - 3*sqrt(3)*((2*d - f)*x^4 + (2*d - f)*x^2 + 2*
d - f)*log(x^2 - x + 1) + 2*((4*d - 8*e + f + 4*g)*x^4 + (4*d - 8
*e + f + 4*g)*x^2 + 4*d - 8*e + f + 4*g)*arctan(1/3*sqrt(3)*(2*x
+ 1)) + 2*((4*d + 8*e + f - 4*g)*x^4 + (4*d + 8*e + f - 4*g)*x^2
+ 4*d + 8*e + f - 4*g)*arctan(1/3*sqrt(3)*(2*x - 1)) - 4*sqrt(3)*
((d - 2*f)*x^3 - (2*e - g)*x^2 - (d + f)*x - e + 2*g))/(x^4 + x^2
 + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((g*x**3+f*x**2+e*x+d)/(x**4+x**2+1)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.27488, size = 192, normalized size = 1.07

$$\frac{1}{36}\sqrt{3}(4\,d+f+4\,g-8\,e)\arctan\left(\frac{1}{3}\sqrt{3}(2\,x+1)\right)+\frac{1}{36}\sqrt{3}(4\,d+f-4\,g+8\,e)\arctan\left(\frac{1}{3}\sqrt{3}(2\,x-1)\right)+\frac{1}{8}(2\,d-f)\ln\left(x^2+x+1\right)-\frac{1}{8}(2\,d-f)\ln\left(x^2-x+1\right)-\frac{dx^3-2\,fx^3+gx^2-2\,x^2e-dx-fx+2\,g-e}{6\,(x^4+x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3 + f*x^2 + e*x + d)/(x^4 + x^2 + 1)^2,x, algorithm="giac")
```

```
[Out] 1/36*sqrt(3)*(4*d + f + 4*g - 8*e)*arctan(1/3*sqrt(3)*(2*x + 1))
+ 1/36*sqrt(3)*(4*d + f - 4*g + 8*e)*arctan(1/3*sqrt(3)*(2*x - 1)
) + 1/8*(2*d - f)*ln(x^2 + x + 1) - 1/8*(2*d - f)*ln(x^2 - x + 1)
 - 1/6*(d*x^3 - 2*f*x^3 + g*x^2 - 2*x^2*e - d*x - f*x + 2*g - e)/
(x^4 + x^2 + 1)
```

3.34 $\int \frac{d+ex+fx^2+gx^3+hx^4}{(1+x^2+x^4)^2}\,dx$

Optimal. Leaf size=187

$$-\frac{1}{8}\log\left(x^2-x+1\right)(2d-f+h)+\frac{1}{8}\log\left(x^2+x+1\right)(2d-f+h)+\frac{x\left(x^2(-(d-2f+h))+d+f-2h\right)}{6\left(x^4+x^2+1\right)}-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)(4d+f+h)}{12\sqrt{3}}+\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)(4d+f+h)}{12\sqrt{3}}+\frac{(2e-g)\tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{x^2(2e-g)+e-2g}{6\left(x^4+x^2+1\right)}$$

```
[Out] (e - 2*g + (2*e - g)*x^2)/(6*(1 + x^2 + x^4)) + (x*(d + f - 2*h -
 (d - 2*f + h)*x^2))/(6*(1 + x^2 + x^4)) - ((4*d + f + h)*ArcTan[
(1 - 2*x)/Sqrt[3]])/(12*Sqrt[3]) + ((4*d + f + h)*ArcTan[(1 + 2*x
)/Sqrt[3]])/(12*Sqrt[3]) + ((2*e - g)*ArcTan[(1 + 2*x^2)/Sqrt[3]]
)/(3*Sqrt[3]) - ((2*d - f + h)*Log[1 - x + x^2])/8 + ((2*d - f +
h)*Log[1 + x + x^2])/8
```

Rubi [A] time = 0.412608, antiderivative size = 187, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 9, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.29$

$$-\frac{1}{8}\log\left(x^2-x+1\right)(2d-f+h)+\frac{1}{8}\log\left(x^2+x+1\right)(2d-f+h)+\frac{x\left(x^2(-(d-2f+h))+d+f-2h\right)}{6\left(x^4+x^2+1\right)}-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)(4d+f+h)}{12\sqrt{3}}+\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)(4d+f+h)}{12\sqrt{3}}+\frac{(2e-g)\tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{x^2(2e-g)+e-2g}{6\left(x^4+x^2+1\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x + f*x^2 + g*x^3 + h*x^4)/(1 + x^2 + x^4)^2,x]
```

```
[Out] (e - 2*g + (2*e - g)*x^2)/(6*(1 + x^2 + x^4)) + (x*(d + f - 2*h -
 (d - 2*f + h)*x^2))/(6*(1 + x^2 + x^4)) - ((4*d + f + h)*ArcTan[
(1 - 2*x)/Sqrt[3]])/(12*Sqrt[3]) + ((4*d + f + h)*ArcTan[(1 + 2*x
)/Sqrt[3]])/(12*Sqrt[3]) + ((2*e - g)*ArcTan[(1 + 2*x^2)/Sqrt[3]]
)/(3*Sqrt[3]) - ((2*d - f + h)*Log[1 - x + x^2])/8 + ((2*d - f +
h)*Log[1 + x + x^2])/8
```

Rubi in Sympy [A] time = 71.1682, size = 167, normalized size = 0.89

$$\frac{x\left(d+f-2h-x^{3}(e-2g)-x^{2}(d-2f+h)+x(e+g)\right)}{6\left(x^{4}+x^{2}+1\right)}+\frac{\sqrt{3}\,(2e-g)\,\mathrm{atan}\left(\sqrt{3}\left(\frac{2x^{2}}{3}+\frac{1}{3}\right)\right)}{9}-\left(\frac{d}{4}-\frac{f}{8}+\frac{h}{8}\right)\log\left(x^{2}-x+1\right)+\left(\frac{d}{4}-\frac{f}{8}+\frac{h}{8}\right)\log\left(x^{2}+x+1\right)+\frac{\sqrt{3}\,(4d+f+h)\,\mathrm{atan}\left(\sqrt{3}\left(\frac{2x}{3}-\frac{1}{3}\right)\right)}{36}+\frac{\sqrt{3}\,(4d+f+h)\,\mathrm{atan}\left(\sqrt{3}\left(\frac{2x}{3}+\frac{1}{3}\right)\right)}{36}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((h*x**4+g*x**3+f*x**2+e*x+d)/(x**4+x**2+1)**2,x)
```

```
[Out] x*(d + f - 2*h - x**3*(e - 2*g) - x**2*(d - 2*f + h) + x*(e + g))
/(6*(x**4 + x**2 + 1)) + sqrt(3)*(2*e - g)*atan(sqrt(3)*(2*x**2/3
 + 1/3))/9 - (d/4 - f/8 + h/8)*log(x**2 - x + 1) + (d/4 - f/8 + h
/8)*log(x**2 + x + 1) + sqrt(3)*(4*d + f + h)*atan(sqrt(3)*(2*x/3
 - 1/3))/36 + sqrt(3)*(4*d + f + h)*atan(sqrt(3)*(2*x/3 + 1/3))/3
6
```

Mathematica [C] time = 1.27862, size = 234, normalized size = 1.25

$$\frac{1}{36}\left(-\frac{6\left(x\left(d\left(x^{2}-1\right)-f\left(2x^{2}+1\right)+h\left(x^{2}+2\right)\right)-e\left(2x^{2}+1\right)+g\left(x^{2}+2\right)\right)}{x^{4}+x^{2}+1}-\frac{\tan^{-1}\left(\frac{1}{2}\left(\sqrt{3}-i\right)x\right)\left(\left(\sqrt{3}-11i\right)d-2\left(\sqrt{3}-2i\right)f+\left(\sqrt{3}-5i\right)h\right)}{\sqrt{\frac{1}{6}\left(1+i\sqrt{3}\right)}}-\frac{\tan^{-1}\left(\frac{1}{2}\left(\sqrt{3}+i\right)x\right)\left(\left(\sqrt{3}+11i\right)d-2\left(\sqrt{3}+2i\right)f+\left(\sqrt{3}+5i\right)h\right)}{\sqrt{\frac{1}{6}\left(1-i\sqrt{3}\right)}}-4\sqrt{3}(2e-g)\tan^{-1}\left(\frac{\sqrt{3}}{2x^{2}+1}\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(d + e*x + f*x^2 + g*x^3 + h*x^4)/(1 + x^2 + x^4)^2,x]
```

```
[Out] ((-6*(g*(2 + x^2) - e*(1 + 2*x^2) + x*(d*(-1 + x^2) + h*(2 + x^2)
 - f*(1 + 2*x^2))))/(1 + x^2 + x^4) - (((-11*I + Sqrt[3])*d - 2*(
-2*I + Sqrt[3])*f + (-5*I + Sqrt[3])*h)*ArcTan[((-I + Sqrt[3])*x)
/2])/Sqrt[(1 + I*Sqrt[3])/6] - (((11*I + Sqrt[3])*d - 2*(2*I + Sq
rt[3])*f + (5*I + Sqrt[3])*h)*ArcTan[((I + Sqrt[3])*x)/2])/Sqrt[(
1 - I*Sqrt[3])/6] - 4*Sqrt[3]*(2*e - g)*ArcTan[Sqrt[3]/(1 + 2*x^2
)])/36
```

Maple [A] time = 0.017, size = 328, normalized size = 1.8

$$
\begin{aligned}
&\frac{1}{4x^2+4x+4}\left(\left(-\frac{d}{3}-\frac{h}{3}+\frac{2f}{3}-\frac{g}{3}-\frac{e}{3}\right)x-\frac{2d}{3}+\frac{h}{3}+\frac{f}{3}-\frac{2g}{3}+\frac{e}{3}\right)\\
&+\frac{d\ln\left(x^2+x+1\right)}{4}-\frac{\ln\left(x^2+x+1\right)f}{8}+\frac{\ln\left(x^2+x+1\right)h}{8}+\frac{d\sqrt{3}}{9}\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\\
&-\frac{2\sqrt{3}e}{9}\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)+\frac{\sqrt{3}f}{36}\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\\
&+\frac{\sqrt{3}g}{9}\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)+\frac{\sqrt{3}h}{36}\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\\
&-\frac{1}{4x^2-4x+4}\left(\left(\frac{d}{3}+\frac{h}{3}-\frac{2f}{3}-\frac{g}{3}-\frac{e}{3}\right)x-\frac{2d}{3}+\frac{h}{3}+\frac{f}{3}+\frac{2g}{3}-\frac{e}{3}\right)\\
&-\frac{d\ln\left(x^2-x+1\right)}{4}+\frac{\ln\left(x^2-x+1\right)f}{8}-\frac{\ln\left(x^2-x+1\right)h}{8}+\frac{d\sqrt{3}}{9}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)\\
&+\frac{2\sqrt{3}e}{9}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)+\frac{\sqrt{3}f}{36}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)\\
&-\frac{\sqrt{3}g}{9}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)+\frac{\sqrt{3}h}{36}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)
\end{aligned}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((h*x^4+g*x^3+f*x^2+e*x+d)/(x^4+x^2+1)^2,x)
```

```
[Out] 1/4*((-1/3*d-1/3*h+2/3*f-1/3*g-1/3*e)*x-2/3*d+1/3*h+1/3*f-2/3*g+1
/3*e)/(x^2+x+1)+1/4*d*ln(x^2+x+1)-1/8*ln(x^2+x+1)*f+1/8*ln(x^2+x+
1)*h+1/9*d*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)-2/9*3^(1/2)*arctan
(1/3*(1+2*x)*3^(1/2))*e+1/36*3^(1/2)*arctan(1/3*(1+2*x)*3^(1/2))*
f+1/9*3^(1/2)*arctan(1/3*(1+2*x)*3^(1/2))*g+1/36*3^(1/2)*arctan(1
/3*(1+2*x)*3^(1/2))*h-1/4*((1/3*d+1/3*h-2/3*f-1/3*g-1/3*e)*x-2/3*
d+1/3*h+1/3*f+2/3*g-1/3*e)/(x^2-x+1)-1/4*d*ln(x^2-x+1)+1/8*ln(x^2
-x+1)*f-1/8*ln(x^2-x+1)*h+1/9*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
*d+2/9*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))*e+1/36*3^(1/2)*arctan(
1/3*(2*x-1)*3^(1/2))*f-1/9*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))*g+
1/36*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))*h
```

Maxima [A] time = 0.781619, size = 193, normalized size = 1.03

$$\frac{1}{36}\sqrt{3}(4d-8e+f+4g+h)\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) + \frac{1}{36}\sqrt{3}(4d+8e+f-4g+h)\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) + \frac{1}{8}(2d-f+h)\log\left(x^2+x+1\right) - \frac{1}{8}(2d-f+h)\log\left(x^2-x+1\right) - \frac{(d-2f+h)x^3-(2e-g)x^2-(d+f-2h)x-e+2g}{6(x^4+x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^4 + g*x^3 + f*x^2 + e*x + d)/(x^4 + x^2 + 1)^2,x, algorithm="maxima
```

```
[Out] 1/36*sqrt(3)*(4*d - 8*e + f + 4*g + h)*arctan(1/3*sqrt(3)*(2*x +
1)) + 1/36*sqrt(3)*(4*d + 8*e + f - 4*g + h)*arctan(1/3*sqrt(3)*(
2*x - 1)) + 1/8*(2*d - f + h)*log(x^2 + x + 1) - 1/8*(2*d - f + h
)*log(x^2 - x + 1) - 1/6*((d - 2*f + h)*x^3 - (2*e - g)*x^2 - (d
+ f - 2*h)*x - e + 2*g)/(x^4 + x^2 + 1)
```

Fricas [A] time = 1.3766, size = 355, normalized size = 1.9

$$\frac{\sqrt{3}\left(3\sqrt{3}\left((2d-f+h)x^4+(2d-f+h)x^2+2d-f+h\right)\log\left(x^2+x+1\right)-3\sqrt{3}\left((2d-f+h)x^4+(2d-f+h)x^2+2d-\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^4 + g*x^3 + f*x^2 + e*x + d)/(x^4 + x^2 + 1)^2,x, algorithm="fricas
```

```
[Out] 1/72*sqrt(3)*(3*sqrt(3)*((2*d - f + h)*x^4 + (2*d - f + h)*x^2 +
2*d - f + h)*log(x^2 + x + 1) - 3*sqrt(3)*((2*d - f + h)*x^4 + (2
*d - f + h)*x^2 + 2*d - f + h)*log(x^2 - x + 1) + 2*((4*d - 8*e +
 f + 4*g + h)*x^4 + (4*d - 8*e + f + 4*g + h)*x^2 + 4*d - 8*e + f
 + 4*g + h)*arctan(1/3*sqrt(3)*(2*x + 1)) + 2*((4*d + 8*e + f - 4
*g + h)*x^4 + (4*d + 8*e + f - 4*g + h)*x^2 + 4*d + 8*e + f - 4*g
 + h)*arctan(1/3*sqrt(3)*(2*x - 1)) - 4*sqrt(3)*((d - 2*f + h)*x^
3 - (2*e - g)*x^2 - (d + f - 2*h)*x - e + 2*g))/(x^4 + x^2 + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((h*x**4+g*x**3+f*x**2+e*x+d)/(x**4+x**2+1)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.275033, size = 209, normalized size = 1.12

$$\frac{1}{36}\sqrt{3}(4d+f+4g+h-8e)\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) + \frac{1}{36}\sqrt{3}(4d+f-4g+h+8e)\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) + \frac{1}{8}(2d-f+h)\ln\left(x^2+x+1\right) - \frac{1}{8}(2d-f+h)\ln\left(x^2-x+1\right) - \frac{dx^3-2fx^3+hx^3+gx^2-2x^2e-dx-fx+2hx+2g-e}{6(x^4+x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^4 + g*x^3 + f*x^2 + e*x + d)/(x^4 + x^2 + 1)^2,x, algorithm="giac")
```

```
[Out] 1/36*sqrt(3)*(4*d + f + 4*g + h - 8*e)*arctan(1/3*sqrt(3)*(2*x +
1)) + 1/36*sqrt(3)*(4*d + f - 4*g + h + 8*e)*arctan(1/3*sqrt(3)*(
2*x - 1)) + 1/8*(2*d - f + h)*ln(x^2 + x + 1) - 1/8*(2*d - f + h)
*ln(x^2 - x + 1) - 1/6*(d*x^3 - 2*f*x^3 + h*x^3 + g*x^2 - 2*x^2*e
 - d*x - f*x + 2*h*x + 2*g - e)/(x^4 + x^2 + 1)
```

3.35 $\int \frac{d+ex+fx^2+gx^3+hx^4+ix^5}{(1+x^2+x^4)^2} dx$

Optimal. Leaf size=194

$$-\frac{1}{8}\log\left(x^2-x+1\right)(2d-f+h)+\frac{1}{8}\log\left(x^2+x+1\right)(2d-f+h)+\frac{x\left(x^2(-(d-2f+h))+d+f-2h\right)}{6\left(x^4+x^2+1\right)}-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)(4d+f+h)}{12\sqrt{3}}+\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)(4d+f+h)}{12\sqrt{3}}+\frac{\tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right)(2e-g+2i)}{3\sqrt{3}}+\frac{x^2(2e-g-i)+e-2g+i}{6\left(x^4+x^2+1\right)}$$

```
[Out] (x*(d + f - 2*h - (d - 2*f + h)*x^2))/(6*(1 + x^2 + x^4)) + (e -
2*g + i + (2*e - g - i)*x^2)/(6*(1 + x^2 + x^4)) - ((4*d + f + h)
*ArcTan[(1 - 2*x)/Sqrt[3]])/(12*Sqrt[3]) + ((4*d + f + h)*ArcTan[
(1 + 2*x)/Sqrt[3]])/(12*Sqrt[3]) + ((2*e - g + 2*i)*ArcTan[(1 + 2
*x^2)/Sqrt[3]])/(3*Sqrt[3]) - ((2*d - f + h)*Log[1 - x + x^2])/8
+ ((2*d - f + h)*Log[1 + x + x^2])/8
```

Rubi [A] time = 0.403186, antiderivative size = 194, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 10, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$

$$-\frac{1}{8}\log\left(x^2-x+1\right)(2d-f+h)+\frac{1}{8}\log\left(x^2+x+1\right)(2d-f+h)+\frac{x\left(x^2(-(d-2f+h))+d+f-2h\right)}{6\left(x^4+x^2+1\right)}-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)(4d+f+h)}{12\sqrt{3}}+\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)(4d+f+h)}{12\sqrt{3}}+\frac{\tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right)(2e-g+2i)}{3\sqrt{3}}+\frac{x^2(2e-g-i)+e-2g+i}{6\left(x^4+x^2+1\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5)/(1 + x^2 + x^4)^2,x]
```


```
[Out] (x*(d + f - 2*h - (d - 2*f + h)*x^2))/(6*(1 + x^2 + x^4)) + (e -
2*g + i + (2*e - g - i)*x^2)/(6*(1 + x^2 + x^4)) - ((4*d + f + h)
*ArcTan[(1 - 2*x)/Sqrt[3]])/(12*Sqrt[3]) + ((4*d + f + h)*ArcTan[
(1 + 2*x)/Sqrt[3]])/(12*Sqrt[3]) + ((2*e - g + 2*i)*ArcTan[(1 + 2
*x^2)/Sqrt[3]])/(3*Sqrt[3]) - ((2*d - f + h)*Log[1 - x + x^2])/8
+ ((2*d - f + h)*Log[1 + x + x^2])/8
```

Rubi in Sympy [A] time = 82.3899, size = 175, normalized size = 0.9

$$\frac{x\left(d+f-2h-x^{3}(e-2g+1)-x^{2}(d-2f+h)-x(-e-g+2)\right)}{6\left(x^{4}+x^{2}+1\right)}-\left(\frac{d}{4}-\frac{f}{8}+\frac{h}{8}\right)\log\left(x^{2}-x+1\right)+\left(\frac{d}{4}-\frac{f}{8}+\frac{h}{8}\right)\log\left(x^{2}+x+1\right)+\frac{\sqrt{3}(4d+f+h)\operatorname{atan}\left(\sqrt{3}\left(\frac{2x}{3}-\frac{1}{3}\right)\right)}{36}+\frac{\sqrt{3}(4d+f+h)\operatorname{atan}\left(\sqrt{3}\left(\frac{2x}{3}+\frac{1}{3}\right)\right)}{36}-\frac{\sqrt{3}(-2e+g-3i+1)\operatorname{atan}\left(\sqrt{3}\left(\frac{2x^{2}}{3}+\frac{1}{3}\right)\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((i*x**5+h*x**4+g*x**3+f*x**2+e*x+d)/(x**4+x**2+1)**2,x)
```

```
[Out] x*(d + f - 2*h - x**3*(e - 2*g + 1) - x**2*(d - 2*f + h) - x*(-e
- g + 2))/(6*(x**4 + x**2 + 1)) - (d/4 - f/8 + h/8)*log(x**2 - x
+ 1) + (d/4 - f/8 + h/8)*log(x**2 + x + 1) + sqrt(3)*(4*d + f + h
)*atan(sqrt(3)*(2*x/3 - 1/3))/36 + sqrt(3)*(4*d + f + h)*atan(sqr
t(3)*(2*x/3 + 1/3))/36 - sqrt(3)*(-2*e + g - 3*i + 1)*atan(sqrt(3
)*(2*x**2/3 + 1/3))/9
```

Mathematica [C] time = 1.42449, size = 243, normalized size = 1.25

$$\frac{1}{36}\left(\frac{6\left(-dx^{3}+dx+2ex^{2}+e+2fx^{3}+fx-g\left(x^{2}+2\right)-hx^{3}-2hx-ix^{2}+i\right)}{x^{4}+x^{2}+1}-\frac{\tan^{-1}\left(\frac{1}{2}\left(\sqrt{3}-i\right)x\right)\left(\left(\sqrt{3}-11i\right)d-2\left(\sqrt{3}-2i\right)f+\left(\sqrt{3}-5i\right)h\right)}{\sqrt{\frac{1}{6}\left(1+i\sqrt{3}\right)}}-\frac{\tan^{-1}\left(\frac{1}{2}\left(\sqrt{3}+i\right)x\right)\left(\left(\sqrt{3}+11i\right)d-2\left(\sqrt{3}+2i\right)f+\left(\sqrt{3}+5i\right)h\right)}{\sqrt{\frac{1}{6}\left(1-i\sqrt{3}\right)}}-4\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}}{2x^{2}+1}\right)(2e-g+2i)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5)/(1 + x^2 + x^4)^2,x]
```

```
[Out] ((6*(e + i + d*x + f*x - 2*h*x + 2*e*x^2 - i*x^2 - d*x^3 + 2*f*x^
3 - h*x^3 - g*(2 + x^2)))/(1 + x^2 + x^4) - (((-11*I + Sqrt[3])*d
 - 2*(-2*I + Sqrt[3])*f + (-5*I + Sqrt[3])*h)*ArcTan[((-I + Sqrt[
3])*x)/2])/Sqrt[(1 + I*Sqrt[3])/6] - (((11*I + Sqrt[3])*d - 2*(2*
I + Sqrt[3])*f + (5*I + Sqrt[3])*h)*ArcTan[((I + Sqrt[3])*x)/2])/
Sqrt[(1 - I*Sqrt[3])/6] - 4*Sqrt[3]*(2*e - g + 2*i)*ArcTan[Sqrt[3
]/(1 + 2*x^2)])/36
```

Maple [B] time = 0.018, size = 374, normalized size = 1.9

$$\frac{1}{4x^2+4x+4}\left(\left(-\frac{d}{3}-\frac{h}{3}-\frac{e}{3}-\frac{g}{3}+\frac{2f}{3}+\frac{2i}{3}\right)x-\frac{2d}{3}+\frac{h}{3}+\frac{e}{3}-\frac{2g}{3}+\frac{f}{3}+\frac{i}{3}\right)$$
$$+\frac{d\ln\left(x^2+x+1\right)}{4}-\frac{\ln\left(x^2+x+1\right)f}{8}+\frac{\ln\left(x^2+x+1\right)h}{8}+\frac{d\sqrt{3}}{9}\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)$$
$$-\frac{2\sqrt{3}e}{9}\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)+\frac{\sqrt{3}f}{36}\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)$$
$$+\frac{\sqrt{3}g}{9}\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)+\frac{\sqrt{3}h}{36}\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)-\frac{2\sqrt{3}i}{9}\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)$$
$$-\frac{1}{4x^2-4x+4}\left(\left(\frac{d}{3}+\frac{h}{3}-\frac{e}{3}-\frac{g}{3}-\frac{2f}{3}+\frac{2i}{3}\right)x-\frac{2d}{3}+\frac{h}{3}-\frac{e}{3}+\frac{2g}{3}+\frac{f}{3}-\frac{i}{3}\right)$$
$$-\frac{d\ln\left(x^2-x+1\right)}{4}+\frac{\ln\left(x^2-x+1\right)f}{8}-\frac{\ln\left(x^2-x+1\right)h}{8}+\frac{d\sqrt{3}}{9}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)$$
$$+\frac{2\sqrt{3}e}{9}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)+\frac{\sqrt{3}f}{36}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)$$
$$-\frac{\sqrt{3}g}{9}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)+\frac{\sqrt{3}h}{36}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)+\frac{2\sqrt{3}i}{9}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4+x^2+1)^2,x)
```

```
[Out] 1/4*((-1/3*d-1/3*h-1/3*e-1/3*g+2/3*f+2/3*i)*x-2/3*d+1/3*h+1/3*e-2
/3*g+1/3*f+1/3*i)/(x^2+x+1)+1/4*d*ln(x^2+x+1)-1/8*ln(x^2+x+1)*f+1
/8*ln(x^2+x+1)*h+1/9*d*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)-2/9*3^
(1/2)*arctan(1/3*(1+2*x)*3^(1/2))*e+1/36*3^(1/2)*arctan(1/3*(1+2*
x)*3^(1/2))*f+1/9*3^(1/2)*arctan(1/3*(1+2*x)*3^(1/2))*g+1/36*3^(1
/2)*arctan(1/3*(1+2*x)*3^(1/2))*h-2/9*3^(1/2)*arctan(1/3*(1+2*x)*
3^(1/2))*i-1/4*((1/3*d+1/3*h-1/3*e-1/3*g-2/3*f+2/3*i)*x-2/3*d+1/3
*h-1/3*e+2/3*g+1/3*f-1/3*i)/(x^2-x+1)-1/4*d*ln(x^2-x+1)+1/8*ln(x^
2-x+1)*f-1/8*ln(x^2-x+1)*h+1/9*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2)
)*d+2/9*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))*e+1/36*3^(1/2)*arctan
(1/3*(2*x-1)*3^(1/2))*f-1/9*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))*g
+1/36*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))*h+2/9*3^(1/2)*arctan(1/
3*(2*x-1)*3^(1/2))*i
```

Maxima [A] time = 0.77894, size = 209, normalized size = 1.08

$$\frac{1}{36}\sqrt{3}(4d-8e+f+4g+h-8i)\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{36}\sqrt{3}(4d+8e+f-4g+h+8i)\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{8}(2d-f+h)\log\left(x^2+x+1\right)-\frac{1}{8}(2d-f+h)\log\left(x^2-x+1\right)-\frac{(d-2f+h)x^3-(2e-g-i)x^2-(d+f-2h)x-e+2g-i}{6(x^4+x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^5 + h*x^4 + g*x^3 + f*x^2 + e*x + d)/(x^4 + x^2 + 1)^2,x, algorithm=
```

```
[Out] 1/36*sqrt(3)*(4*d - 8*e + f + 4*g + h - 8*i)*arctan(1/3*sqrt(3)*(
2*x + 1)) + 1/36*sqrt(3)*(4*d + 8*e + f - 4*g + h + 8*i)*arctan(1
/3*sqrt(3)*(2*x - 1)) + 1/8*(2*d - f + h)*log(x^2 + x + 1) - 1/8*
(2*d - f + h)*log(x^2 - x + 1) - 1/6*((d - 2*f + h)*x^3 - (2*e -
g - i)*x^2 - (d + f - 2*h)*x - e + 2*g - i)/(x^4 + x^2 + 1)
```

Fricas [A] time = 6.3613, size = 387, normalized size = 1.99

$$\frac{\sqrt{3}\Big(3\sqrt{3}\big((2d-f+h)x^4+(2d-f+h)x^2+2d-f+h\big)\log\left(x^2+x+1\right)-3\sqrt{3}\big((2d-f+h)x^4+(2d-f+h)x^2+2d-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^5 + h*x^4 + g*x^3 + f*x^2 + e*x + d)/(x^4 + x^2 + 1)^2,x, algorithm=
```

```
[Out] 1/72*sqrt(3)*(3*sqrt(3)*((2*d - f + h)*x^4 + (2*d - f + h)*x^2 +
2*d - f + h)*log(x^2 + x + 1) - 3*sqrt(3)*((2*d - f + h)*x^4 + (2
*d - f + h)*x^2 + 2*d - f + h)*log(x^2 - x + 1) + 2*((4*d - 8*e +
 f + 4*g + h - 8*i)*x^4 + (4*d - 8*e + f + 4*g + h - 8*i)*x^2 + 4
*d - 8*e + f + 4*g + h - 8*i)*arctan(1/3*sqrt(3)*(2*x + 1)) + 2*(
(4*d + 8*e + f - 4*g + h + 8*i)*x^4 + (4*d + 8*e + f - 4*g + h +
8*i)*x^2 + 4*d + 8*e + f - 4*g + h + 8*i)*arctan(1/3*sqrt(3)*(2*x
 - 1)) - 4*sqrt(3)*((d - 2*f + h)*x^3 - (2*e - g - i)*x^2 - (d +
f - 2*h)*x - e + 2*g - i))/(x^4 + x^2 + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((i*x**5+h*x**4+g*x**3+f*x**2+e*x+d)/(x**4+x**2+1)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.275638, size = 228, normalized size = 1.18

$$\frac{1}{36}\sqrt{3}(4d+f+4g+h-8i-8e)\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{36}\sqrt{3}(4d+f-4g+h+8i+8e)\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{8}(2d-f+h)\ln\left(x^2+x+1\right)-\frac{1}{8}(2d-f+h)\ln\left(x^2-x+1\right)-\frac{dx^3-2fx^3+hx^3+gx^2+ix^2-2x^2e-dx-fx+2hx+2g-i-e}{6\left(x^4+x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^5 + h*x^4 + g*x^3 + f*x^2 + e*x + d)/(x^4 + x^2 + 1)^2,x, algorithm=
```

```
[Out] 1/36*sqrt(3)*(4*d + f + 4*g + h - 8*i - 8*e)*arctan(1/3*sqrt(3)*(
2*x + 1)) + 1/36*sqrt(3)*(4*d + f - 4*g + h + 8*i + 8*e)*arctan(1
/3*sqrt(3)*(2*x - 1)) + 1/8*(2*d - f + h)*ln(x^2 + x + 1) - 1/8*(
2*d - f + h)*ln(x^2 - x + 1) - 1/6*(d*x^3 - 2*f*x^3 + h*x^3 + g*x
^2 + i*x^2 - 2*x^2*e - d*x - f*x + 2*h*x + 2*g - i - e)/(x^4 + x^
2 + 1)
```

3.36 $\int \frac{d+ex}{(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=330

$$\frac{dx\left(-2ac+b^2+bcx^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{c}d\left(b\sqrt{b^2-4ac}-12ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}d\left(-b\sqrt{b^2-4ac}-12ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}a\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{2ce\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}-\frac{e\left(b+2cx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

```
[Out] -(e*(b + 2*c*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (d*x*(
b^2 - 2*a*c + b*c*x^2))/(2*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) +
 (Sqrt[c]*(b^2 - 12*a*c + b*Sqrt[b^2 - 4*a*c])*d*ArcTan[(Sqrt[2]*
Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*(b^2 - 4*a*
c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[c]*(b^2 - 12*a*c -
b*Sqrt[b^2 - 4*a*c])*d*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b
^2 - 4*a*c]]])/(2*Sqrt[2]*a*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2
 - 4*a*c]]) + (2*c*e*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b
^2 - 4*a*c)^(3/2)
```

Rubi [A] time = 1.55996, antiderivative size = 330, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.45

$$\frac{dx\left(-2ac+b^2+bcx^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{c}d\left(b\sqrt{b^2-4ac}-12ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}d\left(-b\sqrt{b^2-4ac}-12ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}a\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{2ce\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}-\frac{e\left(b+2cx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x)/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] -(e*(b + 2*c*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (d*x*(
b^2 - 2*a*c + b*c*x^2))/(2*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) +
```

```
 (Sqrt[c]*(b^2 - 12*a*c + b*Sqrt[b^2 - 4*a*c])*d*ArcTan[(Sqrt[2]*
Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*(b^2 - 4*a*
c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[c]*(b^2 - 12*a*c -
b*Sqrt[b^2 - 4*a*c])*d*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b
^2 - 4*a*c]]])/(2*Sqrt[2]*a*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2
 - 4*a*c]]) + (2*c*e*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b
^2 - 4*a*c)^(3/2)
```

Rubi in Sympy [A] time = 110.156, size = 296, normalized size = 0.9

$$\frac{2ce\operatorname{atanh}\left(\frac{b+2cx^2}{\sqrt{-4ac+b^2}}\right)}{\left(-4ac+b^2\right)^{\frac{3}{2}}}-\frac{\sqrt{2}\sqrt{c}d\left(-12ac+b^2-b\sqrt{-4ac+b^2}\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{-4ac+b^2}}}\right)}{4a\sqrt{b+\sqrt{-4ac+b^2}}\left(-4ac+b^2\right)^{\frac{3}{2}}}+\frac{\sqrt{2}\sqrt{c}d\left(-12ac+b^2+b\sqrt{-4ac+b^2}\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{-4ac+b^2}}}\right)}{4a\sqrt{b-\sqrt{-4ac+b^2}}\left(-4ac+b^2\right)^{\frac{3}{2}}}+\frac{x\left(bcdx^2+bcex^3+d\left(-2ac+b^2\right)+ex\left(-2ac+b^2\right)\right)}{2a\left(-4ac+b^2\right)\left(a+bx^2+cx^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x+d)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] 2*c*e*atanh((b + 2*c*x**2)/sqrt(-4*a*c + b**2))/(-4*a*c + b**2)**
(3/2) - sqrt(2)*sqrt(c)*d*(-12*a*c + b**2 - b*sqrt(-4*a*c + b**2)
)*atan(sqrt(2)*sqrt(c)*x/sqrt(b + sqrt(-4*a*c + b**2)))/(4*a*sqrt
(b + sqrt(-4*a*c + b**2))*(-4*a*c + b**2)**(3/2)) + sqrt(2)*sqrt(
c)*d*(-12*a*c + b**2 + b*sqrt(-4*a*c + b**2))*atan(sqrt(2)*sqrt(c
)*x/sqrt(b - sqrt(-4*a*c + b**2)))/(4*a*sqrt(b - sqrt(-4*a*c + b*
*2))*(-4*a*c + b**2)**(3/2)) + x*(b*c*d*x**2 + b*c*e*x**3 + d*(-2
*a*c + b**2) + e*x*(-2*a*c + b**2))/(2*a*(-4*a*c + b**2)*(a + b*x
**2 + c*x**4))
```

Mathematica [A] time = 1.60811, size = 341, normalized size = 1.03

$$\frac{1}{4}\left(\frac{2abe+4acx(d+ex)-2bdx\left(b+cx^2\right)}{a\left(4ac-b^2\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{2}\sqrt{c}d\left(b\sqrt{b^2-4ac}-12ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{a\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\sqrt{c}d\left(b\sqrt{b^2-4ac}+12ac-b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{a\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}-\frac{4ce\log\left(\sqrt{b^2-4ac}-b-2cx^2\right)}{\left(b^2-4ac\right)^{3/2}}+\frac{4ce\log\left(\sqrt{b^2-4ac}+b+2cx^2\right)}{\left(b^2-4ac\right)^{3/2}}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x)/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((2*a*b*e + 4*a*c*x*(d + e*x) - 2*b*d*x*(b + c*x^2))/(a*(-b^2 + 4
*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[2]*Sqrt[c]*(b^2 - 12*a*c + b*S
qrt[b^2 - 4*a*c])*d*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2
- 4*a*c]]])/(a*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) +
 (Sqrt[2]*Sqrt[c]*(-b^2 + 12*a*c + b*Sqrt[b^2 - 4*a*c])*d*ArcTan[
(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(a*(b^2 - 4*a*c
)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - (4*c*e*Log[-b + Sqrt[b^2 -
 4*a*c] - 2*c*x^2]])/(b^2 - 4*a*c)^(3/2) + (4*c*e*Log[b + Sqrt[b^2
 - 4*a*c] + 2*c*x^2]])/(b^2 - 4*a*c)^(3/2))/4
```

Maple [B] time = 0.143, size = 1241, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x+d)/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] c/(4*a*c-b^2)^2/(x^2+1/2/c*(-4*a*c+b^2)^(1/2)+1/2*b/c)*d*x*(-4*a*
c+b^2)^(1/2)-1/4/(4*a*c-b^2)^2/(x^2+1/2/c*(-4*a*c+b^2)^(1/2)+1/2*
b/c)/a*d*x*b^2*(-4*a*c+b^2)^(1/2)-c/(4*a*c-b^2)^2/(x^2+1/2/c*(-4*
a*c+b^2)^(1/2)+1/2*b/c)*d*x*b+1/4/(4*a*c-b^2)^2/(x^2+1/2/c*(-4*a*
c+b^2)^(1/2)+1/2*b/c)/a*d*x*b^3+2*c/(4*a*c-b^2)^2/(x^2+1/2/c*(-4*
a*c+b^2)^(1/2)+1/2*b/c)*e*a-1/2/(4*a*c-b^2)^2/(x^2+1/2/c*(-4*a*c+
b^2)^(1/2)+1/2*b/c)*e*b^2+c/(4*a*c-b^2)^2*(-4*a*c+b^2)^(1/2)*e*ln
(a*(2*c*x^2+(-4*a*c+b^2)^(1/2)+b))+3*c^2/(4*a*c-b^2)^2*2^(1/2)/((
```

```
b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2
)^(1/2))*c)^(1/2))*(-4*a*c+b^2)^(1/2)*d-1/4*c/(4*a*c-b^2)^2*2^(1/
2)/a/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*
a*c+b^2)^(1/2))*c)^(1/2))*(-4*a*c+b^2)^(1/2)*b^2*d-c^2/(4*a*c-b^2
)^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/(
(b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b*d+1/4*c/(4*a*c-b^2)^2*2^(1/2)/
a/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c
+b^2)^(1/2))*c)^(1/2))*b^3*d-c/(4*a*c-b^2)^2/(x^2+1/2*b/c-1/2/c*(
-4*a*c+b^2)^(1/2))*d*x*(-4*a*c+b^2)^(1/2)+1/4/(4*a*c-b^2)^2/(x^2+
1/2*b/c-1/2/c*(-4*a*c+b^2)^(1/2))/a*d*x*b^2*(-4*a*c+b^2)^(1/2)-c/
(4*a*c-b^2)^2/(x^2+1/2*b/c-1/2/c*(-4*a*c+b^2)^(1/2))*d*x*b+1/4/(4
*a*c-b^2)^2/(x^2+1/2*b/c-1/2/c*(-4*a*c+b^2)^(1/2))/a*d*x*b^3+2*c/
(4*a*c-b^2)^2/(x^2+1/2*b/c-1/2/c*(-4*a*c+b^2)^(1/2))*e*a-1/2/(4*a
*c-b^2)^2/(x^2+1/2*b/c-1/2/c*(-4*a*c+b^2)^(1/2))*e*b^2-c/(4*a*c-b
^2)^2*(-4*a*c+b^2)^(1/2)*e*ln(a*(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b))+
3*c^2/(4*a*c-b^2)^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arc
tanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*(-4*a*c+b^2)^
(1/2)*d-1/4*c/(4*a*c-b^2)^2*2^(1/2)/a/((-b+(-4*a*c+b^2)^(1/2))*c)
^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*(-4
*a*c+b^2)^(1/2)*b^2*d+c^2/(4*a*c-b^2)^2*2^(1/2)/((-b+(-4*a*c+b^2)
^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^
(1/2))*b*d-1/4*c/(4*a*c-b^2)^2*2^(1/2)/a/((-b+(-4*a*c+b^2)^(1/2))
*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*
b^3*d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{bcdx^3 - 2\,acex^2 - abe + \left(b^2 - 2\,ac\right)dx}{2\left((ab^2c - 4\,a^2c^2)x^4 + a^2b^2 - 4\,a^3c + (ab^3 - 4\,a^2bc)x^2\right)} + \frac{\int \frac{bcdx^2 - 4\,acex + \left(b^2 - 6\,ac\right)d}{cx^4 + bx^2 + a}\,dx}{2\left(ab^2 - 4\,a^2c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x + d)/(c*x^4 + b*x^2 + a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(b*c*d*x^3 - 2*a*c*e*x^2 - a*b*e + (b^2 - 2*a*c)*d*x)/((a*b^2
*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2
) + 1/2*integrate((b*c*d*x^2 - 4*a*c*e*x + (b^2 - 6*a*c)*d)/(c*x^
4 + b*x^2 + a), x)/(a*b^2 - 4*a^2*c)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x + d)/(c*x^4 + b*x^2 + a)^2,x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x+d)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x + d)/(c*x^4 + b*x^2 + a)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.37 $\int \frac{d+ex+fx^2}{(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=368

$$\frac{x\left(cx^2(bd-2af)-abf-2acd+b^2d\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{c}\left(\frac{4abf-12acd+b^2d}{\sqrt{b^2-4ac}}-2af+bd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(-\frac{4abf-12acd+b^2d}{\sqrt{b^2-4ac}}-2af+bd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}a\left(b^2-4ac\right)\sqrt{\sqrt{b^2-4ac}+b}}+\frac{2ce\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}-\frac{e\left(b+2cx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

```
[Out] -(e*(b + 2*c*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (x*(b^
2*d - 2*a*c*d - a*b*f + c*(b*d - 2*a*f)*x^2))/(2*a*(b^2 - 4*a*c)*
(a + b*x^2 + c*x^4)) + (Sqrt[c]*(b*d - 2*a*f + (b^2*d - 12*a*c*d
+ 4*a*b*f)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b -
 Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^
2 - 4*a*c]]) + (Sqrt[c]*(b*d - 2*a*f - (b^2*d - 12*a*c*d + 4*a*b*
f)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^
2 - 4*a*c]]])/(2*Sqrt[2]*a*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*
c]]) + (2*c*e*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*
a*c)^(3/2)
```

Rubi [A] time = 1.92523, antiderivative size = 368, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.36$

$$\frac{x\left(cx^2(bd-2af)-abf-2acd+b^2d\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{c}\left(\frac{4abf-12acd+b^2d}{\sqrt{b^2-4ac}}-2af+bd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(-\frac{4abf-12acd+b^2d}{\sqrt{b^2-4ac}}-2af+bd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}a\left(b^2-4ac\right)\sqrt{\sqrt{b^2-4ac}+b}}+\frac{2ce\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}-\frac{e\left(b+2cx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x + f*x^2)/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] -(e*(b + 2*c*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (x*(b^
2*d - 2*a*c*d - a*b*f + c*(b*d - 2*a*f)*x^2))/(2*a*(b^2 - 4*a*c)*
(a + b*x^2 + c*x^4)) + (Sqrt[c]*(b*d - 2*a*f + (b^2*d - 12*a*c*d
+ 4*a*b*f)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b -
 Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^
2 - 4*a*c]]) + (Sqrt[c]*(b*d - 2*a*f - (b^2*d - 12*a*c*d + 4*a*b*
f)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^
2 - 4*a*c]]])/(2*Sqrt[2]*a*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*
c]]) + (2*c*e*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*
a*c)^(3/2)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**2+e*x+d)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 2.53327, size = 398, normalized size = 1.08

$$\frac{1}{4}\left(\frac{2ab(e+fx)+4acx(d+x(e+fx))-2bdx\left(b+cx^2\right)}{a\left(4ac-b^2\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{2}\sqrt{c}\left(b\left(d\sqrt{b^2-4ac}+4af\right)-2a\left(f\sqrt{b^2-4ac}+6cd\right)+b^2d\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{a\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\sqrt{c}\left(bd\sqrt{b^2-4ac}-2af\sqrt{b^2-4ac}-4abf+12acd+b^2(-d)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{a\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}-\frac{4ce\log\left(\sqrt{b^2-4ac}-b-2cx^2\right)}{\left(b^2-4ac\right)^{3/2}}+\frac{4ce\log\left(\sqrt{b^2-4ac}+b+2cx^2\right)}{\left(b^2-4ac\right)^{3/2}}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x + f*x^2)/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((2*a*b*(e + f*x) - 2*b*d*x*(b + c*x^2) + 4*a*c*x*(d + x*(e + f*x
)))/(a*(-b^2 + 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[2]*Sqrt[c]*(b^
2*d + b*(Sqrt[b^2 - 4*a*c]*d + 4*a*f) - 2*a*(6*c*d + Sqrt[b^2 - 4
*a*c]*f))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]]
)/(a*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*
Sqrt[c]*(-(b^2*d) + 12*a*c*d + b*Sqrt[b^2 - 4*a*c]*d - 4*a*b*f -
2*a*Sqrt[b^2 - 4*a*c]*f)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt
[b^2 - 4*a*c]]])/(a*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c
]]) - (4*c*e*Log[-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2])/(b^2 - 4*a*c)
^(3/2) + (4*c*e*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*
c)^(3/2))/4
```

Maple [B] time = 0.163, size = 2851, normalized size = 7.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x^2+e*x+d)/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] 2*c^2/(4*a*c-b^2)^2*2^(1/2)/(4*a*c+3*b^2)/((-b+(-4*a*c+b^2)^(1/2)
)*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))
*b^3*d-3/4*c/(4*a*c-b^2)^2*2^(1/2)/(4*a*c+3*b^2)/a/((b+(-4*a*c+b^
2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^
(1/2))*(-4*a*c+b^2)^(1/2)*b^4*d+4*c^2/(4*a*c-b^2)^2*2^(1/2)/(4*a*
c+3*b^2)*a/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((
b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^2*f-4*c^3/(4*a*c-b^2)^2*2^(1/2)
/(4*a*c+3*b^2)*a/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1
/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b*d+3/4*c/(4*a*c-b^2)^2*2^(
1/2)/(4*a*c+3*b^2)/a/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*
2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^5*d+12*c^3/(4*a*c-b^2
)^2*2^(1/2)/(4*a*c+3*b^2)*a/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arc
tanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*(-4*a*c+b^2)^
(1/2)*d-3*c/(4*a*c-b^2)^2*2^(1/2)/(4*a*c+3*b^2)/((b+(-4*a*c+b^2)^
(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/
2))*(-4*a*c+b^2)^(1/2)*b^3*f+4*c^3/(4*a*c-b^2)^2*2^(1/2)/(4*a*c+3
*b^2)*a/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-
b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b*d-3/4*c/(4*a*c-b^2)^2*2^(1/2)/(
4*a*c+3*b^2)/a/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1
/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^5*d+8*c^2/(4*a*c-b^2)^2*
2^(1/2)/(4*a*c+3*b^2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x
*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*(-4*a*c+b^2)^(1/2)*b^2
*d-3*c/(4*a*c-b^2)^2*2^(1/2)/(4*a*c+3*b^2)/((-b+(-4*a*c+b^2)^(1/2
))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)
)*(-4*a*c+b^2)^(1/2)*b^3*f+8*c^2/(4*a*c-b^2)^2*2^(1/2)/(4*a*c+3*b
^2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-
4*a*c+b^2)^(1/2))*c)^(1/2))*(-4*a*c+b^2)^(1/2)*b^2*d-4*c^2/(4*a*c
-b^2)^2*2^(1/2)/(4*a*c+3*b^2)*a/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)
*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^2*f+12*
```

```
c^3/(4*a*c-b^2)^2*2^(1/2)/(4*a*c+3*b^2)*a/((b+(-4*a*c+b^2)^(1/2))
*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*(-
4*a*c+b^2)^(1/2)*d-1/4/(4*a*c-b^2)^2/(x^2+1/2/c*(-4*a*c+b^2)^(1/2
)+1/2*b/c)/a*d*x*b^2*(-4*a*c+b^2)^(1/2)+c/(4*a*c-b^2)^2/(x^2+1/2/
c*(-4*a*c+b^2)^(1/2)+1/2*b/c)*d*x*(-4*a*c+b^2)^(1/2)-c/(4*a*c-b^2
)^2/(x^2+1/2/c*(-4*a*c+b^2)^(1/2)+1/2*b/c)*d*x*b+1/4/(4*a*c-b^2)^
2/(x^2+1/2/c*(-4*a*c+b^2)^(1/2)+1/2*b/c)/a*d*x*b^3-c/(4*a*c-b^2)^
2/(x^2+1/2*b/c-1/2/c*(-4*a*c+b^2)^(1/2))*d*x*(-4*a*c+b^2)^(1/2)-c
/(4*a*c-b^2)^2/(x^2+1/2*b/c-1/2/c*(-4*a*c+b^2)^(1/2))*d*x*b+1/4/(
4*a*c-b^2)^2/(x^2+1/2*b/c-1/2/c*(-4*a*c+b^2)^(1/2))/a*d*x*b^3-4*c
^2/(4*a*c-b^2)^2*2^(1/2)/(4*a*c+3*b^2)*a/((-b+(-4*a*c+b^2)^(1/2))
*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*
(-4*a*c+b^2)^(1/2)*b*f-3/4*c/(4*a*c-b^2)^2*2^(1/2)/(4*a*c+3*b^2)/
a/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*
a*c+b^2)^(1/2))*c)^(1/2))*(-4*a*c+b^2)^(1/2)*b^4*d-4*c^2/(4*a*c-b
^2)^2*2^(1/2)/(4*a*c+3*b^2)*a/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*ar
ctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*(-4*a*c+b^2)^(
1/2)*b*f+1/4/(4*a*c-b^2)^2/(x^2+1/2*b/c-1/2/c*(-4*a*c+b^2)^(1/2))
/a*d*x*b^2*(-4*a*c+b^2)^(1/2)-8*c^3/(4*a*c-b^2)^2*2^(1/2)/(4*a*c+
3*b^2)*a^2/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/
((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*f+3/2*c/(4*a*c-b^2)^2*2^(1/2)/
(4*a*c+3*b^2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/
2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^4*f+2*c/(4*a*c-b^2)^2/(x^
2+1/2*b/c-1/2/c*(-4*a*c+b^2)^(1/2))*e*a-1/2/(4*a*c-b^2)^2/(x^2+1/
2/c*(-4*a*c+b^2)^(1/2)+1/2*b/c)*e*b^2-1/2/(4*a*c-b^2)^2/(x^2+1/2*
b/c-1/2/c*(-4*a*c+b^2)^(1/2))*e*b^2+2*c/(4*a*c-b^2)^2/(x^2+1/2/c*
(-4*a*c+b^2)^(1/2)+1/2*b/c)*e*a+8*c^3/(4*a*c-b^2)^2*2^(1/2)/(4*a*
c+3*b^2)*a^2/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/
((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*f-3/2*c/(4*a*c-b^2)^2*2^(1/2)/(
4*a*c+3*b^2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/
((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^4*f-2*c^2/(4*a*c-b^2)^2*2^(1/
2)/(4*a*c+3*b^2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1
/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^3*d+2*c/(4*a*c-b^2)^2/(x^
2+1/2*b/c-1/2/c*(-4*a*c+b^2)^(1/2))*a*x*f+2*c/(4*a*c-b^2)^2/(x^2+
1/2/c*(-4*a*c+b^2)^(1/2)+1/2*b/c)*a*x*f+4*c^2/(4*a*c-b^2)^2/(4*a*
c+3*b^2)*a*ln((4*a*c+3*b^2)*a*(2*c*x^2+(-4*a*c+b^2)^(1/2)+b))*(-4
*a*c+b^2)^(1/2)*e+3*c/(4*a*c-b^2)^2/(4*a*c+3*b^2)*ln((4*a*c+3*b^2
)*a*(2*c*x^2+(-4*a*c+b^2)^(1/2)+b))*(-4*a*c+b^2)^(1/2)*b^2*e-4*c^
2/(4*a*c-b^2)^2/(4*a*c+3*b^2)*a*ln((4*a*c+3*b^2)*a*(-2*c*x^2+(-4*
a*c+b^2)^(1/2)-b))*(-4*a*c+b^2)^(1/2)*e-3*c/(4*a*c-b^2)^2/(4*a*c+
3*b^2)*ln((4*a*c+3*b^2)*a*(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b))*(-4*a*
c+b^2)^(1/2)*b^2*e-1/2/(4*a*c-b^2)^2/(x^2+1/2*b/c-1/2/c*(-4*a*c+b
^2)^(1/2))*x*b^2*f-1/2/(4*a*c-b^2)^2/(x^2+1/2/c*(-4*a*c+b^2)^(1/2
)+1/2*b/c)*x*b^2*f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{2\,acex^2 - (bcd - 2\,acf)x^3 + abe + \left(abf - \left(b^2 - 2\,ac\right)d\right)x}{2\left((ab^2c - 4\,a^2c^2)x^4 + a^2b^2 - 4\,a^3c + (ab^3 - 4\,a^2bc)x^2\right)} - \frac{\int \frac{4\,acex - abf - (bcd - 2\,acf)x^2 - \left(b^2 - 6\,ac\right)d}{cx^4 + bx^2 + a}\,dx}{2\left(ab^2 - 4\,a^2c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2 + e*x + d)/(c*x^4 + b*x^2 + a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*a*c*e*x^2 - (b*c*d - 2*a*c*f)*x^3 + a*b*e + (a*b*f - (b^2
 - 2*a*c)*d)*x)/((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c +
(a*b^3 - 4*a^2*b*c)*x^2) - 1/2*integrate((4*a*c*e*x - a*b*f - (b*
c*d - 2*a*c*f)*x^2 - (b^2 - 6*a*c)*d)/(c*x^4 + b*x^2 + a), x)/(a*
b^2 - 4*a^2*c)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2 + e*x + d)/(c*x^4 + b*x^2 + a)^2,x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2 + e*x + d)/(c*x^4 + b*x^2 + a)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.38 $\int \frac{d+ex+fx^2+gx^3}{(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=386

$$\frac{x\left(cx^2(bd-2af)-abf-2acd+b^2d\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{c}\left(\frac{4abf-12acd+b^2d}{\sqrt{b^2-4ac}}-2af+bd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(-\frac{4abf-12acd+b^2d}{\sqrt{b^2-4ac}}-2af+bd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}a\left(b^2-4ac\right)\sqrt{\sqrt{b^2-4ac}+b}}+\frac{(2ce-bg)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}-\frac{-2ag+x^2(2ce-bg)+be}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

```
[Out] (x*(b^2*d - 2*a*c*d - a*b*f + c*(b*d - 2*a*f)*x^2))/(2*a*(b^2 - 4
*a*c)*(a + b*x^2 + c*x^4)) - (b*e - 2*a*g + (2*c*e - b*g)*x^2)/(2
*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[c]*(b*d - 2*a*f + (b^
2*d - 12*a*c*d + 4*a*b*f)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt
[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*(b^2 - 4*a*c)*S
qrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[c]*(b*d - 2*a*f - (b^2*d - 12
*a*c*d + 4*a*b*f)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/S
qrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*(b^2 - 4*a*c)*Sqrt[b +
Sqrt[b^2 - 4*a*c]]) + ((2*c*e - b*g)*ArcTanh[(b + 2*c*x^2)/Sqrt[b
^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rubi [A] time = 1.38711, antiderivative size = 386, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$

$$\frac{x\left(cx^2(bd-2af)-abf-2acd+b^2d\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{c}\left(\frac{4abf-12acd+b^2d}{\sqrt{b^2-4ac}}-2af+bd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(-\frac{4abf-12acd+b^2d}{\sqrt{b^2-4ac}}-2af+bd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}a\left(b^2-4ac\right)\sqrt{\sqrt{b^2-4ac}+b}}+\frac{(2ce-bg)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}-\frac{-2ag+x^2(2ce-bg)+be}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x + f*x^2 + g*x^3)/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (x*(b^2*d - 2*a*c*d - a*b*f + c*(b*d - 2*a*f)*x^2))/(2*a*(b^2 - 4
*a*c)*(a + b*x^2 + c*x^4)) - (b*e - 2*a*g + (2*c*e - b*g)*x^2)/(2
*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[c]*(b*d - 2*a*f + (b^
2*d - 12*a*c*d + 4*a*b*f)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt
[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*(b^2 - 4*a*c)*S
qrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[c]*(b*d - 2*a*f - (b^2*d - 12
*a*c*d + 4*a*b*f)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/S
qrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*(b^2 - 4*a*c)*Sqrt[b +
Sqrt[b^2 - 4*a*c]]) + ((2*c*e - b*g)*ArcTanh[(b + 2*c*x^2)/Sqrt[b
^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((g*x**3+f*x**2+e*x+d)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 2.96732, size = 421, normalized size = 1.09

$$\frac{1}{4}\left(\frac{-4a^2g+2ab(e+x(f-gx))+4acx(d+x(e+fx))-2bdx\left(b+cx^2\right)}{a\left(4ac-b^2\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{2}\sqrt{c}\left(b\left(d\sqrt{b^2-4ac}+4af\right)-2a\left(f\sqrt{b^2-4ac}+6cd\right)+b^2d\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{a\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\sqrt{c}\left(bd\sqrt{b^2-4ac}-2af\sqrt{b^2-4ac}-4abf+12acd+b^2(-d)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{a\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{2(bg-2ce)\log\left(\sqrt{b^2-4ac}-b-2cx^2\right)}{\left(b^2-4ac\right)^{3/2}}-\frac{2(bg-2ce)\log\left(\sqrt{b^2-4ac}+b+2cx^2\right)}{\left(b^2-4ac\right)^{3/2}}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x + f*x^2 + g*x^3)/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((-4*a^2*g - 2*b*d*x*(b + c*x^2) + 4*a*c*x*(d + x*(e + f*x)) + 2*
a*b*(e + x*(f - g*x))))/(a*(-b^2 + 4*a*c)*(a + b*x^2 + c*x^4)) + (
Sqrt[2]*Sqrt[c]*(b^2*d + b*(Sqrt[b^2 - 4*a*c]*d + 4*a*f) - 2*a*(6
*c*d + Sqrt[b^2 - 4*a*c]*f))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b -
Sqrt[b^2 - 4*a*c]]])/(a*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4
*a*c]]) + (Sqrt[2]*Sqrt[c]*(-(b^2*d) + 12*a*c*d + b*Sqrt[b^2 - 4*
a*c]*d - 4*a*b*f - 2*a*Sqrt[b^2 - 4*a*c]*f)*ArcTan[(Sqrt[2]*Sqrt[
c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(a*(b^2 - 4*a*c)^(3/2)*Sqrt[b
 + Sqrt[b^2 - 4*a*c]]) + (2*(-2*c*e + b*g)*Log[-b + Sqrt[b^2 - 4*
a*c] - 2*c*x^2])/(b^2 - 4*a*c)^(3/2) - (2*(-2*c*e + b*g)*Log[b +
Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(3/2))/4
```

Maple [B] time = 0.161, size = 3544, normalized size = 9.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] -2*c/(4*a*c-b^2)^2/(4*a*c+3*b^2)*a*ln((4*a*c+3*b^2)*a*(2*c*x^2+(-
4*a*c+b^2)^(1/2)+b))*(-4*a*c+b^2)^(1/2)*b*g+2*c^2/(4*a*c-b^2)^2*2
^(1/2)/(4*a*c+3*b^2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*
x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^3*d-3/4*c/(4*a*c-b
^2)^2*2^(1/2)/(4*a*c+3*b^2)/a/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*ar
ctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*(-4*a*c+b^2)^(
1/2)*b^4*d+4*c^2/(4*a*c-b^2)^2*2^(1/2)/(4*a*c+3*b^2)*a/((b+(-4*a*
c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))
*c)^(1/2))*b^2*f-4*c^3/(4*a*c-b^2)^2*2^(1/2)/(4*a*c+3*b^2)*a/((b+
(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^
(1/2))*c)^(1/2))*b*d+3/4*c/(4*a*c-b^2)^2*2^(1/2)/(4*a*c+3*b^2)/a/
((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b
^2)^(1/2))*c)^(1/2))*b^5*d+12*c^3/(4*a*c-b^2)^2*2^(1/2)/(4*a*c+3*
b^2)*a/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b
+(-4*a*c+b^2)^(1/2))*c)^(1/2))*(-4*a*c+b^2)^(1/2)*d-3*c/(4*a*c-b^
2)^2*2^(1/2)/(4*a*c+3*b^2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arcta
n(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*(-4*a*c+b^2)^(1/2
)*b^3*f+4*c^3/(4*a*c-b^2)^2*2^(1/2)/(4*a*c+3*b^2)*a/((-b+(-4*a*c+
b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))
*c)^(1/2))*b*d-3/4*c/(4*a*c-b^2)^2*2^(1/2)/(4*a*c+3*b^2)/a/((-b+(
-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)
^(1/2))*c)^(1/2))*b^5*d+8*c^2/(4*a*c-b^2)^2*2^(1/2)/(4*a*c+3*b^2)
/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+
b^2)^(1/2))*c)^(1/2))*(-4*a*c+b^2)^(1/2)*b^2*d-3*c/(4*a*c-b^2)^2*
2^(1/2)/(4*a*c+3*b^2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c
*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*(-4*a*c+b^2)^(1/2)*
b^3*f+8*c^2/(4*a*c-b^2)^2*2^(1/2)/(4*a*c+3*b^2)/((-b+(-4*a*c+b^2)
^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^
(1/2))*(-4*a*c+b^2)^(1/2)*b^2*d-4*c^2/(4*a*c-b^2)^2*2^(1/2)/(4*a*
```

```
c+3*b^2)*a/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/
((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^2*f+12*c^3/(4*a*c-b^2)^2*2^(
1/2)/(4*a*c+3*b^2)*a/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*
2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*(-4*a*c+b^2)^(1/2)*d-1/
4/(4*a*c-b^2)^2/(x^2+1/2/c*(-4*a*c+b^2)^(1/2)+1/2*b/c)/a*d*x*b^2*
(-4*a*c+b^2)^(1/2)+c/(4*a*c-b^2)^2/(x^2+1/2/c*(-4*a*c+b^2)^(1/2)+
1/2*b/c)*d*x*(-4*a*c+b^2)^(1/2)-c/(4*a*c-b^2)^2/(x^2+1/2/c*(-4*a*
c+b^2)^(1/2)+1/2*b/c)*d*x*b+1/4/(4*a*c-b^2)^2/(x^2+1/2/c*(-4*a*c+
b^2)^(1/2)+1/2*b/c)/a*d*x*b^3-c/(4*a*c-b^2)^2/(x^2+1/2*b/c-1/2/c*
(-4*a*c+b^2)^(1/2))*d*x*(-4*a*c+b^2)^(1/2)-c/(4*a*c-b^2)^2/(x^2+1
/2*b/c-1/2/c*(-4*a*c+b^2)^(1/2))*d*x*b+1/4/(4*a*c-b^2)^2/(x^2+1/2
*b/c-1/2/c*(-4*a*c+b^2)^(1/2))/a*d*x*b^3-4*c^2/(4*a*c-b^2)^2*2^(1
/2)/(4*a*c+3*b^2)*a/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x
*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*(-4*a*c+b^2)^(1/2)*b*
f-3/4*c/(4*a*c-b^2)^2*2^(1/2)/(4*a*c+3*b^2)/a/((-b+(-4*a*c+b^2)^(
1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1
/2))*(-4*a*c+b^2)^(1/2)*b^4*d-4*c^2/(4*a*c-b^2)^2*2^(1/2)/(4*a*c+
3*b^2)*a/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+
(-4*a*c+b^2)^(1/2))*c)^(1/2))*(-4*a*c+b^2)^(1/2)*b*f+1/4/(4*a*c-b
^2)^2/(x^2+1/2*b/c-1/2/c*(-4*a*c+b^2)^(1/2))/a*d*x*b^2*(-4*a*c+b^
2)^(1/2)-3/2/(4*a*c-b^2)^2/(4*a*c+3*b^2)*ln((4*a*c+3*b^2)*a*(2*c*
x^2+(-4*a*c+b^2)^(1/2)+b))*(-4*a*c+b^2)^(1/2)*b^3*g+3/2/(4*a*c-b^
2)^2/(4*a*c+3*b^2)*ln((4*a*c+3*b^2)*a*(-2*c*x^2+(-4*a*c+b^2)^(1/2
)-b))*(-4*a*c+b^2)^(1/2)*b^3*g-1/4/c/(4*a*c-b^2)^2/(x^2+1/2*b/c-1
/2/c*(-4*a*c+b^2)^(1/2))*(-4*a*c+b^2)^(1/2)*b^2*g+1/4/c/(4*a*c-b^
2)^2/(x^2+1/2/c*(-4*a*c+b^2)^(1/2)+1/2*b/c)*(-4*a*c+b^2)^(1/2)*b^
2*g-8*c^3/(4*a*c-b^2)^2*2^(1/2)/(4*a*c+3*b^2)*a^2/((-b+(-4*a*c+b^
2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c
)^(1/2))*f+3/2*c/(4*a*c-b^2)^2*2^(1/2)/(4*a*c+3*b^2)/((-b+(-4*a*c
+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2)
)*c)^(1/2))*b^4*f+2*c/(4*a*c-b^2)^2/(x^2+1/2*b/c-1/2/c*(-4*a*c+b^
2)^(1/2))*e*a-1/2/(4*a*c-b^2)^2/(x^2+1/2/c*(-4*a*c+b^2)^(1/2)+1/2
*b/c)*e*b^2-1/2/(4*a*c-b^2)^2/(x^2+1/2*b/c-1/2/c*(-4*a*c+b^2)^(1/
2))*e*b^2+2*c/(4*a*c-b^2)^2/(x^2+1/2/c*(-4*a*c+b^2)^(1/2)+1/2*b/c
)*e*a+1/(4*a*c-b^2)^2/(x^2+1/2*b/c-1/2/c*(-4*a*c+b^2)^(1/2))*(-4*
a*c+b^2)^(1/2)*a*g-1/(4*a*c-b^2)^2/(x^2+1/2*b/c-1/2/c*(-4*a*c+b^2
)^(1/2))*a*b*g+1/4/c/(4*a*c-b^2)^2/(x^2+1/2/c*(-4*a*c+b^2)^(1/2)+
1/2*b/c)*b^3*g+1/4/c/(4*a*c-b^2)^2/(x^2+1/2*b/c-1/2/c*(-4*a*c+b^2
)^(1/2))*b^3*g-1/(4*a*c-b^2)^2/(x^2+1/2/c*(-4*a*c+b^2)^(1/2)+1/2*
b/c)*(-4*a*c+b^2)^(1/2)*a*g-1/(4*a*c-b^2)^2/(x^2+1/2/c*(-4*a*c+b^
2)^(1/2)+1/2*b/c)*a*b*g+8*c^3/(4*a*c-b^2)^2*2^(1/2)/(4*a*c+3*b^2)
*a^2/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*
a*c+b^2)^(1/2))*c)^(1/2))*f-3/2*c/(4*a*c-b^2)^2*2^(1/2)/(4*a*c+3*
b^2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*
a*c+b^2)^(1/2))*c)^(1/2))*b^4*f-2*c^2/(4*a*c-b^2)^2*2^(1/2)/(4*a*
c+3*b^2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+
(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^3*d+2*c/(4*a*c-b^2)^2/(x^2+1/2*b/
c-1/2/c*(-4*a*c+b^2)^(1/2))*a*x*f+2*c/(4*a*c-b^2)^2/(x^2+1/2/c*(-
4*a*c+b^2)^(1/2)+1/2*b/c)*a*x*f+4*c^2/(4*a*c-b^2)^2/(4*a*c+3*b^2)
*a*ln((4*a*c+3*b^2)*a*(2*c*x^2+(-4*a*c+b^2)^(1/2)+b))*(-4*a*c+b^2
)^(1/2)*e+3*c/(4*a*c-b^2)^2/(4*a*c+3*b^2)*ln((4*a*c+3*b^2)*a*(2*c
*x^2+(-4*a*c+b^2)^(1/2)+b))*(-4*a*c+b^2)^(1/2)*b^2*e-4*c^2/(4*a*c
-b^2)^2/(4*a*c+3*b^2)*a*ln((4*a*c+3*b^2)*a*(-2*c*x^2+(-4*a*c+b^2)
^(1/2)-b))*(-4*a*c+b^2)^(1/2)*e-3*c/(4*a*c-b^2)^2/(4*a*c+3*b^2)*l
```

```
n((4*a*c+3*b^2)*a*(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b))*(-4*a*c+b^2)^(
1/2)*b^2*e-1/2/(4*a*c-b^2)^2/(x^2+1/2*b/c-1/2/c*(-4*a*c+b^2)^(1/2
))*x*b^2*f-1/2/(4*a*c-b^2)^2/(x^2+1/2/c*(-4*a*c+b^2)^(1/2)+1/2*b/
c)*x*b^2*f+2*c/(4*a*c-b^2)^2/(4*a*c+3*b^2)*a*ln((4*a*c+3*b^2)*a*(
-2*c*x^2+(-4*a*c+b^2)^(1/2)-b))*(-4*a*c+b^2)^(1/2)*b*g
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{(bcd-2\,acf)x^3-abe+2\,a^2g-(2\,ace-abg)x^2-\left(abf-\left(b^2-2\,ac\right)d\right)x}{2\left((ab^2c-4\,a^2c^2)x^4+a^2b^2-4\,a^3c+(ab^3-4\,a^2bc)x^2\right)}-\frac{-\int\frac{abf+(bcd-2\,acf)x^2+\left(b^2-6\,ac\right)d-2\,(2\,ace-abg)x}{cx^4+bx^2+a}\,dx}{2\left(ab^2-4\,a^2c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3 + f*x^2 + e*x + d)/(c*x^4 + b*x^2 + a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((b*c*d - 2*a*c*f)*x^3 - a*b*e + 2*a^2*g - (2*a*c*e - a*b*g)*
x^2 - (a*b*f - (b^2 - 2*a*c)*d)*x)/((a*b^2*c - 4*a^2*c^2)*x^4 + a
^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2) - 1/2*integrate(-(a*b
*f + (b*c*d - 2*a*c*f)*x^2 + (b^2 - 6*a*c)*d - 2*(2*a*c*e - a*b*g
)*x)/(c*x^4 + b*x^2 + a), x)/(a*b^2 - 4*a^2*c)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3 + f*x^2 + e*x + d)/(c*x^4 + b*x^2 + a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**3+f*x**2+e*x+d)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3 + f*x^2 + e*x + d)/(c*x^4 + b*x^2 + a)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.39 $\int \frac{d+ex+fx^2+gx^3+hx^4}{(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=439

$$\frac{x\left(x^2(abh-2acf+bcd)-abf-2a(cd-ah)+b^2d\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{b^2(cd-ah)+4abcf-4ac(ah+3cd)}{\sqrt{b^2-4ac}}+abh-2acf+bcd\right)}{2\sqrt{2}a\sqrt{c}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-\frac{b^2(cd-ah)+4abcf-4ac(ah+3cd)}{\sqrt{b^2-4ac}}+abh-2acf+bcd\right)}{2\sqrt{2}a\sqrt{c}\left(b^2-4ac\right)\sqrt{\sqrt{b^2-4ac}+b}}+\frac{(2ce-bg)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}-\frac{-2ag+x^2(2ce-bg)+be}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

```
[Out] -(b*e - 2*a*g + (2*c*e - b*g)*x^2)/(2*(b^2 - 4*a*c)*(a + b*x^2 +
c*x^4)) + (x*(b^2*d - a*b*f - 2*a*(c*d - a*h) + (b*c*d - 2*a*c*f
+ a*b*h)*x^2))/(2*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + ((b*c*d
- 2*a*c*f + a*b*h + (4*a*b*c*f + b^2*(c*d - a*h) - 4*a*c*(3*c*d +
 a*h))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqr
t[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b - Sqr
t[b^2 - 4*a*c]]) + ((b*c*d - 2*a*c*f + a*b*h - (4*a*b*c*f + b^2*(
c*d - a*h) - 4*a*c*(3*c*d + a*h))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt
[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*Sqrt[c]
*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + ((2*c*e - b*g)*ArcT
anh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rubi [A] time = 4.41294, antiderivative size = 439, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.229$

$$\frac{x\left(x^2(abh-2acf+bcd)-abf-2a(cd-ah)+b^2d\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{b^2(cd-ah)+4abcf-4ac(ah+3cd)}{\sqrt{b^2-4ac}}+abh-2acf+bcd\right)}{2\sqrt{2}a\sqrt{c}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-\frac{b^2(cd-ah)+4abcf-4ac(ah+3cd)}{\sqrt{b^2-4ac}}+abh-2acf+bcd\right)}{2\sqrt{2}a\sqrt{c}\left(b^2-4ac\right)\sqrt{\sqrt{b^2-4ac}+b}}+\frac{(2ce-bg)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}-\frac{-2ag+x^2(2ce-bg)+be}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x + f*x^2 + g*x^3 + h*x^4)/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] -(b*e - 2*a*g + (2*c*e - b*g)*x^2)/(2*(b^2 - 4*a*c)*(a + b*x^2 +
c*x^4)) + (x*(b^2*d - a*b*f - 2*a*(c*d - a*h) + (b*c*d - 2*a*c*f
+ a*b*h)*x^2))/(2*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + ((b*c*d
- 2*a*c*f + a*b*h + (4*a*b*c*f + b^2*(c*d - a*h) - 4*a*c*(3*c*d +
 a*h))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqr
t[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b - Sqr
t[b^2 - 4*a*c]]) + ((b*c*d - 2*a*c*f + a*b*h - (4*a*b*c*f + b^2*(
c*d - a*h) - 4*a*c*(3*c*d + a*h))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt
[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*Sqrt[c]
*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + ((2*c*e - b*g)*ArcT
anh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((h*x**4+g*x**3+f*x**2+e*x+d)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 4.52876, size = 489, normalized size = 1.11

$$\frac{1}{4}\left(\frac{-4a^2(g+hx)+2ab(e+x(f-x(g+hx)))+4acx(d+x(e+fx))-2bdx\left(b+cx^2\right)}{a\left(4ac-b^2\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(b\left(cd\sqrt{b^2-4ac}+ah\sqrt{b^2-4ac}+4acf\right)-2ac\left(f\sqrt{b^2-4ac}+2ah+6cd\right)+b^2(cd-ah)\right)}{a\sqrt{c}\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(b\left(cd\sqrt{b^2-4ac}+ah\sqrt{b^2-4ac}-4acf\right)+2ac\left(-f\sqrt{b^2-4ac}+2ah+6cd\right)+b^2(ah-cd)\right)}{a\sqrt{c}\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{2(bg-2ce)\log\left(\sqrt{b^2-4ac}-b-2cx^2\right)}{\left(b^2-4ac\right)^{3/2}}-\frac{2(bg-2ce)\log\left(\sqrt{b^2-4ac}+b+2cx^2\right)}{\left(b^2-4ac\right)^{3/2}}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x + f*x^2 + g*x^3 + h*x^4)/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((-4*a^2*(g + h*x) - 2*b*d*x*(b + c*x^2) + 4*a*c*x*(d + x*(e + f*
x)) + 2*a*b*(e + x*(f - x*(g + h*x))))/(a*(-b^2 + 4*a*c)*(a + b*x
^2 + c*x^4)) + (Sqrt[2]*(b^2*(c*d - a*h) - 2*a*c*(6*c*d + Sqrt[b^
2 - 4*a*c]*f + 2*a*h) + b*(c*Sqrt[b^2 - 4*a*c]*d + 4*a*c*f + a*Sq
rt[b^2 - 4*a*c]*h))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2
- 4*a*c]]])/(a*Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*
a*c]]) + (Sqrt[2]*(b^2*(-(c*d) + a*h) + 2*a*c*(6*c*d - Sqrt[b^2 -
 4*a*c]*f + 2*a*h) + b*(c*Sqrt[b^2 - 4*a*c]*d - 4*a*c*f + a*Sqrt[
b^2 - 4*a*c]*h))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4
*a*c]]])/(a*Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c
]]) + (2*(-2*c*e + b*g)*Log[-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2])/(b
^2 - 4*a*c)^(3/2) - (2*(-2*c*e + b*g)*Log[b + Sqrt[b^2 - 4*a*c] +
 2*c*x^2])/(b^2 - 4*a*c)^(3/2))/4
```

Maple [B] time = 0.153, size = 7598, normalized size = 17.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{(bcd-2\,acf+abh)x^3-abe+2\,a^2g-(2\,ace-abg)x^2-\left(abf-2\,a^2h-\left(b^2-2\,ac\right)d\right)x}{2\left((ab^2c-4\,a^2c^2)x^4+a^2b^2-4\,a^3c+(ab^3-4\,a^2bc)x^2\right)}+\frac{\int\frac{abf-2\,a^2h+(bcd-2\,acf+abh)x^2+\left(b^2-6\,ac\right)d-2\,(2\,ace-abg)x}{cx^4+bx^2+a}\,dx}{2\,(ab^2-4\,a^2c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((h*x^4 + g*x^3 + f*x^2 + e*x + d)/(c*x^4 + b*x^2 + a)^2,x, algorithm="ma
```

```
[Out] 1/2*((b*c*d - 2*a*c*f + a*b*h)*x^3 - a*b*e + 2*a^2*g - (2*a*c*e -
 a*b*g)*x^2 - (a*b*f - 2*a^2*h - (b^2 - 2*a*c)*d)*x)/((a*b^2*c -
4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2) + 1
```

```
/2*integrate((a*b*f - 2*a^2*h + (b*c*d - 2*a*c*f + a*b*h)*x^2 + (
b^2 - 6*a*c)*d - 2*(2*a*c*e - a*b*g)*x)/(c*x^4 + b*x^2 + a), x)/(
a*b^2 - 4*a^2*c)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^4 + g*x^3 + f*x^2 + e*x + d)/(c*x^4 + b*x^2 + a)^2,x, algorithm="fr
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((h*x**4+g*x**3+f*x**2+e*x+d)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^4 + g*x^3 + f*x^2 + e*x + d)/(c*x^4 + b*x^2 + a)^2,x, algorithm="gi
```

```
[Out] Exception raised: TypeError
```

3.40 $\int \frac{d+ex+fx^2+gx^3+hx^4+ix^5}{(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=468

$$\frac{x^2\left(-\left(-2aci+b^2i-bcg+2c^2e\right)\right)-b(ai+ce)+2acg}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{x\left(x^2(abh-2acf+bcd)-abf-2a(cd-ah)+b^2d\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{b^2(cd-ah)+4abcf-4ac(ah+3cd)}{\sqrt{b^2-4ac}}+abh-2acf+bcd\right)}{2\sqrt{2}a\sqrt{c}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-\frac{b^2(cd-ah)+4abcf-4ac(ah+3cd)}{\sqrt{b^2-4ac}}+abh-2acf+bcd\right)}{2\sqrt{2}a\sqrt{c}\left(b^2-4ac\right)\sqrt{\sqrt{b^2-4ac}+b}}+\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)(2ai-bg+2ce)}{\left(b^2-4ac\right)^{3/2}}$$

```
[Out] (x*(b^2*d - a*b*f - 2*a*(c*d - a*h) + (b*c*d - 2*a*c*f + a*b*h)*x
^2))/(2*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (2*a*c*g - b*(c*e
+ a*i) - (2*c^2*e - b*c*g + b^2*i - 2*a*c*i)*x^2)/(2*c*(b^2 - 4*a
*c)*(a + b*x^2 + c*x^4)) + ((b*c*d - 2*a*c*f + a*b*h + (4*a*b*c*f
 + b^2*(c*d - a*h) - 4*a*c*(3*c*d + a*h))/Sqrt[b^2 - 4*a*c])*ArcT
an[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a
*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((b*c*d - 2
*a*c*f + a*b*h - (4*a*b*c*f + b^2*(c*d - a*h) - 4*a*c*(3*c*d + a*
h))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b
^2 - 4*a*c]]])/(2*Sqrt[2]*a*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b
^2 - 4*a*c]]) + ((2*c*e - b*g + 2*a*i)*ArcTanh[(b + 2*c*x^2)/Sqrt
[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rubi [A] time = 3.87628, antiderivative size = 468, normalized size of antiderivative = 1., number of

steps used = 10, number of rules used = 9, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.225$

$$\frac{x^2\left(-\left(-2aci+b^2i-bcg+2c^2e\right)\right)-b(ai+ce)+2acg}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{x\left(x^2(abh-2acf+bcd)-abf-2a(cd-ah)+b^2d\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{b^2(cd-ah)+4abcf-4ac(ah+3cd)}{\sqrt{b^2-4ac}}+abh-2acf+bcd\right)}{2\sqrt{2}a\sqrt{c}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-\frac{b^2(cd-ah)+4abcf-4ac(ah+3cd)}{\sqrt{b^2-4ac}}+abh-2acf+bcd\right)}{2\sqrt{2}a\sqrt{c}\left(b^2-4ac\right)\sqrt{\sqrt{b^2-4ac}+b}} + \frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)(2ai-bg+2ce)}{\left(b^2-4ac\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5)/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (x*(b^2*d - a*b*f - 2*a*(c*d - a*h) + (b*c*d - 2*a*c*f + a*b*h)*x
^2))/(2*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (2*a*c*g - b*(c*e
+ a*i) - (2*c^2*e - b*c*g + b^2*i - 2*a*c*i)*x^2)/(2*c*(b^2 - 4*a
*c)*(a + b*x^2 + c*x^4)) + ((b*c*d - 2*a*c*f + a*b*h + (4*a*b*c*f
 + b^2*(c*d - a*h) - 4*a*c*(3*c*d + a*h))/Sqrt[b^2 - 4*a*c])*ArcT
an[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a
*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((b*c*d - 2
*a*c*f + a*b*h - (4*a*b*c*f + b^2*(c*d - a*h) - 4*a*c*(3*c*d + a*
h))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b
^2 - 4*a*c]]])/(2*Sqrt[2]*a*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b
^2 - 4*a*c]]) + ((2*c*e - b*g + 2*a*i)*ArcTanh[(b + 2*c*x^2)/Sqrt
[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((i*x**5+h*x**4+g*x**3+f*x**2+e*x+d)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 5.20845, size = 524, normalized size = 1.12

$$\frac{1}{4}\left(\frac{2\left(a^2(bi-2c(g+x(h+ix)))+a\left(b^2ix^2+bc(e+x(f-x(g+hx)))+2c^2x(d+x(e+fx))\right)-bcdx\left(b+cx^2\right)\right)}{ac\left(4ac-b^2\right)\left(a+bx^2+cx^4\right)}\right.$$
$$+\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(b\left(cd\sqrt{b^2-4ac}+ah\sqrt{b^2-4ac}+4acf\right)-2ac\left(f\sqrt{b^2-4ac}+2ah+6cd\right)+b^2(cd-ah)\right)}{a\sqrt{c}\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}$$
$$+\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(b\left(cd\sqrt{b^2-4ac}+ah\sqrt{b^2-4ac}-4acf\right)+2ac\left(-f\sqrt{b^2-4ac}+2ah+6cd\right)+b^2(ah-cd)\right)}{a\sqrt{c}\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}$$
$$\left.+\frac{2\log\left(\sqrt{b^2-4ac}-b-2cx^2\right)(-2ai+bg-2ce)}{\left(b^2-4ac\right)^{3/2}}+\frac{2\log\left(\sqrt{b^2-4ac}+b+2cx^2\right)(2ai-bg+2ce)}{\left(b^2-4ac\right)^{3/2}}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5)/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((2*(-(b*c*d*x*(b + c*x^2)) + a^2*(b*i - 2*c*(g + x*(h + i*x))) +
 a*(b^2*i*x^2 + 2*c^2*x*(d + x*(e + f*x)) + b*c*(e + x*(f - x*(g
+ h*x))))))/(a*c*(-b^2 + 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[2]*(
b^2*(c*d - a*h) - 2*a*c*(6*c*d + Sqrt[b^2 - 4*a*c]*f + 2*a*h) + b
*(c*Sqrt[b^2 - 4*a*c]*d + 4*a*c*f + a*Sqrt[b^2 - 4*a*c]*h))*ArcTa
n[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(a*Sqrt[c]*(b
^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*(b^2*(-
(c*d) + a*h) + 2*a*c*(6*c*d - Sqrt[b^2 - 4*a*c]*f + 2*a*h) + b*(c
*Sqrt[b^2 - 4*a*c]*d - 4*a*c*f + a*Sqrt[b^2 - 4*a*c]*h))*ArcTan[(
Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(a*Sqrt[c]*(b^2
- 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + (2*(-2*c*e + b*g -
2*a*i)*Log[-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2])/(b^2 - 4*a*c)^(3/2)
 + (2*(2*c*e - b*g + 2*a*i)*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])
/(b^2 - 4*a*c)^(3/2))/4
```

Maple [B] time = 0.093, size = 8189, normalized size = 17.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{abce - 2\,a^2cg + a^2bi - \left(bc^2d - 2\,ac^2f + abch\right)x^3 + \left(2\,ac^2e - abcg + \left(ab^2 - 2\,a^2c\right)i\right)x^2 + \left(abcf - 2\,a^2ch - \left(b^2c - 2\,ac^2\right)d}{2\,(a^2b^2c - 4\,a^3c^2 + (ab^2c^2 - 4\,a^2c^3)x^4 + (ab^3c - 4\,a^2bc^2)x^2)}$$
$$+\frac{\int \frac{abf - 2\,a^2h + (bcd - 2\,acf + abh)x^2 + \left(b^2 - 6\,ac\right)d - 2\,\left(2\,ace - abg + 2\,a^2i\right)x}{cx^4 + bx^2 + a}\,dx}{2\,(ab^2 - 4\,a^2c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^5 + h*x^4 + g*x^3 + f*x^2 + e*x + d)/(c*x^4 + b*x^2 + a)^2,x, algori
```

```
[Out] -1/2*(a*b*c*e - 2*a^2*c*g + a^2*b*i - (b*c^2*d - 2*a*c^2*f + a*b*
c*h)*x^3 + (2*a*c^2*e - a*b*c*g + (a*b^2 - 2*a^2*c)*i)*x^2 + (a*b
*c*f - 2*a^2*c*h - (b^2*c - 2*a*c^2)*d)*x)/(a^2*b^2*c - 4*a^3*c^2
 + (a*b^2*c^2 - 4*a^2*c^3)*x^4 + (a*b^3*c - 4*a^2*b*c^2)*x^2) + 1
/2*integrate((a*b*f - 2*a^2*h + (b*c*d - 2*a*c*f + a*b*h)*x^2 + (
b^2 - 6*a*c)*d - 2*(2*a*c*e - a*b*g + 2*a^2*i)*x)/(c*x^4 + b*x^2
+ a), x)/(a*b^2 - 4*a^2*c)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^5 + h*x^4 + g*x^3 + f*x^2 + e*x + d)/(c*x^4 + b*x^2 + a)^2,x, algori
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x**5+h*x**4+g*x**3+f*x**2+e*x+d)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^5 + h*x^4 + g*x^3 + f*x^2 + e*x + d)/(c*x^4 + b*x^2 + a)^2,x, algori
```

```
[Out] Exception raised: TypeError
```

3.41 $\int \frac{d+ex+fx^2+gx^3+hx^4+jx^5+kx^6+lx^7+mx^8}{(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=770

$$-\frac{x\left(b^2\left(-\left(a^2m+c^2d\right)\right)+x^2\left(-bc\left(-3a^2m+ach+c^2d\right)-ab^3m+ab^2ck+2ac^2(cf-ak)\right)+2ac\left(a^2m-ach+c^2d\right)+abc(a\right.}{2ac^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(-\frac{-b^2c\left(-19a^2m-ach+c^2d\right)+4ac^2\left(-5a^2m+ach+3c^2d\right)-3ab^4m+ab^3ck-4abc^2(2ak+cf)}{\sqrt{b^2-4ac}}+bc\left(13a^2m+ach+c^2d\right)-3a\right.}{2\sqrt{2}ac^{5/2}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}$$

$$+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(\frac{-b^2c\left(-19a^2m-ach+c^2d\right)+4ac^2\left(-5a^2m+ach+3c^2d\right)-3ab^4m+ab^3ck-4abc^2(2ak+cf)}{\sqrt{b^2-4ac}}+bc\left(13a^2m+ach+c^2d\right)-3ab\right.}{2\sqrt{2}ac^{5/2}\left(b^2-4ac\right)\sqrt{\sqrt{b^2-4ac}+b}}$$

$$+\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)\left(-c^2(2bg-4aj)-6abcl+b^3l+4c^3e\right)}{2c^2\left(b^2-4ac\right)^{3/2}}$$

$$-\frac{x^2\left(-c^2(2aj+bg)+bc(3al+bj)+b^3(-l)+2c^3e\right)-ab^2l+bc(aj+ce)-2ac(cg-al)}{2c^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$+\frac{l\log\left(a+bx^2+cx^4\right)}{4c^2}+\frac{mx}{c^2}$$

```
[Out] (m*x)/c^2 - (b*c*(c*e + a*j) - a*b^2*l - 2*a*c*(c*g - a*l) + (2*c
^3*e - c^2*(b*g + 2*a*j) - b^3*l + b*c*(b*j + 3*a*l))*x^2)/(2*c^2
*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (x*(a*b*c*(c*f + a*k) - b^2
*(c^2*d + a^2*m) + 2*a*c*(c^2*d - a*c*h + a^2*m) + (a*b^2*c*k + 2
*a*c^2*(c*f - a*k) - a*b^3*m - b*c*(c^2*d + a*c*h - 3*a^2*m))*x^2
))/(2*a*c^2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + ((a*b^2*c*k - 2*
a*c^2*(c*f + 3*a*k) - 3*a*b^3*m + b*c*(c^2*d + a*c*h + 13*a^2*m)
- (a*b^3*c*k - 4*a*b*c^2*(c*f + 2*a*k) - 3*a*b^4*m - b^2*c*(c^2*d
 - a*c*h - 19*a^2*m) + 4*a*c^2*(3*c^2*d + a*c*h - 5*a^2*m))/Sqrt[
b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*
c]]])/(2*Sqrt[2]*a*c^(5/2)*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*
c]]) + ((a*b^2*c*k - 2*a*c^2*(c*f + 3*a*k) - 3*a*b^3*m + b*c*(c^2
*d + a*c*h + 13*a^2*m) + (a*b^3*c*k - 4*a*b*c^2*(c*f + 2*a*k) - 3
*a*b^4*m - b^2*c*(c^2*d - a*c*h - 19*a^2*m) + 4*a*c^2*(3*c^2*d +
a*c*h - 5*a^2*m))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/S
qrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*c^(5/2)*(b^2 - 4*a*c)*S
qrt[b + Sqrt[b^2 - 4*a*c]]) + ((4*c^3*e - c^2*(2*b*g - 4*a*j) + b
^3*l - 6*a*b*c*l)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c^
2*(b^2 - 4*a*c)^(3/2)) + (l*Log[a + b*x^2 + c*x^4])/(4*c^2)
```

Rubi [A] time = 25.763, antiderivative size = 770, normalized size of antiderivative = 1., number of

steps used = 13, number of rules used = 11, integrand size = 55, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$-\frac{x\left(b^2\left(-\left(a^2m+c^2d\right)\right)+x^2\left(-bc\left(-3a^2m+ach+c^2d\right)-ab^3m+ab^2ck+2ac^2(cf-ak)\right)+2ac\left(a^2m-ach+c^2d\right)+abc(a\cdots}{2ac^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$
$$+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(-\frac{-b^2c\left(-19a^2m-ach+c^2d\right)+4ac^2\left(-5a^2m+ach+3c^2d\right)-3ab^4m+ab^3ck-4abc^2(2ak+cf)}{\sqrt{b^2-4ac}}+bc\left(13a^2m+ach+c^2d\right)-3a\cdots\right.}{2\sqrt{2}ac^{5/2}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}$$
$$+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(\frac{-b^2c\left(-19a^2m-ach+c^2d\right)+4ac^2\left(-5a^2m+ach+3c^2d\right)-3ab^4m+ab^3ck-4abc^2(2ak+cf)}{\sqrt{b^2-4ac}}+bc\left(13a^2m+ach+c^2d\right)-3ab\cdots\right.}{2\sqrt{2}ac^{5/2}\left(b^2-4ac\right)\sqrt{\sqrt{b^2-4ac}+b}}$$
$$+\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)\left(-c^2(2bg-4aj)-6abcl+b^3l+4c^3e\right)}{2c^2\left(b^2-4ac\right)^{3/2}}$$
$$-\frac{x^2\left(-c^2(2aj+bg)+bc(3al+bj)+b^3(-l)+2c^3e\right)-ab^2l+bc(aj+ce)-2ac(cg-al)}{2c^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$
$$+\frac{l\log\left(a+bx^2+cx^4\right)}{4c^2}+\frac{mx}{c^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x + f*x^2 + g*x^3 + h*x^4 + j*x^5 + k*x^6 + l*x^7 + m*x^8)/(a + b*x^2 + c*
```

```
[Out] (m*x)/c^2 - (b*c*(c*e + a*j) - a*b^2*l - 2*a*c*(c*g - a*l) + (2*c
^3*e - c^2*(b*g + 2*a*j) - b^3*l + b*c*(b*j + 3*a*l))*x^2)/(2*c^2
*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (x*(a*b*c*(c*f + a*k) - b^2
*(c^2*d + a^2*m) + 2*a*c*(c^2*d - a*c*h + a^2*m) + (a*b^2*c*k + 2
*a*c^2*(c*f - a*k) - a*b^3*m - b*c*(c^2*d + a*c*h - 3*a^2*m))*x^2
))/(2*a*c^2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + ((a*b^2*c*k - 2*
a*c^2*(c*f + 3*a*k) - 3*a*b^3*m + b*c*(c^2*d + a*c*h + 13*a^2*m)
- (a*b^3*c*k - 4*a*b*c^2*(c*f + 2*a*k) - 3*a*b^4*m - b^2*c*(c^2*d
 - a*c*h - 19*a^2*m) + 4*a*c^2*(3*c^2*d + a*c*h - 5*a^2*m))/Sqrt[
b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*
c]]])/(2*Sqrt[2]*a*c^(5/2)*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*
c]]) + ((a*b^2*c*k - 2*a*c^2*(c*f + 3*a*k) - 3*a*b^3*m + b*c*(c^2
*d + a*c*h + 13*a^2*m) + (a*b^3*c*k - 4*a*b*c^2*(c*f + 2*a*k) - 3
*a*b^4*m - b^2*c*(c^2*d - a*c*h - 19*a^2*m) + 4*a*c^2*(3*c^2*d +
a*c*h - 5*a^2*m))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/S
qrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*c^(5/2)*(b^2 - 4*a*c)*S
qrt[b + Sqrt[b^2 - 4*a*c]]) + ((4*c^3*e - c^2*(2*b*g - 4*a*j) + b
^3*l - 6*a*b*c*l)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c^
2*(b^2 - 4*a*c)^(3/2)) + (l*Log[a + b*x^2 + c*x^4])/(4*c^2)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((m*x**8+l*x**7+k*x**6+j*x**5+h*x**4+g*x**3+f*x**2+e*x+d)/(c*x**
```

```
[Out] Timed out
```

Mathematica [A] time = 8.76673, size = 1109, normalized size = 1.44

$$\frac{mx}{c^2} - \frac{\left(-3amb^4 + ackb^3 + 3a\sqrt{b^2-4ac}mb^3 - c^3db^2 + ac^2hb^2 - ac\sqrt{b^2-4ac}kb^2 + 19a^2cmb^2 - c^3\sqrt{b^2-4ac}db - 4ac^3fb - ac^2\sqrt{}\right.}{2\sqrt{2}ac^{5/2}(b^2}$$

$$- \frac{\left(3amb^4 - ackb^3 + 3a\sqrt{b^2-4ac}mb^3 + c^3db^2 - ac^2hb^2 - ac\sqrt{b^2-4ac}kb^2 - 19a^2cmb^2 - c^3\sqrt{b^2-4ac}db + 4ac^3fb - ac^2\sqrt{}\right.}{2\sqrt{2}ac^{5/2}(b^2}$$

$$- \frac{\left(lb^3 - \sqrt{b^2-4ac}lb^2 - 2c^2gb - 6aclb + 4c^3e + 4ac^2j + 4ac\sqrt{b^2-4ac}l\right)\log\left(-2cx^2 - b + \sqrt{b^2-4ac}\right)}{4c^2\left(b^2-4ac\right)^{3/2}}$$

$$- \frac{\left(-lb^3 - \sqrt{b^2-4ac}lb^2 + 2c^2gb + 6aclb - 4c^3e - 4ac^2j + 4ac\sqrt{b^2-4ac}l\right)\log\left(2cx^2 + b + \sqrt{b^2-4ac}\right)}{4c^2\left(b^2-4ac\right)^{3/2}}$$

$$+ \frac{2cla^3 + 2cmxa^3 - 2c^2kx^3a^2 + 3bcmx^3a^2 - 2c^2jx^2a^2 + 3bclx^2a^2 - 2c^2ga^2 + bcja^2 - b^2la^2 - 2c^2hxa^2 + bckxa^2 - b^2mxa^2 +}{2ac^2(4ac -}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2 + g*x^3 + h*x^4 + j*x^5 + k*x^6 + l*x^7 + m*x^8)/(a + b*x
```

```
[Out] (m*x)/c^2 + (a*b*c^2*e - 2*a^2*c^2*g + a^2*b*c*j - a^2*b^2*l + 2*
a^3*c*l - b^2*c^2*d*x + 2*a*c^3*d*x + a*b*c^2*f*x - 2*a^2*c^2*h*x
 + a^2*b*c*k*x - a^2*b^2*m*x + 2*a^3*c*m*x + 2*a*c^3*e*x^2 - a*b*
c^2*g*x^2 + a*b^2*c*j*x^2 - 2*a^2*c^2*j*x^2 - a*b^3*l*x^2 + 3*a^2
*b*c*l*x^2 - b*c^3*d*x^3 + 2*a*c^3*f*x^3 - a*b*c^2*h*x^3 + a*b^2*
c*k*x^3 - 2*a^2*c^2*k*x^3 - a*b^3*m*x^3 + 3*a^2*b*c*m*x^3)/(2*a*c
^2*(-b^2 + 4*a*c)*(a + b*x^2 + c*x^4)) - ((-(b^2*c^3*d) + 12*a*c^
4*d - b*c^3*Sqrt[b^2 - 4*a*c]*d - 4*a*b*c^3*f + 2*a*c^3*Sqrt[b^2
- 4*a*c]*f + a*b^2*c^2*h + 4*a^2*c^3*h - a*b*c^2*Sqrt[b^2 - 4*a*c
]*h + a*b^3*c*k - 8*a^2*b*c^2*k - a*b^2*c*Sqrt[b^2 - 4*a*c]*k + 6
*a^2*c^2*Sqrt[b^2 - 4*a*c]*k - 3*a*b^4*m + 19*a^2*b^2*c*m - 20*a^
3*c^2*m + 3*a*b^3*Sqrt[b^2 - 4*a*c]*m - 13*a^2*b*c*Sqrt[b^2 - 4*a
*c]*m)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(
2*Sqrt[2]*a*c^(5/2)*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c
]]) - ((b^2*c^3*d - 12*a*c^4*d - b*c^3*Sqrt[b^2 - 4*a*c]*d + 4*a*
b*c^3*f + 2*a*c^3*Sqrt[b^2 - 4*a*c]*f - a*b^2*c^2*h - 4*a^2*c^3*h
```

```
 - a*b*c^2*Sqrt[b^2 - 4*a*c]*h - a*b^3*c*k + 8*a^2*b*c^2*k - a*b^
2*c*Sqrt[b^2 - 4*a*c]*k + 6*a^2*c^2*Sqrt[b^2 - 4*a*c]*k + 3*a*b^4
*m - 19*a^2*b^2*c*m + 20*a^3*c^2*m + 3*a*b^3*Sqrt[b^2 - 4*a*c]*m
- 13*a^2*b*c*Sqrt[b^2 - 4*a*c]*m)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt
[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*c^(5/2)*(b^2 - 4*a*c)^(3/2
)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - ((4*c^3*e - 2*b*c^2*g + 4*a*c^2*
j + b^3*l - 6*a*b*c*l - b^2*Sqrt[b^2 - 4*a*c]*l + 4*a*c*Sqrt[b^2
- 4*a*c]*l)*Log[-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2])/(4*c^2*(b^2 -
4*a*c)^(3/2)) - ((-4*c^3*e + 2*b*c^2*g - 4*a*c^2*j - b^3*l + 6*a*
b*c*l - b^2*Sqrt[b^2 - 4*a*c]*l + 4*a*c*Sqrt[b^2 - 4*a*c]*l)*Log[
b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(4*c^2*(b^2 - 4*a*c)^(3/2))
```

Maple [B] time = 0.151, size = 16517, normalized size = 21.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((m*x^8+l*x^7+k*x^6+j*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{abc^2e - 2a^2c^2g + a^2bcj - \left(bc^3d - 2ac^3f + abc^2h - \left(ab^2c - 2a^2c^2\right)k + \left(ab^3 - 3a^2bc\right)m\right)x^3 + \left(2ac^3e - abc^2g + \left(ab^2c - 2 \right.\right.}{2\left(a^2b^2c^2 - 4a^3c^3 + \left(ab^2c^3 - 4a^2c\right.\right.}$$
$$+\frac{mx}{c^2}$$
$$-\frac{-\int \frac{abc^2f-2a^2c^2h+a^2bck+2\left(ab^2c-4a^2c^2\right)lx^3+\left(bc^3d-2ac^3f+abc^2h+\left(ab^2c-6a^2c^2\right)k-\left(3ab^3-13a^2bc\right)m\right)x^2+\left(b^2c^2-6ac^3\right)d-\left(3a^2b^2-10a^3c\right)m-2\left(2\right.}{cx^4+bx^2+a}}{2\left(ab^2c^2-4a^2c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((m*x^8 + l*x^7 + k*x^6 + j*x^5 + h*x^4 + g*x^3 + f*x^2 + e*x + d)/(c*x^4 +
```

```
[Out] -1/2*(a*b*c^2*e - 2*a^2*c^2*g + a^2*b*c*j - (b*c^3*d - 2*a*c^3*f
+ a*b*c^2*h - (a*b^2*c - 2*a^2*c^2)*k + (a*b^3 - 3*a^2*b*c)*m)*x^
3 + (2*a*c^3*e - a*b*c^2*g + (a*b^2*c - 2*a^2*c^2)*j - (a*b^3 - 3
*a^2*b*c)*l)*x^2 - (a^2*b^2 - 2*a^3*c)*l + (a*b*c^2*f - 2*a^2*c^2
*h + a^2*b*c*k - (b^2*c^2 - 2*a*c^3)*d - (a^2*b^2 - 2*a^3*c)*m)*x
)/(a^2*b^2*c^2 - 4*a^3*c^3 + (a*b^2*c^3 - 4*a^2*c^4)*x^4 + (a*b^3
*c^2 - 4*a^2*b*c^3)*x^2) + m*x/c^2 - 1/2*integrate(-(a*b*c^2*f -
2*a^2*c^2*h + a^2*b*c*k + 2*(a*b^2*c - 4*a^2*c^2)*l*x^3 + (b*c^3*
d - 2*a*c^3*f + a*b*c^2*h + (a*b^2*c - 6*a^2*c^2)*k - (3*a*b^3 -
```

```
13*a^2*b*c)*m)*x^2 + (b^2*c^2 - 6*a*c^3)*d - (3*a^2*b^2 - 10*a^3*
c)*m - 2*(2*a*c^3*e - a*b*c^2*g + 2*a^2*c^2*j - a^2*b*c*l)*x)/(c*
x^4 + b*x^2 + a), x)/(a*b^2*c^2 - 4*a^2*c^3)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((m*x^8 + l*x^7 + k*x^6 + j*x^5 + h*x^4 + g*x^3 + f*x^2 + e*x + d)/(c*x^4 +
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((m*x**8+l*x**7+k*x**6+j*x**5+h*x**4+g*x**3+f*x**2+e*x+d)/(c*x**4+b*x
```

```
[Out] Timed out
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((m*x^8 + l*x^7 + k*x^6 + j*x^5 + h*x^4 + g*x^3 + f*x^2 + e*x + d)/(c*x^4 +
```

```
[Out] Exception raised: TypeError
```

3.42 $\int \frac{d+ex}{(4-5x^2+x^4)^3}\,dx$

Optimal. Leaf size=143

$$-\frac{dx\left(59-35x^2\right)}{3456\left(x^4-5x^2+4\right)}+\frac{dx\left(17-5x^2\right)}{144\left(x^4-5x^2+4\right)^2}-\frac{313d\tanh^{-1}\left(\frac{x}{2}\right)}{20736}+\frac{13}{648}d\tanh^{-1}(x)-\frac{1}{81}e\log\left(1-x^2\right)+\frac{1}{81}e\log\left(4-x^2\right)-\frac{e\left(5-2x^2\right)}{54\left(x^4-5x^2+4\right)}+\frac{e\left(5-2x^2\right)}{36\left(x^4-5x^2+4\right)^2}$$

```
[Out] (d*x*(17 - 5*x^2))/(144*(4 - 5*x^2 + x^4)^2) + (e*(5 - 2*x^2))/(3
6*(4 - 5*x^2 + x^4)^2) - (d*x*(59 - 35*x^2))/(3456*(4 - 5*x^2 + x
^4)) - (e*(5 - 2*x^2))/(54*(4 - 5*x^2 + x^4)) - (313*d*ArcTanh[x/
2])/20736 + (13*d*ArcTanh[x])/648 - (e*Log[1 - x^2])/81 + (e*Log[
4 - x^2])/81
```

Rubi [A] time = 0.175132, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556

$$-\frac{dx\left(59-35x^2\right)}{3456\left(x^4-5x^2+4\right)}+\frac{dx\left(17-5x^2\right)}{144\left(x^4-5x^2+4\right)^2}-\frac{313d\tanh^{-1}\left(\frac{x}{2}\right)}{20736}+\frac{13}{648}d\tanh^{-1}(x)-\frac{1}{81}e\log\left(1-x^2\right)+\frac{1}{81}e\log\left(4-x^2\right)-\frac{e\left(5-2x^2\right)}{54\left(x^4-5x^2+4\right)}+\frac{e\left(5-2x^2\right)}{36\left(x^4-5x^2+4\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x)/(4 - 5*x^2 + x^4)^3,x]
```

```
[Out] (d*x*(17 - 5*x^2))/(144*(4 - 5*x^2 + x^4)^2) + (e*(5 - 2*x^2))/(3
6*(4 - 5*x^2 + x^4)^2) - (d*x*(59 - 35*x^2))/(3456*(4 - 5*x^2 + x
^4)) - (e*(5 - 2*x^2))/(54*(4 - 5*x^2 + x^4)) - (313*d*ArcTanh[x/
2])/20736 + (13*d*ArcTanh[x])/648 - (e*Log[1 - x^2])/81 + (e*Log[
4 - x^2])/81
```

Rubi in Sympy [A] time = 48.4808, size = 110, normalized size = 0.77

$$-\frac{313d\,\mathrm{atanh}\left(\frac{x}{2}\right)}{20736}+\frac{13d\,\mathrm{atanh}(x)}{648}-\frac{e\log\left(-x^2+1\right)}{81}+\frac{e\log\left(-x^2+4\right)}{81}-\frac{x\left(-105dx^2+177d-150ex^3+366ex\right)}{10368\left(x^4-5x^2+4\right)}+\frac{x\left(-5dx^2+17d-5ex^3+17ex\right)}{144\left(x^4-5x^2+4\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x+d)/(x**4-5*x**2+4)**3,x)
```

```
[Out] -313*d*atanh(x/2)/20736 + 13*d*atanh(x)/648 - e*log(-x**2 + 1)/81
 + e*log(-x**2 + 4)/81 - x*(-105*d*x**2 + 177*d - 150*e*x**3 + 36
6*e*x)/(10368*(x**4 - 5*x**2 + 4)) + x*(-5*d*x**2 + 17*d - 5*e*x*
*3 + 17*e*x)/(144*(x**4 - 5*x**2 + 4)**2)
```

Mathematica [A] time = 0.177604, size = 128, normalized size = 0.9

$$\frac{\frac{288\left(dx\left(17-5x^2\right)+e\left(20-8x^2\right)\right)}{\left(x^4-5x^2+4\right)^2}+\frac{12\left(dx\left(35x^2-59\right)+64e\left(2x^2-5\right)\right)}{x^4-5x^2+4}-32(13d+16e)\log(1-x)+(313d+512e)\log(2-x)+32(13d-16e)\,\mathrm{lo}}{41472}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x)/(4 - 5*x^2 + x^4)^3,x]
```

```
[Out] ((288*(e*(20 - 8*x^2) + d*x*(17 - 5*x^2)))/(4 - 5*x^2 + x^4)^2 +
(12*(64*e*(-5 + 2*x^2) + d*x*(-59 + 35*x^2)))/(4 - 5*x^2 + x^4) -
 32*(13*d + 16*e)*Log[1 - x] + (313*d + 512*e)*Log[2 - x] + 32*(1
3*d - 16*e)*Log[1 + x] + (-313*d + 512*e)*Log[2 + x])/41472
```

Maple [A] time = 0.029, size = 186, normalized size = 1.3

$$-\frac{313\ln(2+x)d}{41472}+\frac{\ln(2+x)e}{81}+\frac{19d}{13824+6912x}-\frac{17e}{6912+3456x}+\frac{d}{3456(2+x)^2}-\frac{e}{1728(2+x)^2}$$
$$-\frac{13\ln(-1+x)d}{1296}-\frac{\ln(-1+x)e}{81}+\frac{d}{-432+432x}+\frac{e}{-144+144x}+\frac{d}{432(-1+x)^2}+\frac{e}{432(-1+x)^2}$$
$$+\frac{d}{432+432x}-\frac{e}{144+144x}-\frac{d}{432(1+x)^2}+\frac{e}{432(1+x)^2}+\frac{13\ln(1+x)d}{1296}-\frac{\ln(1+x)e}{81}$$
$$+\frac{19d}{6912x-13824}+\frac{17e}{3456x-6912}-\frac{d}{3456(x-2)^2}-\frac{e}{1728(x-2)^2}+\frac{313\ln(x-2)d}{41472}+\frac{\ln(x-2)e}{81}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x+d)/(x^4-5*x^2+4)^3,x)
```

```
[Out] -313/41472*ln(2+x)*d+1/81*ln(2+x)*e+19/6912/(2+x)*d-17/3456/(2+x)
*e+1/3456/(2+x)^2*d-1/1728/(2+x)^2*e-13/1296*ln(-1+x)*d-1/81*ln(-
1+x)*e+1/432/(-1+x)*d+1/144/(-1+x)*e+1/432/(-1+x)^2*d+1/432/(-1+x
)^2*e+1/432/(1+x)*d-1/144/(1+x)*e-1/432/(1+x)^2*d+1/432/(1+x)^2*e
+13/1296*ln(1+x)*d-1/81*ln(1+x)*e+19/6912/(x-2)*d+17/3456/(x-2)*e
-1/3456/(x-2)^2*d-1/1728/(x-2)^2*e+313/41472*ln(x-2)*d+1/81*ln(x-
```

```
2)*e
```

Maxima [A] time = 0.710577, size = 163, normalized size = 1.14

$$-\frac{1}{41472}(313d-512e)\log(x+2)+\frac{1}{1296}(13d-16e)\log(x+1)-\frac{1}{1296}(13d+16e)\log(x-1)+\frac{1}{41472}(313d+512e)\log(x-2)+\frac{35dx^7+128ex^6-234dx^5-960ex^4+315dx^3+1920ex^2+172dx-800e}{3456(x^8-10x^6+33x^4-40x^2+16)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x + d)/(x^4 - 5*x^2 + 4)^3,x, algorithm="maxima")
```

```
[Out] -1/41472*(313*d - 512*e)*log(x + 2) + 1/1296*(13*d - 16*e)*log(x
+ 1) - 1/1296*(13*d + 16*e)*log(x - 1) + 1/41472*(313*d + 512*e)*
log(x - 2) + 1/3456*(35*d*x^7 + 128*e*x^6 - 234*d*x^5 - 960*e*x^4
 + 315*d*x^3 + 1920*e*x^2 + 172*d*x - 800*e)/(x^8 - 10*x^6 + 33*x
^4 - 40*x^2 + 16)
```

Fricas [A] time = 0.303163, size = 414, normalized size = 2.9

$$\frac{420dx^7+1536ex^6-2808dx^5-11520ex^4+3780dx^3+23040ex^2+2064dx-\left((313d-512e)x^8-10(313d-512e)x^6+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x + d)/(x^4 - 5*x^2 + 4)^3,x, algorithm="fricas")
```

```
[Out] 1/41472*(420*d*x^7 + 1536*e*x^6 - 2808*d*x^5 - 11520*e*x^4 + 3780
*d*x^3 + 23040*e*x^2 + 2064*d*x - ((313*d - 512*e)*x^8 - 10*(313*
d - 512*e)*x^6 + 33*(313*d - 512*e)*x^4 - 40*(313*d - 512*e)*x^2
+ 5008*d - 8192*e)*log(x + 2) + 32*((13*d - 16*e)*x^8 - 10*(13*d
- 16*e)*x^6 + 33*(13*d - 16*e)*x^4 - 40*(13*d - 16*e)*x^2 + 208*d
 - 256*e)*log(x + 1) - 32*((13*d + 16*e)*x^8 - 10*(13*d + 16*e)*x
^6 + 33*(13*d + 16*e)*x^4 - 40*(13*d + 16*e)*x^2 + 208*d + 256*e)
*log(x - 1) + ((313*d + 512*e)*x^8 - 10*(313*d + 512*e)*x^6 + 33*
(313*d + 512*e)*x^4 - 40*(313*d + 512*e)*x^2 + 5008*d + 8192*e)*l
og(x - 2) - 9600*e)/(x^8 - 10*x^6 + 33*x^4 - 40*x^2 + 16)
```

Sympy [A] time = 9.17654, size = 668, normalized size = 4.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x+d)/(x**4-5*x**2+4)**3,x)
```

```
[Out] (13*d - 16*e)*log(x + (-1106258459719280*d**4*e - 13113710954343*
d**4*(13*d - 16*e) - 817263343042560*d**2*e**3 + 153628968222720*
d**2*e**2*(13*d - 16*e) + 9530197557248*d**2*e*(13*d - 16*e)**2 +
 88038005760*d**2*(13*d - 16*e)**3 + 5035763255214080*e**5 + 1426
61633703936*e**4*(13*d - 16*e) - 19670950215680*e**3*(13*d - 16*e
)**2 - 557272006656*e**2*(13*d - 16*e)**3)/(22941256248261*d**5 -
 2312740746035200*d**3*e**2 + 4473912813420544*d*e**4))/1296 - (1
3*d + 16*e)*log(x + (-1106258459719280*d**4*e + 13113710954343*d*
*4*(13*d + 16*e) - 817263343042560*d**2*e**3 - 153628968222720*d*
*2*e**2*(13*d + 16*e) + 9530197557248*d**2*e*(13*d + 16*e)**2 - 8
8038005760*d**2*(13*d + 16*e)**3 + 5035763255214080*e**5 - 142661
633703936*e**4*(13*d + 16*e) - 19670950215680*e**3*(13*d + 16*e)*
*2 + 557272006656*e**2*(13*d + 16*e)**3)/(22941256248261*d**5 - 2
312740746035200*d**3*e**2 + 4473912813420544*d*e**4))/1296 - (313
*d - 512*e)*log(x + (-1106258459719280*d**4*e + 13113710954343*d*
*4*(313*d - 512*e)/32 - 817263343042560*d**2*e**3 - 4800905256960
*d**2*e**2*(313*d - 512*e) + 9306833552*d**2*e*(313*d - 512*e)**2
 - 85974615*d**2*(313*d - 512*e)**3/32 + 5035763255214080*e**5 -
4458176053248*e**4*(313*d - 512*e) - 19209912320*e**3*(313*d - 51
2*e)**2 + 17006592*e**2*(313*d - 512*e)**3)/(22941256248261*d**5
- 2312740746035200*d**3*e**2 + 4473912813420544*d*e**4))/41472 +
(313*d + 512*e)*log(x + (-1106258459719280*d**4*e - 1311371095434
3*d**4*(313*d + 512*e)/32 - 817263343042560*d**2*e**3 + 480090525
6960*d**2*e**2*(313*d + 512*e) + 9306833552*d**2*e*(313*d + 512*e
)**2 + 85974615*d**2*(313*d + 512*e)**3/32 + 5035763255214080*e**
5 + 4458176053248*e**4*(313*d + 512*e) - 19209912320*e**3*(313*d
+ 512*e)**2 - 17006592*e**2*(313*d + 512*e)**3)/(22941256248261*d
**5 - 2312740746035200*d**3*e**2 + 4473912813420544*d*e**4))/4147
2 + (35*d*x**7 - 234*d*x**5 + 315*d*x**3 + 172*d*x + 128*e*x**6 -
 960*e*x**4 + 1920*e*x**2 - 800*e)/(3456*x**8 - 34560*x**6 + 1140
48*x**4 - 138240*x**2 + 55296)
```

GIAC/XCAS [A] time = 0.279685, size = 166, normalized size = 1.16

$$-\frac{1}{41472}(313\,d-512\,e)\ln\left(|x+2|\right)+\frac{1}{1296}(13\,d-16\,e)\ln\left(|x+1|\right)-\frac{1}{1296}(13\,d+16\,e)\ln\left(|x-1|\right)+\frac{1}{41472}(313\,d+512\,e)\ln\left(|x-2|\right)+\frac{35\,dx^7+128\,x^6e-234\,dx^5-960\,x^4e+315\,dx^3+1920\,x^2e+172\,dx-800\,e}{3456\,(x^4-5\,x^2+4)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x + d)/(x^4 - 5*x^2 + 4)^3,x, algorithm="giac")
```

```
[Out] -1/41472*(313*d - 512*e)*ln(abs(x + 2)) + 1/1296*(13*d - 16*e)*ln
(abs(x + 1)) - 1/1296*(13*d + 16*e)*ln(abs(x - 1)) + 1/41472*(313
*d + 512*e)*ln(abs(x - 2)) + 1/3456*(35*d*x^7 + 128*x^6*e - 234*d
*x^5 - 960*x^4*e + 315*d*x^3 + 1920*x^2*e + 172*d*x - 800*e)/(x^4
 - 5*x^2 + 4)^2
```

3.43 $\int \frac{d+ex+fx^2}{(4-5x^2+x^4)^3}\,dx$

Optimal. Leaf size=175

$$-\frac{x\left(-35x^2(d+4f)+59d+380f\right)}{3456\left(x^4-5x^2+4\right)}+\frac{x\left(x^2(-(5d+8f))+17d+20f\right)}{144\left(x^4-5x^2+4\right)^2}-\frac{(313d+820f)\tanh^{-1}\left(\frac{x}{2}\right)}{20736}+\frac{1}{648}(13d+25f)\tanh^{-1}(x)-\frac{1}{81}e\log\left(1-x^2\right)+\frac{1}{81}e\log\left(4-x^2\right)-\frac{e\left(5-2x^2\right)}{54\left(x^4-5x^2+4\right)}+\frac{e\left(5-2x^2\right)}{36\left(x^4-5x^2+4\right)^2}$$

```
[Out] (e*(5 - 2*x^2))/(36*(4 - 5*x^2 + x^4)^2) + (x*(17*d + 20*f - (5*d
 + 8*f)*x^2))/(144*(4 - 5*x^2 + x^4)^2) - (e*(5 - 2*x^2))/(54*(4
- 5*x^2 + x^4)) - (x*(59*d + 380*f - 35*(d + 4*f)*x^2))/(3456*(4
- 5*x^2 + x^4)) - ((313*d + 820*f)*ArcTanh[x/2])/20736 + ((13*d +
 25*f)*ArcTanh[x])/648 - (e*Log[1 - x^2])/81 + (e*Log[4 - x^2])/8
1
```

Rubi [A] time = 0.452485, antiderivative size = 175, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$

$$-\frac{x\left(-35x^2(d+4f)+59d+380f\right)}{3456\left(x^4-5x^2+4\right)}+\frac{x\left(x^2(-(5d+8f))+17d+20f\right)}{144\left(x^4-5x^2+4\right)^2}-\frac{(313d+820f)\tanh^{-1}\left(\frac{x}{2}\right)}{20736}+\frac{1}{648}(13d+25f)\tanh^{-1}(x)-\frac{1}{81}e\log\left(1-x^2\right)+\frac{1}{81}e\log\left(4-x^2\right)-\frac{e\left(5-2x^2\right)}{54\left(x^4-5x^2+4\right)}+\frac{e\left(5-2x^2\right)}{36\left(x^4-5x^2+4\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x + f*x^2)/(4 - 5*x^2 + x^4)^3,x]
```

```
[Out] (e*(5 - 2*x^2))/(36*(4 - 5*x^2 + x^4)^2) + (x*(17*d + 20*f - (5*d
 + 8*f)*x^2))/(144*(4 - 5*x^2 + x^4)^2) - (e*(5 - 2*x^2))/(54*(4
- 5*x^2 + x^4)) - (x*(59*d + 380*f - 35*(d + 4*f)*x^2))/(3456*(4
- 5*x^2 + x^4)) - ((313*d + 820*f)*ArcTanh[x/2])/20736 + ((13*d +
 25*f)*ArcTanh[x])/648 - (e*Log[1 - x^2])/81 + (e*Log[4 - x^2])/8
1
```

Rubi in Sympy [A] time = 62.9354, size = 134, normalized size = 0.77

$$-\frac{e\log\left(-x^2+1\right)}{81}+\frac{e\log\left(-x^2+4\right)}{81}-\frac{x\left(177d-150ex^3+366ex+1140f-x^2\left(105d+420f\right)\right)}{10368\left(x^4-5x^2+4\right)}+\frac{x\left(17d-5ex^3+17ex+20f-x^2\left(5d+8f\right)\right)}{144\left(x^4-5x^2+4\right)^2}-\left(\frac{313d}{20736}+\frac{205f}{5184}\right)\operatorname{atanh}\left(\frac{x}{2}\right)+\left(\frac{13d}{648}+\frac{25f}{648}\right)\operatorname{atanh}\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**2+e*x+d)/(x**4-5*x**2+4)**3,x)
```

```
[Out] -e*log(-x**2 + 1)/81 + e*log(-x**2 + 4)/81 - x*(177*d - 150*e*x**
3 + 366*e*x + 1140*f - x**2*(105*d + 420*f))/(10368*(x**4 - 5*x**
2 + 4)) + x*(17*d - 5*e*x**3 + 17*e*x + 20*f - x**2*(5*d + 8*f))/
(144*(x**4 - 5*x**2 + 4)**2) - (313*d/20736 + 205*f/5184)*atanh(x
/2) + (13*d/648 + 25*f/648)*atanh(x)
```

Mathematica [A] time = 0.264777, size = 161, normalized size = 0.92

$$\frac{\frac{12\left(dx\left(35x^2-59\right)+64e\left(2x^2-5\right)+20fx\left(7x^2-19\right)\right)}{x^4-5x^2+4}+\frac{288\left(-5dx^3+17dx+e\left(20-8x^2\right)-8fx^3+20fx\right)}{\left(x^4-5x^2+4\right)^2}-32\log(1-x)(13d+16e+25f)+\log(2-x)(31}{41472}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x + f*x^2)/(4 - 5*x^2 + x^4)^3,x]
```

```
[Out] ((288*(17*d*x + 20*f*x - 5*d*x^3 - 8*f*x^3 + e*(20 - 8*x^2)))/(4
- 5*x^2 + x^4)^2 + (12*(64*e*(-5 + 2*x^2) + 20*f*x*(-19 + 7*x^2)
+ d*x*(-59 + 35*x^2)))/(4 - 5*x^2 + x^4) - 32*(13*d + 16*e + 25*f
)*Log[1 - x] + (313*d + 512*e + 820*f)*Log[2 - x] + 32*(13*d - 16
*e + 25*f)*Log[1 + x] + (-313*d + 512*e - 820*f)*Log[2 + x])/4147
2
```

Maple [A] time = 0.029, size = 278, normalized size = 1.6

$$\frac{d}{432(-1+x)^2}+\frac{e}{432(-1+x)^2}-\frac{d}{432(1+x)^2}+\frac{e}{432(1+x)^2}-\frac{d}{3456(x-2)^2}-\frac{e}{1728(x-2)^2}+\frac{d}{3456(2+x)^2}-\frac{e}{1728(2+x)^2}-\frac{f}{432(1+x)^2}-\frac{f}{864(x-2)^2}+\frac{f}{864(2+x)^2}+\frac{f}{432(-1+x)^2}+\frac{5f}{432+432x}+\frac{d}{432+432x}-\frac{e}{144+144x}+\frac{19d}{6912x-13824}+\frac{17e}{3456x-6912}+\frac{5f}{576x-1152}+\frac{5f}{-432+432x}+\frac{19d}{13824+6912x}-\frac{17e}{6912+3456x}+\frac{d}{-432+432x}+\frac{e}{-144+144x}+\frac{5f}{1152+576x}+\frac{13\ln(1+x)d}{1296}-\frac{\ln(1+x)e}{81}-\frac{13\ln(-1+x)d}{1296}-\frac{\ln(-1+x)e}{81}+\frac{313\ln(x-2)d}{41472}+\frac{\ln(x-2)e}{81}+\frac{\ln(2+x)e}{81}+\frac{205\ln(x-2)f}{10368}-\frac{313\ln(2+x)d}{41472}+\frac{25\ln(1+x)f}{1296}-\frac{25\ln(-1+x)f}{1296}-\frac{205\ln(2+x)f}{10368}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x^2+e*x+d)/(x^4-5*x^2+4)^3,x)
```

```
[Out] 1/432/(-1+x)^2*d+1/432/(-1+x)^2*e-1/432/(1+x)^2*d+1/432/(1+x)^2*e
-1/3456/(x-2)^2*d-1/1728/(x-2)^2*e+1/3456/(2+x)^2*d-1/1728/(2+x)^
2*e-1/432/(1+x)^2*f-1/864/(x-2)^2*f+1/864/(2+x)^2*f+1/432/(-1+x)^
2*f+5/432/(1+x)*f+1/432/(1+x)*d-1/144/(1+x)*e+19/6912/(x-2)*d+17/
3456/(x-2)*e+5/576/(x-2)*f+5/432/(-1+x)*f+19/6912/(2+x)*d-17/3456
/(2+x)*e+1/432/(-1+x)*d+1/144/(-1+x)*e+5/576/(2+x)*f+13/1296*ln(1
+x)*d-1/81*ln(1+x)*e-13/1296*ln(-1+x)*d-1/81*ln(-1+x)*e+313/41472
*ln(x-2)*d+1/81*ln(x-2)*e+1/81*ln(2+x)*e+205/10368*ln(x-2)*f-313/
41472*ln(2+x)*d+25/1296*ln(1+x)*f-25/1296*ln(-1+x)*f-205/10368*ln
(2+x)*f
```

Maxima [A] time = 0.705283, size = 209, normalized size = 1.19

$$-\frac{1}{41472}(313d-512e+820f)\log(x+2)+\frac{1}{1296}(13d-16e+25f)\log(x+1)-\frac{1}{1296}(13d+16e+25f)\log(x-1)+\frac{1}{41472}(313d+512e+820f)\log(x-2)+\frac{35(d+4f)x^7+128ex^6-18(13d+60f)x^5-960ex^4+63(5d+36f)x^3+1920ex^2+4(43d-260f)x-800e}{3456(x^8-10x^6+33x^4-40x^2+16)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2 + e*x + d)/(x^4 - 5*x^2 + 4)^3,x, algorithm="maxima")
```

```
[Out] -1/41472*(313*d - 512*e + 820*f)*log(x + 2) + 1/1296*(13*d - 16*e
 + 25*f)*log(x + 1) - 1/1296*(13*d + 16*e + 25*f)*log(x - 1) + 1/
41472*(313*d + 512*e + 820*f)*log(x - 2) + 1/3456*(35*(d + 4*f)*x
^7 + 128*e*x^6 - 18*(13*d + 60*f)*x^5 - 960*e*x^4 + 63*(5*d + 36*
f)*x^3 + 1920*e*x^2 + 4*(43*d - 260*f)*x - 800*e)/(x^8 - 10*x^6 +
 33*x^4 - 40*x^2 + 16)
```

Fricas [A] time = 0.349195, size = 525, normalized size = 3.

$$\frac{420\,(d + 4\,f)x^7 + 1536\,ex^6 - 216\,(13\,d + 60\,f)x^5 - 11520\,ex^4 + 756\,(5\,d + 36\,f)x^3 + 23040\,ex^2 + 48\,(43\,d - 260\,f)x - \big((313\,d}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2 + e*x + d)/(x^4 - 5*x^2 + 4)^3,x, algorithm="fricas")
```

```
[Out] 1/41472*(420*(d + 4*f)*x^7 + 1536*e*x^6 - 216*(13*d + 60*f)*x^5 -
 11520*e*x^4 + 756*(5*d + 36*f)*x^3 + 23040*e*x^2 + 48*(43*d - 26
0*f)*x - ((313*d - 512*e + 820*f)*x^8 - 10*(313*d - 512*e + 820*f
)*x^6 + 33*(313*d - 512*e + 820*f)*x^4 - 40*(313*d - 512*e + 820*
f)*x^2 + 5008*d - 8192*e + 13120*f)*log(x + 2) + 32*((13*d - 16*e
 + 25*f)*x^8 - 10*(13*d - 16*e + 25*f)*x^6 + 33*(13*d - 16*e + 25
*f)*x^4 - 40*(13*d - 16*e + 25*f)*x^2 + 208*d - 256*e + 400*f)*lo
g(x + 1) - 32*((13*d + 16*e + 25*f)*x^8 - 10*(13*d + 16*e + 25*f)
*x^6 + 33*(13*d + 16*e + 25*f)*x^4 - 40*(13*d + 16*e + 25*f)*x^2
+ 208*d + 256*e + 400*f)*log(x - 1) + ((313*d + 512*e + 820*f)*x^
8 - 10*(313*d + 512*e + 820*f)*x^6 + 33*(313*d + 512*e + 820*f)*x
^4 - 40*(313*d + 512*e + 820*f)*x^2 + 5008*d + 8192*e + 13120*f)*
log(x - 2) - 9600*e)/(x^8 - 10*x^6 + 33*x^4 - 40*x^2 + 16)
```

Sympy [A] time = 118.807, size = 2822, normalized size = 16.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)/(x**4-5*x**2+4)**3,x)
```

```
[Out] (13*d - 16*e + 25*f)*log(x + (-1106258459719280*d**5*e - 13113710
954343*d**5*(13*d - 16*e + 25*f) - 12929482401572800*d**4*e*f - 1
07063904267900*d**4*f*(13*d - 16*e + 25*f) - 817263343042560*d**3
*e**3 + 153628968222720*d**3*e**2*(13*d - 16*e + 25*f) - 59478343
838144000*d**3*e*f**2 + 9530197557248*d**3*e*(13*d - 16*e + 25*f)
**2 - 324891412840800*d**3*f**2*(13*d - 16*e + 25*f) + 8803800576
```

0*d**3*(13*d - 16*e + 25*f)**3 - 2885705898393600*d**2*e**3*f + 1
014848673546240*d**2*e**2*f*(13*d - 16*e + 25*f) - 13490528680832
0000*d**2*e*f**3 + 63469758382080*d**2*e*f*(13*d - 16*e + 25*f)**
2 - 422972724528000*d**2*f**3*(13*d - 16*e + 25*f) + 364616847360
*d**2*f*(13*d - 16*e + 25*f)**3 + 5035763255214080*d*e**5 + 14266
1633703936*d*e**4*(13*d - 16*e + 25*f) - 2138314899456000*d*e**3*
f**2 - 19670950215680*d*e**3*(13*d - 16*e + 25*f)**2 + 2257033730
457600*d*e**2*f**2*(13*d - 16*e + 25*f) - 557272006656*d*e**2*(13
*d - 16*e + 25*f)**3 - 15108264559360000*d*e*f**4 + 141056507904
000*d*e*f**2*(13*d - 16*e + 25*f)**2 - 167683154400000*d*f**4*(13
*d - 16*e + 25*f) + 339373670400*d*f**2*(13*d - 16*e + 25*f)**3 +
10643272556871680*e**5*f + 214404767416320*e**4*f*(13*d - 16*e +
25*f) + 529992253440000*e**3*f**3 - 41575283425280*e**3*f*(13*d
- 16*e + 25*f)**2 + 1671759396864000*e**2*f**3*(13*d - 16*e + 25*
f) - 837518622720*e**2*f*(13*d - 16*e + 25*f)**3 - 66895452108800
000*e*f**5 + 104485486592000*e*f**3*(13*d - 16*e + 25*f)**2 + 510
41923200000*f**5*(13*d - 16*e + 25*f) - 80289792000*f**3*(13*d -
16*e + 25*f)**3)/(22941256248261*d**6 + 197271407316645*d**5*f -
2312740746035200*d**4*e**2 + 612862910928900*d**4*f**2 - 20566607
354920960*d**3*e**2*f + 767363353812000*d**3*f**3 + 4473912813420
544*d**2*e**4 - 68552762169753600*d**2*e**2*f**2 + 19749922200000
0*d**2*f**4 + 20324472439439360*d*e**4*f - 101559983669248000*d*e
2*f3 - 182883938400000*d*f**5 + 22539988369408000*e**4*f**2 -
56422196838400000*e**2*f**4 + 21520080000000*f**6))/1296 - (13*d
+ 16*e + 25*f)*log(x + (-1106258459719280*d**5*e + 1311371095434
3*d**5*(13*d + 16*e + 25*f) - 12929482401572800*d**4*e*f + 107063
904267900*d**4*f*(13*d + 16*e + 25*f) - 817263343042560*d**3*e**3
- 153628968222720*d**3*e**2*(13*d + 16*e + 25*f) - 5947834383814
4000*d**3*e*f**2 + 9530197557248*d**3*e*(13*d + 16*e + 25*f)**2 +
324891412840800*d**3*f**2*(13*d + 16*e + 25*f) - 88038005760*d**
3*(13*d + 16*e + 25*f)**3 - 2885705898393600*d**2*e**3*f - 101484
8673546240*d**2*e**2*f*(13*d + 16*e + 25*f) - 134905286808320000*
d**2*e*f**3 + 63469758382080*d**2*e*f*(13*d + 16*e + 25*f)**2 + 4
22972724528000*d**2*f**3*(13*d + 16*e + 25*f) - 364616847360*d**2
f(13*d + 16*e + 25*f)**3 + 5035763255214080*d*e**5 - 1426616337
03936*d*e**4*(13*d + 16*e + 25*f) - 2138314899456000*d*e**3*f**2
- 19670950215680*d*e**3*(13*d + 16*e + 25*f)**2 - 225703373045760
0*d*e**2*f**2*(13*d + 16*e + 25*f) + 557272006656*d*e**2*(13*d +
16*e + 25*f)**3 - 15108264559360000*d*e*f**4 + 141056507904000*d
*e*f**2*(13*d + 16*e + 25*f)**2 + 167683154400000*d*f**4*(13*d +
16*e + 25*f) - 339373670400*d*f**2*(13*d + 16*e + 25*f)**3 + 1064
3272556871680*e**5*f - 214404767416320*e**4*f*(13*d + 16*e + 25*f
) + 529992253440000*e**3*f**3 - 41575283425280*e**3*f*(13*d + 16*
e + 25*f)**2 - 1671759396864000*e**2*f**3*(13*d + 16*e + 25*f) +
837518622720*e**2*f*(13*d + 16*e + 25*f)**3 - 66895452108800000*e
*f**5 + 104485486592000*e*f**3*(13*d + 16*e + 25*f)**2 - 51041923
200000*f**5*(13*d + 16*e + 25*f) + 80289792000*f**3*(13*d + 16*e
+ 25*f)**3)/(22941256248261*d**6 + 197271407316645*d**5*f - 23127
40746035200*d**4*e**2 + 612862910928900*d**4*f**2 - 2056660735492
0960*d**3*e**2*f + 767363353812000*d**3*f**3 + 4473912813420544*d
2*e4 - 68552762169753600*d**2*e**2*f**2 + 197499222000000*d**
2*f**4 + 20324472439439360*d*e**4*f - 101559983669248000*d*e**2*f
3 - 182883938400000*d*f5 + 22539988369408000*e**4*f**2 - 5642
2196838400000*e**2*f**4 + 21520080000000*f**6))/1296 - (313*d - 5

```
12*e + 820*f)*log(x + (-1106258459719280*d**5*e + 13113710954343*
d**5*(313*d - 512*e + 820*f)/32 - 12929482401572800*d**4*e*f + 26
765976066975*d**4*f*(313*d - 512*e + 820*f)/8 - 817263343042560*d
**3*e**3 - 4800905256960*d**3*e**2*(313*d - 512*e + 820*f) - 5947
8343838144000*d**3*e*f**2 + 9306833552*d**3*e*(313*d - 512*e + 82
0*f)**2 + 10152856651275*d**3*f**2*(313*d - 512*e + 820*f) - 8597
4615*d**3*(313*d - 512*e + 820*f)**3/32 - 2885705898393600*d**2*e
**3*f - 31714021048320*d**2*e**2*f*(313*d - 512*e + 820*f) - 1349
05286808320000*d**2*e*f**3 + 61982185920*d**2*e*f*(313*d - 512*e
+ 820*f)**2 + 13217897641500*d**2*f**3*(313*d - 512*e + 820*f) -
89017785*d**2*f*(313*d - 512*e + 820*f)**3/8 + 5035763255214080*d
*e**5 - 4458176053248*d*e**4*(313*d - 512*e + 820*f) - 2138314899
456000*d*e**3*f**2 - 19209912320*d*e**3*(313*d - 512*e + 820*f)**
2 - 70532304076800*d*e**2*f**2*(313*d - 512*e + 820*f) + 17006592
*d*e**2*(313*d - 512*e + 820*f)**3 - 151082645593600000*d*e*f**4
+ 137750496000*d*e*f**2*(313*d - 512*e + 820*f)**2 + 524009857500
0*d*f**4*(313*d - 512*e + 820*f) - 20713725*d*f**2*(313*d - 512*e
 + 820*f)**3/2 + 10643272556871680*e**5*f - 6700148981760*e**4*f*
(313*d - 512*e + 820*f) + 529992253440000*e**3*f**3 - 40600862720
*e**3*f*(313*d - 512*e + 820*f)**2 - 52242481152000*e**2*f**3*(31
3*d - 512*e + 820*f) + 25559040*e**2*f*(313*d - 512*e + 820*f)**3
 - 66895452108800000*e*f**5 + 102036608000*e*f**3*(313*d - 512*e
+ 820*f)**2 - 1595060100000*f**5*(313*d - 512*e + 820*f) + 245025
0*f**3*(313*d - 512*e + 820*f)**3)/(22941256248261*d**6 + 1972714
07316645*d**5*f - 2312740746035200*d**4*e**2 + 612862910928900*d*
*4*f**2 - 20566607354920960*d**3*e**2*f + 767363353812000*d**3*f*
*3 + 4473912813420544*d**2*e**4 - 68552762169753600*d**2*e**2*f**
2 + 197499222000000*d**2*f**4 + 20324472439439360*d*e**4*f - 1015
59983669248000*d*e**2*f**3 - 182883938400000*d*f**5 + 22539988369
408000*e**4*f**2 - 56422196838400000*e**2*f**4 + 21520080000000*f
**6))/41472 + (313*d + 512*e + 820*f)*log(x + (-1106258459719280*
d**5*e - 13113710954343*d**5*(313*d + 512*e + 820*f)/32 - 1292948
2401572800*d**4*e*f - 26765976066975*d**4*f*(313*d + 512*e + 820*
f)/8 - 817263343042560*d**3*e**3 + 4800905256960*d**3*e**2*(313*d
 + 512*e + 820*f) - 59478343838144000*d**3*e*f**2 + 9306833552*d*
*3*e*(313*d + 512*e + 820*f)**2 - 10152856651275*d**3*f**2*(313*d
 + 512*e + 820*f) + 85974615*d**3*(313*d + 512*e + 820*f)**3/32 -
 2885705898393600*d**2*e**3*f + 31714021048320*d**2*e**2*f*(313*d
 + 512*e + 820*f) - 134905286808320000*d**2*e*f**3 + 61982185920*
d**2*e*f*(313*d + 512*e + 820*f)**2 - 13217897641500*d**2*f**3*(3
13*d + 512*e + 820*f) + 89017785*d**2*f*(313*d + 512*e + 820*f)**
3/8 + 5035763255214080*d*e**5 + 4458176053248*d*e**4*(313*d + 512
*e + 820*f) - 2138314899456000*d*e**3*f**2 - 19209912320*d*e**3*(
313*d + 512*e + 820*f)**2 + 70532304076800*d*e**2*f**2*(313*d + 5
12*e + 820*f) - 17006592*d*e**2*(313*d + 512*e + 820*f)**3 - 1510
82645593600000*d*e*f**4 + 137750496000*d*e*f**2*(313*d + 512*e +
820*f)**2 - 5240098575000*d*f**4*(313*d + 512*e + 820*f) + 207137
25*d*f**2*(313*d + 512*e + 820*f)**3/2 + 10643272556871680*e**5*f
 + 6700148981760*e**4*f*(313*d + 512*e + 820*f) + 529992253440000
*e**3*f**3 - 40600862720*e**3*f*(313*d + 512*e + 820*f)**2 + 5224
2481152000*e**2*f**3*(313*d + 512*e + 820*f) - 25559040*e**2*f*(3
13*d + 512*e + 820*f)**3 - 66895452108800000*e*f**5 + 10203660800
0*e*f**3*(313*d + 512*e + 820*f)**2 + 1595060100000*f**5*(313*d +
 512*e + 820*f) - 2450250*f**3*(313*d + 512*e + 820*f)**3)/(22941
```

```
256248261*d**6 + 197271407316645*d**5*f - 2312740746035200*d**4*e
**2 + 612862910928900*d**4*f**2 - 20566607354920960*d**3*e**2*f +
 767363353812000*d**3*f**3 + 4473912813420544*d**2*e**4 - 6855276
2169753600*d**2*e**2*f**2 + 197499222000000*d**2*f**4 + 203244724
39439360*d*e**4*f - 101559983669248000*d*e**2*f**3 - 182883938400
000*d*f**5 + 22539988369408000*e**4*f**2 - 56422196838400000*e**2
*f**4 + 21520080000000*f**6))/41472 + (128*e*x**6 - 960*e*x**4 +
1920*e*x**2 - 800*e + x**7*(35*d + 140*f) + x**5*(-234*d - 1080*f
) + x**3*(315*d + 2268*f) + x*(172*d - 1040*f))/(3456*x**8 - 3456
0*x**6 + 114048*x**4 - 138240*x**2 + 55296)
```

GIAC/XCAS [A] time = 0.271668, size = 212, normalized size = 1.21

$$-\frac{1}{41472}(313\,d+820\,f-512\,e)\ln\left(|x+2|\right)+\frac{1}{1296}(13\,d+25\,f-16\,e)\ln\left(|x+1|\right)$$
$$-\frac{1}{1296}(13\,d+25\,f+16\,e)\ln\left(|x-1|\right)+\frac{1}{41472}(313\,d+820\,f+512\,e)\ln\left(|x-2|\right)$$
$$+\frac{35\,dx^7+140\,fx^7+128\,x^6e-234\,dx^5-1080\,fx^5-960\,x^4e+315\,dx^3+2268\,fx^3+1920\,x^2e+172\,dx-1040\,fx-800\,e}{3456\,(x^4-5\,x^2+4)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^2 + e*x + d)/(x^4 - 5*x^2 + 4)^3,x, algorithm="giac")
```

```
[Out] -1/41472*(313*d + 820*f - 512*e)*ln(abs(x + 2)) + 1/1296*(13*d +
25*f - 16*e)*ln(abs(x + 1)) - 1/1296*(13*d + 25*f + 16*e)*ln(abs(
x - 1)) + 1/41472*(313*d + 820*f + 512*e)*ln(abs(x - 2)) + 1/3456
*(35*d*x^7 + 140*f*x^7 + 128*x^6*e - 234*d*x^5 - 1080*f*x^5 - 960
*x^4*e + 315*d*x^3 + 2268*f*x^3 + 1920*x^2*e + 172*d*x - 1040*f*x
 - 800*e)/(x^4 - 5*x^2 + 4)^2
```

3.44 $\int \frac{d+ex+fx^2+gx^3}{(4-5x^2+x^4)^3} dx$

Optimal. Leaf size=204

$$-\frac{x\left(-35x^2(d+4f)+59d+380f\right)}{3456\left(x^4-5x^2+4\right)}+\frac{x\left(x^2(-(5d+8f))+17d+20f\right)}{144\left(x^4-5x^2+4\right)^2}-\frac{(313d+820f)\tanh^{-1}\left(\frac{x}{2}\right)}{20736}+\frac{1}{648}(13d+25f)\tanh^{-1}(x)-\frac{1}{162}(2e+5g)\log\left(1-x^2\right)+\frac{1}{162}(2e+5g)\log\left(4-x^2\right)-\frac{\left(5-2x^2\right)(2e+5g)}{108\left(x^4-5x^2+4\right)}+\frac{x^2(-(2e+5g))+5e+8g}{36\left(x^4-5x^2+4\right)^2}$$

```
[Out] (x*(17*d + 20*f - (5*d + 8*f)*x^2))/(144*(4 - 5*x^2 + x^4)^2) + (
5*e + 8*g - (2*e + 5*g)*x^2)/(36*(4 - 5*x^2 + x^4)^2) - ((2*e + 5
*g)*(5 - 2*x^2))/(108*(4 - 5*x^2 + x^4)) - (x*(59*d + 380*f - 35*
(d + 4*f)*x^2))/(3456*(4 - 5*x^2 + x^4)) - ((313*d + 820*f)*ArcTa
nh[x/2])/20736 + ((13*d + 25*f)*ArcTanh[x])/648 - ((2*e + 5*g)*Lo
g[1 - x^2])/162 + ((2*e + 5*g)*Log[4 - x^2])/162
```

Rubi [A] time = 0.5138, antiderivative size = 204, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.321

$$-\frac{x\left(-35x^2(d+4f)+59d+380f\right)}{3456\left(x^4-5x^2+4\right)}+\frac{x\left(x^2(-(5d+8f))+17d+20f\right)}{144\left(x^4-5x^2+4\right)^2}-\frac{(313d+820f)\tanh^{-1}\left(\frac{x}{2}\right)}{20736}+\frac{1}{648}(13d+25f)\tanh^{-1}(x)-\frac{1}{162}(2e+5g)\log\left(1-x^2\right)+\frac{1}{162}(2e+5g)\log\left(4-x^2\right)-\frac{\left(5-2x^2\right)(2e+5g)}{108\left(x^4-5x^2+4\right)}+\frac{x^2(-(2e+5g))+5e+8g}{36\left(x^4-5x^2+4\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x + f*x^2 + g*x^3)/(4 - 5*x^2 + x^4)^3,x]
```

```
[Out] (x*(17*d + 20*f - (5*d + 8*f)*x^2))/(144*(4 - 5*x^2 + x^4)^2) + (
5*e + 8*g - (2*e + 5*g)*x^2)/(36*(4 - 5*x^2 + x^4)^2) - ((2*e + 5
*g)*(5 - 2*x^2))/(108*(4 - 5*x^2 + x^4)) - (x*(59*d + 380*f - 35*
(d + 4*f)*x^2))/(3456*(4 - 5*x^2 + x^4)) - ((313*d + 820*f)*ArcTa
nh[x/2])/20736 + ((13*d + 25*f)*ArcTanh[x])/648 - ((2*e + 5*g)*Lo
g[1 - x^2])/162 + ((2*e + 5*g)*Log[4 - x^2])/162
```

Rubi in Sympy [A] time = 75.3119, size = 158, normalized size = 0.77

$$-\frac{x\left(177d+1140f-x^3(150e+456g)-x^2(105d+420f)+x(366e+1320g)\right)}{10368(x^4-5x^2+4)}+\frac{x\left(17d+20f-x^3(5e+8g)-x^2(5d+8f)+x(17e+20g)\right)}{144(x^4-5x^2+4)^2}-\left(\frac{313d}{20736}+\frac{205f}{5184}\right)\operatorname{atanh}\left(\frac{x}{2}\right)+\left(\frac{13d}{648}+\frac{25f}{648}\right)\operatorname{atanh}(x)-\left(\frac{e}{81}+\frac{5g}{162}\right)\log\left(-x^2+1\right)+\left(\frac{e}{81}+\frac{5g}{162}\right)\log\left(-x^2+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((g*x**3+f*x**2+e*x+d)/(x**4-5*x**2+4)**3,x)
```

```
[Out] -x*(177*d + 1140*f - x**3*(150*e + 456*g) - x**2*(105*d + 420*f)
+ x*(366*e + 1320*g))/(10368*(x**4 - 5*x**2 + 4)) + x*(17*d + 20*
f - x**3*(5*e + 8*g) - x**2*(5*d + 8*f) + x*(17*e + 20*g))/(144*(
x**4 - 5*x**2 + 4)**2) - (313*d/20736 + 205*f/5184)*atanh(x/2) +
(13*d/648 + 25*f/648)*atanh(x) - (e/81 + 5*g/162)*log(-x**2 + 1)
+ (e/81 + 5*g/162)*log(-x**2 + 4)
```

Mathematica [A] time = 0.165335, size = 193, normalized size = 0.95

$$\frac{\frac{12\left(dx(35x^2-59)+64e(2x^2-5)+20fx(7x^2-19)+160g(2x^2-5)\right)}{x^4-5x^2+4}+\frac{288\left(-5dx^3+17dx+e(20-8x^2)-8fx^3+20fx-4g(5x^2-8)\right)}{(x^4-5x^2+4)^2}-32\log(1-x)(13d+16e+}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x + f*x^2 + g*x^3)/(4 - 5*x^2 + x^4)^3,x]
```

```
[Out] ((288*(17*d*x + 20*f*x - 5*d*x^3 - 8*f*x^3 + e*(20 - 8*x^2) - 4*g
*(-8 + 5*x^2)))/(4 - 5*x^2 + x^4)^2 + (12*(64*e*(-5 + 2*x^2) + 16
0*g*(-5 + 2*x^2) + 20*f*x*(-19 + 7*x^2) + d*x*(-59 + 35*x^2)))/(4
 - 5*x^2 + x^4) - 32*(13*d + 16*e + 25*f + 40*g)*Log[1 - x] + (31
3*d + 512*e + 820*f + 1280*g)*Log[2 - x] + 32*(13*d - 16*e + 25*f
 - 40*g)*Log[1 + x] + (-313*d + 512*e - 820*f + 1280*g)*Log[2 + x
])/41472
```

Maple [A] time = 0.031, size = 370, normalized size = 1.8

$$-\frac{g}{432 (x-2)^2}+\frac{g}{432 (1+x)^2}+\frac{g}{432 (-1+x)^2}-\frac{g}{432 (2+x)^2}+\frac{d}{432 (-1+x)^2}+\frac{e}{432 (-1+x)^2}-\frac{d}{432 (1+x)^2}+\frac{e}{432 (1+x)^2}-\frac{d}{3456 (x-2)^2}-\frac{e}{1728 (x-2)^2}+\frac{d}{3456 (2+x)^2}-\frac{e}{1728 (2+x)^2}-\frac{f}{432 (1+x)^2}-\frac{f}{864 (x-2)^2}+\frac{f}{864 (2+x)^2}+\frac{f}{432 (-1+x)^2}-\frac{7 g}{432+432 x}+\frac{7 g}{-432+432 x}-\frac{13 g}{1728+864 x}+\frac{13 g}{864 x-1728}+\frac{5 f}{432+432 x}+\frac{d}{432+432 x}-\frac{e}{144+144 x}+\frac{19 d}{6912 x-13824}+\frac{17 e}{3456 x-6912}+\frac{5 f}{576 x-1152}+\frac{5 f}{-432+432 x}+\frac{19 d}{13824+6912 x}-\frac{17 e}{6912+3456 x}+\frac{d}{-432+432 x}+\frac{e}{-144+144 x}+\frac{5 f}{1152+576 x}+\frac{13 \ln (1+x) d}{1296}-\frac{\ln (1+x) e}{81}-\frac{13 \ln (-1+x) d}{1296}-\frac{\ln (-1+x) e}{81}-\frac{5 \ln (1+x) g}{162}+\frac{5 \ln (x-2) g}{162}-\frac{5 \ln (-1+x) g}{162}+\frac{5 \ln (2+x) g}{162}+\frac{313 \ln (x-2) d}{41472}+\frac{\ln (x-2) e}{81}+\frac{\ln (2+x) e}{81}+\frac{205 \ln (x-2) f}{10368}-\frac{313 \ln (2+x) d}{41472}+\frac{25 \ln (1+x) f}{1296}-\frac{25 \ln (-1+x) f}{1296}-\frac{205 \ln (2+x) f}{10368}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int((g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^3,x)
```

```
[Out] -1/432/(x-2)^2*g+1/432/(1+x)^2*g+1/432/(-1+x)^2*g-1/432/(2+x)^2*g
+1/432/(-1+x)^2*d+1/432/(-1+x)^2*e-1/432/(1+x)^2*d+1/432/(1+x)^2*
e-1/3456/(x-2)^2*d-1/1728/(x-2)^2*e+1/3456/(2+x)^2*d-1/1728/(2+x)
^2*e-1/432/(1+x)^2*f-1/864/(x-2)^2*f+1/864/(2+x)^2*f+1/432/(-1+x)
^2*f-7/432/(1+x)*g+7/432/(-1+x)*g-13/864/(2+x)*g+13/864/(x-2)*g+5
/432/(1+x)*f+1/432/(1+x)*d-1/144/(1+x)*e+19/6912/(x-2)*d+17/3456/
(x-2)*e+5/576/(x-2)*f+5/432/(-1+x)*f+19/6912/(2+x)*d-17/3456/(2+x
)*e+1/432/(-1+x)*d+1/144/(-1+x)*e+5/576/(2+x)*f+13/1296*ln(1+x)*d
-1/81*ln(1+x)*e-13/1296*ln(-1+x)*d-1/81*ln(-1+x)*e-5/162*ln(1+x)*
g+5/162*ln(x-2)*g-5/162*ln(-1+x)*g+5/162*ln(2+x)*g+313/41472*ln(x
-2)*d+1/81*ln(x-2)*e+1/81*ln(2+x)*e+205/10368*ln(x-2)*f-313/41472
*ln(2+x)*d+25/1296*ln(1+x)*f-25/1296*ln(-1+x)*f-205/10368*ln(2+x)
*f
```

Maxima [A] time = 0.710456, size = 254, normalized size = 1.25

$$-\frac{1}{41472}\left(313\,d-512\,e+820\,f-1280\,g\right)\log\left(x+2\right)+\frac{1}{1296}\left(13\,d-16\,e+25\,f-40\,g\right)\log\left(x+1\right)$$
$$-\frac{1}{1296}\left(13\,d+16\,e+25\,f+40\,g\right)\log\left(x-1\right)+\frac{1}{41472}\left(313\,d+512\,e+820\,f+1280\,g\right)\log\left(x-2\right)$$
$$+\frac{35\,(d+4\,f)x^{7}+64\,(2\,e+5\,g)x^{6}-18\,(13\,d+60\,f)x^{5}-480\,(2\,e+5\,g)x^{4}+63\,(5\,d+36\,f)x^{3}+960\,(2\,e+5\,g)x^{2}+4\,(43\,d-26}{3456\,(x^{8}-10\,x^{6}+33\,x^{4}-40\,x^{2}+16)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3 + f*x^2 + e*x + d)/(x^4 - 5*x^2 + 4)^3,x, algorithm="maxima")
```

```
[Out] -1/41472*(313*d - 512*e + 820*f - 1280*g)*log(x + 2) + 1/1296*(13
*d - 16*e + 25*f - 40*g)*log(x + 1) - 1/1296*(13*d + 16*e + 25*f
+ 40*g)*log(x - 1) + 1/41472*(313*d + 512*e + 820*f + 1280*g)*log
(x - 2) + 1/3456*(35*(d + 4*f)*x^7 + 64*(2*e + 5*g)*x^6 - 18*(13*
d + 60*f)*x^5 - 480*(2*e + 5*g)*x^4 + 63*(5*d + 36*f)*x^3 + 960*(
2*e + 5*g)*x^2 + 4*(43*d - 260*f)*x - 800*e - 2432*g)/(x^8 - 10*x
^6 + 33*x^4 - 40*x^2 + 16)
```

Fricas [A] time = 0.627975, size = 635, normalized size = 3.11

$$\frac{420\,(d+4\,f)x^{7}+768\,(2\,e+5\,g)x^{6}-216\,(13\,d+60\,f)x^{5}-5760\,(2\,e+5\,g)x^{4}+756\,(5\,d+36\,f)x^{3}+11520\,(2\,e+5\,g)x^{2}+48\,(4}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3 + f*x^2 + e*x + d)/(x^4 - 5*x^2 + 4)^3,x, algorithm="fricas")
```

```
[Out] 1/41472*(420*(d + 4*f)*x^7 + 768*(2*e + 5*g)*x^6 - 216*(13*d + 60
*f)*x^5 - 5760*(2*e + 5*g)*x^4 + 756*(5*d + 36*f)*x^3 + 11520*(2*
e + 5*g)*x^2 + 48*(43*d - 260*f)*x - ((313*d - 512*e + 820*f - 12
80*g)*x^8 - 10*(313*d - 512*e + 820*f - 1280*g)*x^6 + 33*(313*d -
 512*e + 820*f - 1280*g)*x^4 - 40*(313*d - 512*e + 820*f - 1280*g
)*x^2 + 5008*d - 8192*e + 13120*f - 20480*g)*log(x + 2) + 32*((13
*d - 16*e + 25*f - 40*g)*x^8 - 10*(13*d - 16*e + 25*f - 40*g)*x^6
 + 33*(13*d - 16*e + 25*f - 40*g)*x^4 - 40*(13*d - 16*e + 25*f -
40*g)*x^2 + 208*d - 256*e + 400*f - 640*g)*log(x + 1) - 32*((13*d
 + 16*e + 25*f + 40*g)*x^8 - 10*(13*d + 16*e + 25*f + 40*g)*x^6 +
 33*(13*d + 16*e + 25*f + 40*g)*x^4 - 40*(13*d + 16*e + 25*f + 40
*g)*x^2 + 208*d + 256*e + 400*f + 640*g)*log(x - 1) + ((313*d + 5
12*e + 820*f + 1280*g)*x^8 - 10*(313*d + 512*e + 820*f + 1280*g)*
x^6 + 33*(313*d + 512*e + 820*f + 1280*g)*x^4 - 40*(313*d + 512*e
 + 820*f + 1280*g)*x^2 + 5008*d + 8192*e + 13120*f + 20480*g)*log
(x - 2) - 9600*e - 29184*g)/(x^8 - 10*x^6 + 33*x^4 - 40*x^2 + 16)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((g*x**3+f*x**2+e*x+d)/(x**4-5*x**2+4)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.265825, size = 257, normalized size = 1.26

$$-\frac{1}{41472}\,(313\,d+820\,f-1280\,g-512\,e)\ln\left(|x+2|\right)+\frac{1}{1296}\,(13\,d+25\,f-40\,g-16\,e)\ln\left(|x+1|\right)-\frac{1}{1296}\,(13\,d+25\,f+40\,g+16\,e)\ln\left(|x-1|\right)+\frac{1}{41472}\,(313\,d+820\,f+1280\,g+512\,e)\ln\left(|x-2|\right)+\frac{35\,dx^7+140\,fx^7+320\,gx^6+128\,x^6e-234\,dx^5-1080\,fx^5-2400\,gx^4-960\,x^4e+315\,dx^3+2268\,fx^3+4800\,gx^2+1920}{3456\,(x^4-5\,x^2+4)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((g*x^3 + f*x^2 + e*x + d)/(x^4 - 5*x^2 + 4)^3,x, algorithm="giac")
```

```
[Out] -1/41472*(313*d + 820*f - 1280*g - 512*e)*ln(abs(x + 2)) + 1/1296
*(13*d + 25*f - 40*g - 16*e)*ln(abs(x + 1)) - 1/1296*(13*d + 25*f
 + 40*g + 16*e)*ln(abs(x - 1)) + 1/41472*(313*d + 820*f + 1280*g
+ 512*e)*ln(abs(x - 2)) + 1/3456*(35*d*x^7 + 140*f*x^7 + 320*g*x^
6 + 128*x^6*e - 234*d*x^5 - 1080*f*x^5 - 2400*g*x^4 - 960*x^4*e +
 315*d*x^3 + 2268*f*x^3 + 4800*g*x^2 + 1920*x^2*e + 172*d*x - 104
0*f*x - 2432*g - 800*e)/(x^4 - 5*x^2 + 4)^2
```

3.45 $\int \frac{d+ex+fx^2+gx^3+hx^4}{(4-5x^2+x^4)^3}\,dx$

Optimal. Leaf size=224

$$-\frac{x\left(-5x^2(7d+28f+64h)+59d+380f+848h\right)}{3456\left(x^4-5x^2+4\right)}+\frac{x\left(x^2(-(5d+8f+20h))+17d+20f+32h\right)}{144\left(x^4-5x^2+4\right)^2}-\frac{\tanh^{-1}\left(\frac{x}{2}\right)(313d+820f+1936h)}{20736}+\frac{1}{648}\tanh^{-1}(x)(13d+25f+61h)-\frac{1}{162}(2e+5g)\log\left(1-x^2\right)+\frac{1}{162}(2e+5g)\log\left(4-x^2\right)-\frac{\left(5-2x^2\right)(2e+5g)}{108\left(x^4-5x^2+4\right)}+\frac{x^2(-(2e+5g))+5e+8g}{36\left(x^4-5x^2+4\right)^2}$$

```
[Out] (5*e + 8*g - (2*e + 5*g)*x^2)/(36*(4 - 5*x^2 + x^4)^2) + (x*(17*d
 + 20*f + 32*h - (5*d + 8*f + 20*h)*x^2))/(144*(4 - 5*x^2 + x^4)^
2) - ((2*e + 5*g)*(5 - 2*x^2))/(108*(4 - 5*x^2 + x^4)) - (x*(59*d
 + 380*f + 848*h - 5*(7*d + 28*f + 64*h)*x^2))/(3456*(4 - 5*x^2 +
 x^4)) - ((313*d + 820*f + 1936*h)*ArcTanh[x/2])/20736 + ((13*d +
 25*f + 61*h)*ArcTanh[x])/648 - ((2*e + 5*g)*Log[1 - x^2])/162 +
((2*e + 5*g)*Log[4 - x^2])/162
```

Rubi [A] time = 0.626121, antiderivative size = 224, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 10, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.303$

$$-\frac{x\left(-5x^2(7d+28f+64h)+59d+380f+848h\right)}{3456\left(x^4-5x^2+4\right)}+\frac{x\left(x^2(-(5d+8f+20h))+17d+20f+32h\right)}{144\left(x^4-5x^2+4\right)^2}-\frac{\tanh^{-1}\left(\frac{x}{2}\right)(313d+820f+1936h)}{20736}+\frac{1}{648}\tanh^{-1}(x)(13d+25f+61h)-\frac{1}{162}(2e+5g)\log\left(1-x^2\right)+\frac{1}{162}(2e+5g)\log\left(4-x^2\right)-\frac{\left(5-2x^2\right)(2e+5g)}{108\left(x^4-5x^2+4\right)}+\frac{x^2(-(2e+5g))+5e+8g}{36\left(x^4-5x^2+4\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x + f*x^2 + g*x^3 + h*x^4)/(4 - 5*x^2 + x^4)^3,x]
```


```
[Out] (5*e + 8*g - (2*e + 5*g)*x^2)/(36*(4 - 5*x^2 + x^4)^2) + (x*(17*d
 + 20*f + 32*h - (5*d + 8*f + 20*h)*x^2))/(144*(4 - 5*x^2 + x^4)^
2) - ((2*e + 5*g)*(5 - 2*x^2))/(108*(4 - 5*x^2 + x^4)) - (x*(59*d
 + 380*f + 848*h - 5*(7*d + 28*f + 64*h)*x^2))/(3456*(4 - 5*x^2 +
 x^4)) - ((313*d + 820*f + 1936*h)*ArcTanh[x/2])/20736 + ((13*d +
 25*f + 61*h)*ArcTanh[x])/648 - ((2*e + 5*g)*Log[1 - x^2])/162 +
((2*e + 5*g)*Log[4 - x^2])/162
```

Rubi in Sympy [A] time = 88.8783, size = 182, normalized size = 0.81

$$-\frac{x\left(4425d+28500f+63600h-x^3(3750e+11400g)-x^2(2625d+10500f+24000h)+x(9150e+33000g)\right)}{259200(x^4-5x^2+4)}+\frac{x\left(425d+500f+800h-x^3(125e+200g)-x^2(125d+200f+500h)+x(425e+500g)\right)}{3600(x^4-5x^2+4)^2}-\left(\frac{e}{81}+\frac{5g}{162}\right)\log\left(-x^2+1\right)+\left(\frac{e}{81}+\frac{5g}{162}\right)\log\left(-x^2+4\right)-\left(\frac{313d}{20736}+\frac{205f}{5184}+\frac{121h}{1296}\right)\operatorname{atanh}\left(\frac{x}{2}\right)+\left(\frac{13d}{648}+\frac{25f}{648}+\frac{61h}{648}\right)\operatorname{atanh}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((h*x**4+g*x**3+f*x**2+e*x+d)/(x**4-5*x**2+4)**3,x)
```

```
[Out] -x*(4425*d + 28500*f + 63600*h - x**3*(3750*e + 11400*g) - x**2*(
2625*d + 10500*f + 24000*h) + x*(9150*e + 33000*g))/(259200*(x**4
 - 5*x**2 + 4)) + x*(425*d + 500*f + 800*h - x**3*(125*e + 200*g)
 - x**2*(125*d + 200*f + 500*h) + x*(425*e + 500*g))/(3600*(x**4
- 5*x**2 + 4)**2) - (e/81 + 5*g/162)*log(-x**2 + 1) + (e/81 + 5*g
/162)*log(-x**2 + 4) - (313*d/20736 + 205*f/5184 + 121*h/1296)*at
anh(x/2) + (13*d/648 + 25*f/648 + 61*h/648)*atanh(x)
```

Mathematica [A] time = 0.236482, size = 231, normalized size = 1.03

$$\frac{-5dx^3+17dx-8ex^2+20e-8fx^3+20fx-20gx^2+32g-20hx^3+32hx}{144(x^4-5x^2+4)^2}+\frac{35dx^3-59dx+128ex^2-320e+140fx^3-380fx+320gx^2-800g+320hx^3-848hx}{3456(x^4-5x^2+4)}+\frac{\log(1-x)(-13d-16e-25f-40g-61h)}{1296}+\frac{\log(2-x)(313d+512e+820f+1280g+1936h)}{41472}+\frac{\log(x+1)(13d-16e+25f-40g+61h)}{1296}+\frac{\log(x+2)(-313d+512e-820f+1280g-1936h)}{41472}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x + f*x^2 + g*x^3 + h*x^4)/(4 - 5*x^2 + x^4)^3,x]
```

```
[Out] (20*e + 32*g + 17*d*x + 20*f*x + 32*h*x - 8*e*x^2 - 20*g*x^2 - 5*
d*x^3 - 8*f*x^3 - 20*h*x^3)/(144*(4 - 5*x^2 + x^4)^2) + (-320*e -
 800*g - 59*d*x - 380*f*x - 848*h*x + 128*e*x^2 + 320*g*x^2 + 35*
d*x^3 + 140*f*x^3 + 320*h*x^3)/(3456*(4 - 5*x^2 + x^4)) + ((-13*d
 - 16*e - 25*f - 40*g - 61*h)*Log[1 - x])/1296 + ((313*d + 512*e
+ 820*f + 1280*g + 1936*h)*Log[2 - x])/41472 + ((13*d - 16*e + 25
```

```
*f - 40*g + 61*h)*Log[1 + x])/1296 + ((-313*d + 512*e - 820*f + 1
280*g - 1936*h)*Log[2 + x])/41472
```

Maple [B] time = 0.032, size = 462, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^3,x)
```

```
[Out] -1/216/(x-2)^2*h+1/432/(-1+x)^2*h-1/432/(1+x)^2*h+1/216/(2+x)^2*h
-1/432/(x-2)^2*g+1/432/(1+x)^2*g+1/432/(-1+x)^2*g-1/432/(2+x)^2*g
+1/432/(-1+x)^2*d+1/432/(-1+x)^2*e-1/432/(1+x)^2*d+1/432/(1+x)^2*
e-1/3456/(x-2)^2*d-1/1728/(x-2)^2*e+1/3456/(2+x)^2*d-1/1728/(2+x)
^2*e-1/432/(1+x)^2*f-1/864/(x-2)^2*f+1/864/(2+x)^2*f+1/432/(-1+x)
^2*f+11/432/(x-2)*h+1/48/(1+x)*h+1/48/(-1+x)*h+11/432/(2+x)*h-7/4
32/(1+x)*g+7/432/(-1+x)*g-13/864/(2+x)*g+13/864/(x-2)*g+5/432/(1+
x)*f+1/432/(1+x)*d-1/144/(1+x)*e+19/6912/(x-2)*d+17/3456/(x-2)*e+
5/576/(x-2)*f+5/432/(-1+x)*f+19/6912/(2+x)*d-17/3456/(2+x)*e+1/43
2/(-1+x)*d+1/144/(-1+x)*e+5/576/(2+x)*f+13/1296*ln(1+x)*d-1/81*ln
(1+x)*e-13/1296*ln(-1+x)*d-1/81*ln(-1+x)*e+121/2592*ln(x-2)*h+61/
1296*ln(1+x)*h-121/2592*ln(2+x)*h-61/1296*ln(-1+x)*h-5/162*ln(1+x
)*g+5/162*ln(x-2)*g-5/162*ln(-1+x)*g+5/162*ln(2+x)*g+313/41472*ln
(x-2)*d+1/81*ln(x-2)*e+1/81*ln(2+x)*e+205/10368*ln(x-2)*f-313/414
72*ln(2+x)*d+25/1296*ln(1+x)*f-25/1296*ln(-1+x)*f-205/10368*ln(2+
x)*f
```

Maxima [A] time = 0.70701, size = 289, normalized size = 1.29

$$-\frac{1}{41472}\left(313\,d - 512\,e + 820\,f - 1280\,g + 1936\,h\right)\log\left(x + 2\right)$$
$$+\frac{1}{1296}\left(13\,d - 16\,e + 25\,f - 40\,g + 61\,h\right)\log\left(x + 1\right) - \frac{1}{1296}\left(13\,d + 16\,e + 25\,f + 40\,g + 61\,h\right)\log\left(x - 1\right)$$
$$+\frac{1}{41472}\left(313\,d + 512\,e + 820\,f + 1280\,g + 1936\,h\right)\log\left(x - 2\right)$$
$$+\frac{5\left(7\,d + 28\,f + 64\,h\right)x^7 + 64\left(2\,e + 5\,g\right)x^6 - 18\left(13\,d + 60\,f + 136\,h\right)x^5 - 480\left(2\,e + 5\,g\right)x^4 + 63\left(5\,d + 36\,f + 80\,h\right)x^3 + 960\left(2\,e\right.}{3456\left(x^8 - 10\,x^6 + 33\,x^4 - 40\,x^2 + 16\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((h*x^4 + g*x^3 + f*x^2 + e*x + d)/(x^4 - 5*x^2 + 4)^3,x, algorithm="maxi
```

```
[Out] -1/41472*(313*d - 512*e + 820*f - 1280*g + 1936*h)*log(x + 2) + 1
/1296*(13*d - 16*e + 25*f - 40*g + 61*h)*log(x + 1) - 1/1296*(13*
```

```
d + 16*e + 25*f + 40*g + 61*h)*log(x - 1) + 1/41472*(313*d + 512*
e + 820*f + 1280*g + 1936*h)*log(x - 2) + 1/3456*(5*(7*d + 28*f +
 64*h)*x^7 + 64*(2*e + 5*g)*x^6 - 18*(13*d + 60*f + 136*h)*x^5 -
480*(2*e + 5*g)*x^4 + 63*(5*d + 36*f + 80*h)*x^3 + 960*(2*e + 5*g
)*x^2 + 4*(43*d - 260*f - 656*h)*x - 800*e - 2432*g)/(x^8 - 10*x^
6 + 33*x^4 - 40*x^2 + 16)
```

Fricas [A] time = 2.01395, size = 734, normalized size = 3.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^4 + g*x^3 + f*x^2 + e*x + d)/(x^4 - 5*x^2 + 4)^3,x, algorithm="fric
```

```
[Out] 1/41472*(60*(7*d + 28*f + 64*h)*x^7 + 768*(2*e + 5*g)*x^6 - 216*(
13*d + 60*f + 136*h)*x^5 - 5760*(2*e + 5*g)*x^4 + 756*(5*d + 36*f
 + 80*h)*x^3 + 11520*(2*e + 5*g)*x^2 + 48*(43*d - 260*f - 656*h)*
x - ((313*d - 512*e + 820*f - 1280*g + 1936*h)*x^8 - 10*(313*d -
512*e + 820*f - 1280*g + 1936*h)*x^6 + 33*(313*d - 512*e + 820*f
- 1280*g + 1936*h)*x^4 - 40*(313*d - 512*e + 820*f - 1280*g + 193
6*h)*x^2 + 5008*d - 8192*e + 13120*f - 20480*g + 30976*h)*log(x +
 2) + 32*((13*d - 16*e + 25*f - 40*g + 61*h)*x^8 - 10*(13*d - 16*
e + 25*f - 40*g + 61*h)*x^6 + 33*(13*d - 16*e + 25*f - 40*g + 61*
h)*x^4 - 40*(13*d - 16*e + 25*f - 40*g + 61*h)*x^2 + 208*d - 256*
e + 400*f - 640*g + 976*h)*log(x + 1) - 32*((13*d + 16*e + 25*f +
 40*g + 61*h)*x^8 - 10*(13*d + 16*e + 25*f + 40*g + 61*h)*x^6 + 3
3*(13*d + 16*e + 25*f + 40*g + 61*h)*x^4 - 40*(13*d + 16*e + 25*f
 + 40*g + 61*h)*x^2 + 208*d + 256*e + 400*f + 640*g + 976*h)*log(
x - 1) + ((313*d + 512*e + 820*f + 1280*g + 1936*h)*x^8 - 10*(313
*d + 512*e + 820*f + 1280*g + 1936*h)*x^6 + 33*(313*d + 512*e + 8
20*f + 1280*g + 1936*h)*x^4 - 40*(313*d + 512*e + 820*f + 1280*g
+ 1936*h)*x^2 + 5008*d + 8192*e + 13120*f + 20480*g + 30976*h)*lo
g(x - 2) - 9600*e - 29184*g)/(x^8 - 10*x^6 + 33*x^4 - 40*x^2 + 16
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((h*x**4+g*x**3+f*x**2+e*x+d)/(x**4-5*x**2+4)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.267177, size = 302, normalized size = 1.35

$$-\frac{1}{41472}\,(313\,d + 820\,f - 1280\,g + 1936\,h - 512\,e)\ln\left(|x+2|\right) + \frac{1}{1296}\,(13\,d + 25\,f - 40\,g + 61\,h - 16\,e)\ln\left(|x+1|\right) - \frac{1}{1296}\,(13\,d + 25\,f + 40\,g + 61\,h + 16\,e)\ln\left(|x-1|\right) + \frac{1}{41472}\,(313\,d + 820\,f + 1280\,g + 1936\,h + 512\,e)\ln\left(|x-2|\right) + \frac{35\,dx^7 + 140\,fx^7 + 320\,hx^7 + 320\,gx^6 + 128\,x^6e - 234\,dx^5 - 1080\,fx^5 - 2448\,hx^5 - 2400\,gx^4 - 960\,x^4e + 315\,dx^3 + 2268\,f}{3456\,(x^4 - 5\,x^2 + 4)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((h*x^4 + g*x^3 + f*x^2 + e*x + d)/(x^4 - 5*x^2 + 4)^3,x, algorithm="giac
```

```
[Out] -1/41472*(313*d + 820*f - 1280*g + 1936*h - 512*e)*ln(abs(x + 2))
 + 1/1296*(13*d + 25*f - 40*g + 61*h - 16*e)*ln(abs(x + 1)) - 1/1
296*(13*d + 25*f + 40*g + 61*h + 16*e)*ln(abs(x - 1)) + 1/41472*(
313*d + 820*f + 1280*g + 1936*h + 512*e)*ln(abs(x - 2)) + 1/3456*
(35*d*x^7 + 140*f*x^7 + 320*h*x^7 + 320*g*x^6 + 128*x^6*e - 234*d
*x^5 - 1080*f*x^5 - 2448*h*x^5 - 2400*g*x^4 - 960*x^4*e + 315*d*x
^3 + 2268*f*x^3 + 5040*h*x^3 + 4800*g*x^2 + 1920*x^2*e + 172*d*x
- 1040*f*x - 2624*h*x - 2432*g - 800*e)/(x^4 - 5*x^2 + 4)^2
```

3.46 $\int \frac{d+ex+fx^2+gx^3+hx^4+ix^5}{(4-5x^2+x^4)^3}\,dx$

Optimal. Leaf size=239

$$-\frac{x\left(-5x^2(7d+28f+64h)+59d+380f+848h\right)}{3456\left(x^4-5x^2+4\right)}+\frac{x\left(x^2(-(5d+8f+20h))+17d+20f+32h\right)}{144\left(x^4-5x^2+4\right)^2}-\frac{\tanh^{-1}\left(\frac{x}{2}\right)(313d+820f+1936h)}{20736}+\frac{1}{648}\tanh^{-1}(x)(13d+25f+61h)-\frac{1}{162}\log\left(1-x^2\right)(2e+5g+11i)+\frac{1}{162}\log\left(4-x^2\right)(2e+5g+11i)-\frac{\left(5-2x^2\right)(2e+5g+11i)}{108\left(x^4-5x^2+4\right)}+\frac{x^2(-(2e+5g+17i))+5e+8g+20i}{36\left(x^4-5x^2+4\right)^2}$$

```
[Out] (x*(17*d + 20*f + 32*h - (5*d + 8*f + 20*h)*x^2))/(144*(4 - 5*x^2
 + x^4)^2) + (5*e + 8*g + 20*i - (2*e + 5*g + 17*i)*x^2)/(36*(4 -
 5*x^2 + x^4)^2) - ((2*e + 5*g + 11*i)*(5 - 2*x^2))/(108*(4 - 5*x
^2 + x^4)) - (x*(59*d + 380*f + 848*h - 5*(7*d + 28*f + 64*h)*x^2
))/(3456*(4 - 5*x^2 + x^4)) - ((313*d + 820*f + 1936*h)*ArcTanh[x
/2])/20736 + ((13*d + 25*f + 61*h)*ArcTanh[x])/648 - ((2*e + 5*g
+ 11*i)*Log[1 - x^2])/162 + ((2*e + 5*g + 11*i)*Log[4 - x^2])/162
```

Rubi [A] time = 0.660687, antiderivative size = 239, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 11, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.29$

$$-\frac{x\left(-5x^2(7d+28f+64h)+59d+380f+848h\right)}{3456\left(x^4-5x^2+4\right)}+\frac{x\left(x^2(-(5d+8f+20h))+17d+20f+32h\right)}{144\left(x^4-5x^2+4\right)^2}-\frac{\tanh^{-1}\left(\frac{x}{2}\right)(313d+820f+1936h)}{20736}+\frac{1}{648}\tanh^{-1}(x)(13d+25f+61h)-\frac{1}{162}\log\left(1-x^2\right)(2e+5g+11i)+\frac{1}{162}\log\left(4-x^2\right)(2e+5g+11i)-\frac{\left(5-2x^2\right)(2e+5g+11i)}{108\left(x^4-5x^2+4\right)}+\frac{x^2(-(2e+5g+17i))+5e+8g+20i}{36\left(x^4-5x^2+4\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5)/(4 - 5*x^2 + x^4)^3,x]
```


```
[Out] (x*(17*d + 20*f + 32*h - (5*d + 8*f + 20*h)*x^2))/(144*(4 - 5*x^2
 + x^4)^2) + (5*e + 8*g + 20*i - (2*e + 5*g + 17*i)*x^2)/(36*(4 -
 5*x^2 + x^4)^2) - ((2*e + 5*g + 11*i)*(5 - 2*x^2))/(108*(4 - 5*x
^2 + x^4)) - (x*(59*d + 380*f + 848*h - 5*(7*d + 28*f + 64*h)*x^2
))/(3456*(4 - 5*x^2 + x^4)) - ((313*d + 820*f + 1936*h)*ArcTanh[x
/2])/20736 + ((13*d + 25*f + 61*h)*ArcTanh[x])/648 - ((2*e + 5*g
```

```
+ 11*i)*Log[1 - x^2])/162 + ((2*e + 5*g + 11*i)*Log[4 - x^2])/162
```

Rubi in Sympy [A] time = 104.153, size = 196, normalized size = 0.82

$$-\frac{x\left(22125d+142500f+318000h-x^3\left(18750e+57000g+120000\right)-x^2\left(13125d+52500f+120000h\right)+x\left(45750e+1650\right.\right.}{1296000\left(x^4-5x^2+4\right)}$$
$$+\frac{x\left(2125d+2500f+4000h-x^3\left(625e+1000g+2500\right)-x^2\left(625d+1000f+2500h\right)+x\left(2125e+2500g+4000\right)\right)}{18000\left(x^4-5x^2+4\right)^2}$$
$$-\left(\frac{313d}{20736}+\frac{205f}{5184}+\frac{121h}{1296}\right)\operatorname{atanh}\left(\frac{x}{2}\right)+\left(\frac{13d}{648}+\frac{25f}{648}+\frac{61h}{648}\right)\operatorname{atanh}\left(x\right)$$
$$-\left(\frac{e}{81}+\frac{5g}{162}+\frac{11}{162}\right)\log\left(-x^2+1\right)+\left(\frac{e}{81}+\frac{5g}{162}+\frac{11}{162}\right)\log\left(-x^2+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((i*x**5+h*x**4+g*x**3+f*x**2+e*x+d)/(x**4-5*x**2+4)**3,x)
```

```
[Out] -x*(22125*d + 142500*f + 318000*h - x**3*(18750*e + 57000*g + 120
000) - x**2*(13125*d + 52500*f + 120000*h) + x*(45750*e + 165000*
g + 336000))/(1296000*(x**4 - 5*x**2 + 4)) + x*(2125*d + 2500*f +
 4000*h - x**3*(625*e + 1000*g + 2500) - x**2*(625*d + 1000*f + 2
500*h) + x*(2125*e + 2500*g + 4000))/(18000*(x**4 - 5*x**2 + 4)**
2) - (313*d/20736 + 205*f/5184 + 121*h/1296)*atanh(x/2) + (13*d/6
48 + 25*f/648 + 61*h/648)*atanh(x) - (e/81 + 5*g/162 + 11/162)*lo
g(-x**2 + 1) + (e/81 + 5*g/162 + 11/162)*log(-x**2 + 4)
```

Mathematica [A] time = 0.304763, size = 261, normalized size = 1.09

$$\frac{-5dx^3+17dx-8ex^2+20e-8fx^3+20fx-20gx^2+32g-20hx^3+32hx-68ix^2+80i}{144\left(x^4-5x^2+4\right)^2}$$
$$+\frac{35dx^3-59dx+128ex^2-320e+140fx^3-380fx+320gx^2-800g+320hx^3-848hx+704ix^2-1760i}{3456\left(x^4-5x^2+4\right)}$$
$$+\frac{\log(1-x)(-13d-16e-25f-40g-61h-88i)}{1296}$$
$$+\frac{\log(2-x)(313d+512e+820f+1280g+1936h+2816i)}{41472}$$
$$+\frac{\log(x+1)(13d-16e+25f-40g+61h-88i)}{1296}$$
$$+\frac{\log(x+2)(-313d+512e-820f+1280g-1936h+2816i)}{41472}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5)/(4 - 5*x^2 + x^4)^3,x]
```

```
[Out] (20*e + 32*g + 80*i + 17*d*x + 20*f*x + 32*h*x - 8*e*x^2 - 20*g*x
^2 - 68*i*x^2 - 5*d*x^3 - 8*f*x^3 - 20*h*x^3)/(144*(4 - 5*x^2 + x
^4)^2) + (-320*e - 800*g - 1760*i - 59*d*x - 380*f*x - 848*h*x +
128*e*x^2 + 320*g*x^2 + 704*i*x^2 + 35*d*x^3 + 140*f*x^3 + 320*h*
x^3)/(3456*(4 - 5*x^2 + x^4)) + ((-13*d - 16*e - 25*f - 40*g - 61
*h - 88*i)*Log[1 - x])/1296 + ((313*d + 512*e + 820*f + 1280*g +
1936*h + 2816*i)*Log[2 - x])/41472 + ((13*d - 16*e + 25*f - 40*g
+ 61*h - 88*i)*Log[1 + x])/1296 + ((-313*d + 512*e - 820*f + 1280
*g - 1936*h + 2816*i)*Log[2 + x])/41472
```

Maple [B] time = 0.034, size = 554, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^3,x)
```

```
[Out] -1/108/(2+x)^2*i-1/108/(x-2)^2*i+1/432/(1+x)^2*i+1/432/(-1+x)^2*i
-1/216/(x-2)^2*h+1/432/(-1+x)^2*h-1/432/(1+x)^2*h+1/216/(2+x)^2*h
-1/432/(x-2)^2*g+1/432/(1+x)^2*g+1/432/(-1+x)^2*g-1/432/(2+x)^2*g
+1/432/(-1+x)^2*d+1/432/(-1+x)^2*e-1/432/(1+x)^2*d+1/432/(1+x)^2*
e-1/3456/(x-2)^2*d-1/1728/(x-2)^2*e+1/3456/(2+x)^2*d-1/1728/(2+x)
^2*e-1/432/(1+x)^2*f-1/864/(x-2)^2*f+1/864/(2+x)^2*f+1/432/(-1+x)
^2*f+1/24/(x-2)*i-11/432/(1+x)*i+11/432/(-1+x)*i-1/24/(2+x)*i+11/
432/(x-2)*h+1/48/(1+x)*h+1/48/(-1+x)*h+11/432/(2+x)*h-7/432/(1+x)
*g+7/432/(-1+x)*g-13/864/(2+x)*g+13/864/(x-2)*g+5/432/(1+x)*f+1/4
32/(1+x)*d-1/144/(1+x)*e+19/6912/(x-2)*d+17/3456/(x-2)*e+5/576/(x
-2)*f+5/432/(-1+x)*f+19/6912/(2+x)*d-17/3456/(2+x)*e+1/432/(-1+x)
*d+1/144/(-1+x)*e+5/576/(2+x)*f+13/1296*ln(1+x)*d-1/81*ln(1+x)*e-
13/1296*ln(-1+x)*d-1/81*ln(-1+x)*e+11/162*ln(x-2)*i-11/162*ln(1+x
)*i-11/162*ln(-1+x)*i+11/162*ln(2+x)*i+121/2592*ln(x-2)*h+61/1296
*ln(1+x)*h-121/2592*ln(2+x)*h-61/1296*ln(-1+x)*h-5/162*ln(1+x)*g+
5/162*ln(x-2)*g-5/162*ln(-1+x)*g+5/162*ln(2+x)*g+313/41472*ln(x-2
)*d+1/81*ln(x-2)*e+1/81*ln(2+x)*e+205/10368*ln(x-2)*f-313/41472*l
n(2+x)*d+25/1296*ln(1+x)*f-25/1296*ln(-1+x)*f-205/10368*ln(2+x)*f
```

Maxima [A] time = 0.709746, size = 321, normalized size = 1.34

$$-\frac{1}{41472}\left(313\,d-512\,e+820\,f-1280\,g+1936\,h-2816\,i\right)\log\left(x+2\right)$$
$$+\frac{1}{1296}\left(13\,d-16\,e+25\,f-40\,g+61\,h-88\,i\right)\log\left(x+1\right)$$
$$-\frac{1}{1296}\left(13\,d+16\,e+25\,f+40\,g+61\,h+88\,i\right)\log\left(x-1\right)$$
$$+\frac{1}{41472}\left(313\,d+512\,e+820\,f+1280\,g+1936\,h+2816\,i\right)\log\left(x-2\right)$$
$$+\frac{5\left(7\,d+28\,f+64\,h\right)x^{7}+64\left(2\,e+5\,g+11\,i\right)x^{6}-18\left(13\,d+60\,f+136\,h\right)x^{5}-480\left(2\,e+5\,g+11\,i\right)x^{4}+63\left(5\,d+36\,f+80\,h\right)}{3456\left(x^{8}-10\,x^{6}+33\,x^{4}-40\,x^{2}+16\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^5 + h*x^4 + g*x^3 + f*x^2 + e*x + d)/(x^4 - 5*x^2 + 4)^3,x, algorith
```

```
[Out] -1/41472*(313*d - 512*e + 820*f - 1280*g + 1936*h - 2816*i)*log(x
 + 2) + 1/1296*(13*d - 16*e + 25*f - 40*g + 61*h - 88*i)*log(x +
1) - 1/1296*(13*d + 16*e + 25*f + 40*g + 61*h + 88*i)*log(x - 1)
+ 1/41472*(313*d + 512*e + 820*f + 1280*g + 1936*h + 2816*i)*log(
x - 2) + 1/3456*(5*(7*d + 28*f + 64*h)*x^7 + 64*(2*e + 5*g + 11*i
)*x^6 - 18*(13*d + 60*f + 136*h)*x^5 - 480*(2*e + 5*g + 11*i)*x^4
 + 63*(5*d + 36*f + 80*h)*x^3 + 192*(10*e + 25*g + 52*i)*x^2 + 4*
(43*d - 260*f - 656*h)*x - 800*e - 2432*g - 5120*i)/(x^8 - 10*x^6
 + 33*x^4 - 40*x^2 + 16)
```

Fricas [A] time = 8.88531, size = 832, normalized size = 3.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^5 + h*x^4 + g*x^3 + f*x^2 + e*x + d)/(x^4 - 5*x^2 + 4)^3,x, algorith
```

```
[Out] 1/41472*(60*(7*d + 28*f + 64*h)*x^7 + 768*(2*e + 5*g + 11*i)*x^6
- 216*(13*d + 60*f + 136*h)*x^5 - 5760*(2*e + 5*g + 11*i)*x^4 + 7
56*(5*d + 36*f + 80*h)*x^3 + 2304*(10*e + 25*g + 52*i)*x^2 + 48*(
43*d - 260*f - 656*h)*x - ((313*d - 512*e + 820*f - 1280*g + 1936
*h - 2816*i)*x^8 - 10*(313*d - 512*e + 820*f - 1280*g + 1936*h -
2816*i)*x^6 + 33*(313*d - 512*e + 820*f - 1280*g + 1936*h - 2816*
i)*x^4 - 40*(313*d - 512*e + 820*f - 1280*g + 1936*h - 2816*i)*x^
2 + 5008*d - 8192*e + 13120*f - 20480*g + 30976*h - 45056*i)*log(
x + 2) + 32*((13*d - 16*e + 25*f - 40*g + 61*h - 88*i)*x^8 - 10*(
13*d - 16*e + 25*f - 40*g + 61*h - 88*i)*x^6 + 33*(13*d - 16*e +
25*f - 40*g + 61*h - 88*i)*x^4 - 40*(13*d - 16*e + 25*f - 40*g +
61*h - 88*i)*x^2 + 208*d - 256*e + 400*f - 640*g + 976*h - 1408*i
```

```
)*log(x + 1) - 32*((13*d + 16*e + 25*f + 40*g + 61*h + 88*i)*x^8
- 10*(13*d + 16*e + 25*f + 40*g + 61*h + 88*i)*x^6 + 33*(13*d + 1
6*e + 25*f + 40*g + 61*h + 88*i)*x^4 - 40*(13*d + 16*e + 25*f + 4
0*g + 61*h + 88*i)*x^2 + 208*d + 256*e + 400*f + 640*g + 976*h +
1408*i)*log(x - 1) + ((313*d + 512*e + 820*f + 1280*g + 1936*h +
2816*i)*x^8 - 10*(313*d + 512*e + 820*f + 1280*g + 1936*h + 2816*
i)*x^6 + 33*(313*d + 512*e + 820*f + 1280*g + 1936*h + 2816*i)*x^
4 - 40*(313*d + 512*e + 820*f + 1280*g + 1936*h + 2816*i)*x^2 + 5
008*d + 8192*e + 13120*f + 20480*g + 30976*h + 45056*i)*log(x - 2
) - 9600*e - 29184*g - 61440*i)/(x^8 - 10*x^6 + 33*x^4 - 40*x^2 +
 16)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x**5+h*x**4+g*x**3+f*x**2+e*x+d)/(x**4-5*x**2+4)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.273092, size = 347, normalized size = 1.45

$$-\frac{1}{41472}\,(313\,d + 820\,f - 1280\,g + 1936\,h - 2816\,i - 512\,e)\ln\left(|x + 2|\right) + \frac{1}{1296}\,(13\,d + 25\,f - 40\,g + 61\,h - 88\,i - 16\,e)\ln\left(|x + 1|\right) - \frac{1}{1296}\,(13\,d + 25\,f + 40\,g + 61\,h + 88\,i + 16\,e)\ln\left(|x - 1|\right) + \frac{1}{41472}\,(313\,d + 820\,f + 1280\,g + 1936\,h + 2816\,i + 512\,e)\ln\left(|x - 2|\right) + \frac{35\,dx^7 + 140\,fx^7 + 320\,hx^7 + 320\,gx^6 + 704\,ix^6 + 128\,x^6e - 234\,dx^5 - 1080\,fx^5 - 2448\,hx^5 - 2400\,gx^4 - 5280\,ix^4 - 960\,x}{3456\,(x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^5 + h*x^4 + g*x^3 + f*x^2 + e*x + d)/(x^4 - 5*x^2 + 4)^3,x, algorith
```

```
[Out] -1/41472*(313*d + 820*f - 1280*g + 1936*h - 2816*i - 512*e)*ln(ab
s(x + 2)) + 1/1296*(13*d + 25*f - 40*g + 61*h - 88*i - 16*e)*ln(a
bs(x + 1)) - 1/1296*(13*d + 25*f + 40*g + 61*h + 88*i + 16*e)*ln(
abs(x - 1)) + 1/41472*(313*d + 820*f + 1280*g + 1936*h + 2816*i +
 512*e)*ln(abs(x - 2)) + 1/3456*(35*d*x^7 + 140*f*x^7 + 320*h*x^7
```

```
 + 320*g*x^6 + 704*i*x^6 + 128*x^6*e - 234*d*x^5 - 1080*f*x^5 - 2
448*h*x^5 - 2400*g*x^4 - 5280*i*x^4 - 960*x^4*e + 315*d*x^3 + 226
8*f*x^3 + 5040*h*x^3 + 4800*g*x^2 + 9984*i*x^2 + 1920*x^2*e + 172
*d*x - 1040*f*x - 2624*h*x - 2432*g - 5120*i - 800*e)/(x^4 - 5*x^
2 + 4)^2
```

3.47 $\int \frac{d+ex}{(1+x^2+x^4)^3}\,dx$

Optimal. Leaf size=185

$$-\frac{9}{32}d\log\left(x^2-x+1\right)+\frac{9}{32}d\log\left(x^2+x+1\right)+\frac{dx\left(2-7x^2\right)}{24\left(x^4+x^2+1\right)}+\frac{dx\left(1-x^2\right)}{12\left(x^4+x^2+1\right)^2}-\frac{13d\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{48\sqrt{3}}+\frac{13d\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{48\sqrt{3}}+\frac{2e\tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{e\left(2x^2+1\right)}{6\left(x^4+x^2+1\right)}+\frac{e\left(2x^2+1\right)}{12\left(x^4+x^2+1\right)^2}$$

```
[Out] (d*x*(1 - x^2))/(12*(1 + x^2 + x^4)^2) + (e*(1 + 2*x^2))/(12*(1 +
 x^2 + x^4)^2) + (d*x*(2 - 7*x^2))/(24*(1 + x^2 + x^4)) + (e*(1 +
 2*x^2))/(6*(1 + x^2 + x^4)) - (13*d*ArcTan[(1 - 2*x)/Sqrt[3]])/(
48*Sqrt[3]) + (13*d*ArcTan[(1 + 2*x)/Sqrt[3]])/(48*Sqrt[3]) + (2*
e*ArcTan[(1 + 2*x^2)/Sqrt[3]])/(3*Sqrt[3]) - (9*d*Log[1 - x + x^2
])/32 + (9*d*Log[1 + x + x^2])/32
```

Rubi [A] time = 0.258473, antiderivative size = 185, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 11, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.688$

$$-\frac{9}{32}d\log\left(x^2-x+1\right)+\frac{9}{32}d\log\left(x^2+x+1\right)+\frac{dx\left(2-7x^2\right)}{24\left(x^4+x^2+1\right)}+\frac{dx\left(1-x^2\right)}{12\left(x^4+x^2+1\right)^2}-\frac{13d\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{48\sqrt{3}}+\frac{13d\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{48\sqrt{3}}+\frac{2e\tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{e\left(2x^2+1\right)}{6\left(x^4+x^2+1\right)}+\frac{e\left(2x^2+1\right)}{12\left(x^4+x^2+1\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x)/(1 + x^2 + x^4)^3,x]
```

```
[Out] (d*x*(1 - x^2))/(12*(1 + x^2 + x^4)^2) + (e*(1 + 2*x^2))/(12*(1 +
 x^2 + x^4)^2) + (d*x*(2 - 7*x^2))/(24*(1 + x^2 + x^4)) + (e*(1 +
 2*x^2))/(6*(1 + x^2 + x^4)) - (13*d*ArcTan[(1 - 2*x)/Sqrt[3]])/(
48*Sqrt[3]) + (13*d*ArcTan[(1 + 2*x)/Sqrt[3]])/(48*Sqrt[3]) + (2*
e*ArcTan[(1 + 2*x^2)/Sqrt[3]])/(3*Sqrt[3]) - (9*d*Log[1 - x + x^2
])/32 + (9*d*Log[1 + x + x^2])/32
```

Rubi in Sympy [A] time = 57.2801, size = 168, normalized size = 0.91

$$-\frac{9d\log\left(x^2-x+1\right)}{32}+\frac{9d\log\left(x^2+x+1\right)}{32}+\frac{13\sqrt{3}d\operatorname{atan}\left(\sqrt{3}\left(\frac{2x}{3}-\frac{1}{3}\right)\right)}{144}+\frac{13\sqrt{3}d\operatorname{atan}\left(\sqrt{3}\left(\frac{2x}{3}+\frac{1}{3}\right)\right)}{144}+\frac{2\sqrt{3}e\operatorname{atan}\left(\sqrt{3}\left(\frac{2x^2}{3}+\frac{1}{3}\right)\right)}{9}+\frac{x\left(-21dx^2+6d-18ex^3+6ex\right)}{72\left(x^4+x^2+1\right)}+\frac{x\left(-dx^2+d-ex^3+ex\right)}{12\left(x^4+x^2+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x+d)/(x**4+x**2+1)**3,x)
```

```
[Out] -9*d*log(x**2 - x + 1)/32 + 9*d*log(x**2 + x + 1)/32 + 13*sqrt(3)
*d*atan(sqrt(3)*(2*x/3 - 1/3))/144 + 13*sqrt(3)*d*atan(sqrt(3)*(2
*x/3 + 1/3))/144 + 2*sqrt(3)*e*atan(sqrt(3)*(2*x**2/3 + 1/3))/9 +
 x*(-21*d*x**2 + 6*d - 18*e*x**3 + 6*e*x)/(72*(x**4 + x**2 + 1))
+ x*(-d*x**2 + d - e*x**3 + e*x)/(12*(x**4 + x**2 + 1)**2)
```

Mathematica [C] time = 2.18715, size = 186, normalized size = 1.01

$$\frac{1}{144}\left(\frac{6\left(dx\left(2-7x^2\right)+e\left(8x^2+4\right)\right)}{x^4+x^2+1}+\frac{12\left(d\left(x-x^3\right)+2ex^2+e\right)}{\left(x^4+x^2+1\right)^2}-\frac{\left(7\sqrt{3}-47i\right)d\tan^{-1}\left(\frac{1}{2}\left(\sqrt{3}-i\right)x\right)}{\sqrt{\frac{1}{6}\left(1+i\sqrt{3}\right)}}-\frac{\left(7\sqrt{3}+47i\right)d\tan^{-1}\left(\frac{1}{2}\left(\sqrt{3}+i\right)x\right)}{\sqrt{\frac{1}{6}\left(1-i\sqrt{3}\right)}}-32\sqrt{3}e\tan^{-1}\left(\frac{\sqrt{3}}{2x^2+1}\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(d + e*x)/(1 + x^2 + x^4)^3,x]
```

```
[Out] ((6*(d*x*(2 - 7*x^2) + e*(4 + 8*x^2)))/(1 + x^2 + x^4) + (12*(e +
 2*e*x^2 + d*(x - x^3)))/(1 + x^2 + x^4)^2 - ((-47*I + 7*Sqrt[3])
*d*ArcTan[((-I + Sqrt[3])*x)/2])/Sqrt[(1 + I*Sqrt[3])/6] - ((47*I
 + 7*Sqrt[3])*d*ArcTan[((I + Sqrt[3])*x)/2])/Sqrt[(1 - I*Sqrt[3])
```

```
/6] - 32*Sqrt[3]*e*ArcTan[Sqrt[3]/(1 + 2*x^2)])/144
```

Maple [A] time = 0.023, size = 180, normalized size = 1.

$$\frac{1}{16\,(x^2+x+1)^2}\left(\left(-\frac{7\,d}{3}-\frac{4\,e}{3}\right)x^3-6\,dx^2+\left(-\frac{20\,d}{3}+\frac{e}{3}\right)x-4\,d+2\,e\right)+\frac{9\,d\ln\left(x^2+x+1\right)}{32}+\frac{13\,d\sqrt{3}}{144}\arctan\left(\frac{(1+2\,x)\sqrt{3}}{3}\right)-\frac{2\sqrt{3}e}{9}\arctan\left(\frac{(1+2\,x)\sqrt{3}}{3}\right)-\frac{1}{16\,(x^2-x+1)^2}\left(\left(\frac{7\,d}{3}-\frac{4\,e}{3}\right)x^3-6\,dx^2+\left(\frac{20\,d}{3}+\frac{e}{3}\right)x-4\,d-2\,e\right)-\frac{9\,d\ln\left(x^2-x+1\right)}{32}+\frac{13\,d\sqrt{3}}{144}\arctan\left(\frac{(2\,x-1)\sqrt{3}}{3}\right)+\frac{2\sqrt{3}e}{9}\arctan\left(\frac{(2\,x-1)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x+d)/(x^4+x^2+1)^3,x)
```

```
[Out] 1/16*((-7/3*d-4/3*e)*x^3-6*d*x^2+(-20/3*d+1/3*e)*x-4*d+2*e)/(x^2+
x+1)^2+9/32*d*ln(x^2+x+1)+13/144*d*arctan(1/3*(1+2*x)*3^(1/2))*3^
(1/2)-2/9*3^(1/2)*arctan(1/3*(1+2*x)*3^(1/2))*e-1/16*((7/3*d-4/3*
e)*x^3-6*d*x^2+(20/3*d+1/3*e)*x-4*d-2*e)/(x^2-x+1)^2-9/32*d*ln(x^
2-x+1)+13/144*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))*d+2/9*3^(1/2)*a
rctan(1/3*(2*x-1)*3^(1/2))*e
```

Maxima [A] time = 0.775462, size = 185, normalized size = 1.

$$\frac{1}{144}\sqrt{3}(13\,d-32\,e)\arctan\left(\frac{1}{3}\sqrt{3}(2\,x+1)\right)+\frac{1}{144}\sqrt{3}(13\,d+32\,e)\arctan\left(\frac{1}{3}\sqrt{3}(2\,x-1)\right)+\frac{9}{32}\,d\log\left(x^2+x+1\right)-\frac{9}{32}\,d\log\left(x^2-x+1\right)-\frac{7\,dx^7-8\,ex^6+5\,dx^5-12\,ex^4+7\,dx^3-16\,ex^2-4\,dx-6\,e}{24\,(x^8+2\,x^6+3\,x^4+2\,x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x + d)/(x^4 + x^2 + 1)^3,x, algorithm="maxima")
```

```
[Out] 1/144*sqrt(3)*(13*d - 32*e)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/144
*sqrt(3)*(13*d + 32*e)*arctan(1/3*sqrt(3)*(2*x - 1)) + 9/32*d*log
(x^2 + x + 1) - 9/32*d*log(x^2 - x + 1) - 1/24*(7*d*x^7 - 8*e*x^6
 + 5*d*x^5 - 12*e*x^4 + 7*d*x^3 - 16*e*x^2 - 4*d*x - 6*e)/(x^8 +
```

```
2*x^6 + 3*x^4 + 2*x^2 + 1)
```

Fricas [A] time = 0.277566, size = 387, normalized size = 2.09

$$\frac{\sqrt{3}\left(27\sqrt{3}\left(dx^8 + 2\,dx^6 + 3\,dx^4 + 2\,dx^2 + d\right)\log\left(x^2 + x + 1\right) - 27\sqrt{3}\left(dx^8 + 2\,dx^6 + 3\,dx^4 + 2\,dx^2 + d\right)\log\left(x^2 - x + 1\right) + 2\,(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x + d)/(x^4 + x^2 + 1)^3,x, algorithm="fricas")
```

```
[Out] 1/288*sqrt(3)*(27*sqrt(3)*(d*x^8 + 2*d*x^6 + 3*d*x^4 + 2*d*x^2 +
d)*log(x^2 + x + 1) - 27*sqrt(3)*(d*x^8 + 2*d*x^6 + 3*d*x^4 + 2*d
*x^2 + d)*log(x^2 - x + 1) + 2*((13*d - 32*e)*x^8 + 2*(13*d - 32*
e)*x^6 + 3*(13*d - 32*e)*x^4 + 2*(13*d - 32*e)*x^2 + 13*d - 32*e)
*arctan(1/3*sqrt(3)*(2*x + 1)) + 2*((13*d + 32*e)*x^8 + 2*(13*d +
 32*e)*x^6 + 3*(13*d + 32*e)*x^4 + 2*(13*d + 32*e)*x^2 + 13*d + 3
2*e)*arctan(1/3*sqrt(3)*(2*x - 1)) - 4*sqrt(3)*(7*d*x^7 - 8*e*x^6
 + 5*d*x^5 - 12*e*x^4 + 7*d*x^3 - 16*e*x^2 - 4*d*x - 6*e))/(x^8 +
 2*x^6 + 3*x^4 + 2*x^2 + 1)
```

Sympy [A] time = 9.10324, size = 1103, normalized size = 5.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x+d)/(x**4+x**2+1)**3,x)
```

```
[Out] (-9*d/32 - sqrt(3)*I*(13*d + 32*e)/288)*log(x + (-1025428432*d**4
*e - 334752912*d**4*(-9*d/32 - sqrt(3)*I*(13*d + 32*e)/288) - 431
308800*d**2*e**3 - 3143688192*d**2*e**2*(-9*d/32 - sqrt(3)*I*(13*
d + 32*e)/288) + 9917005824*d**2*e*(-9*d/32 - sqrt(3)*I*(13*d + 3
2*e)/288)**2 + 11878244352*d**2*(-9*d/32 - sqrt(3)*I*(13*d + 32*e
)/288)**3 + 142606336*e**5 + 754974720*e**4*(-9*d/32 - sqrt(3)*I*
(13*d + 32*e)/288) + 3850371072*e**3*(-9*d/32 - sqrt(3)*I*(13*d +
 32*e)/288)**2 + 20384317440*e**2*(-9*d/32 - sqrt(3)*I*(13*d + 32
*e)/288)**3)/(217696167*d**5 - 1217128448*d**3*e**2 - 617611264*d
*e**4)) + (-9*d/32 + sqrt(3)*I*(13*d + 32*e)/288)*log(x + (-10254
28432*d**4*e - 334752912*d**4*(-9*d/32 + sqrt(3)*I*(13*d + 32*e)/
288) - 431308800*d**2*e**3 - 3143688192*d**2*e**2*(-9*d/32 + sqrt
(3)*I*(13*d + 32*e)/288) + 9917005824*d**2*e*(-9*d/32 + sqrt(3)*I
*(13*d + 32*e)/288)**2 + 11878244352*d**2*(-9*d/32 + sqrt(3)*I*(1
3*d + 32*e)/288)**3 + 142606336*e**5 + 754974720*e**4*(-9*d/32 +
```

```
sqrt(3)*I*(13*d + 32*e)/288) + 3850371072*e**3*(-9*d/32 + sqrt(3)
*I*(13*d + 32*e)/288)**2 + 20384317440*e**2*(-9*d/32 + sqrt(3)*I*
(13*d + 32*e)/288)**3)/(217696167*d**5 - 1217128448*d**3*e**2 - 6
17611264*d*e**4)) + (9*d/32 - sqrt(3)*I*(13*d - 32*e)/288)*log(x
+ (-1025428432*d**4*e - 334752912*d**4*(9*d/32 - sqrt(3)*I*(13*d
- 32*e)/288) - 431308800*d**2*e**3 - 3143688192*d**2*e**2*(9*d/32
 - sqrt(3)*I*(13*d - 32*e)/288) + 9917005824*d**2*e*(9*d/32 - sqr
t(3)*I*(13*d - 32*e)/288)**2 + 11878244352*d**2*(9*d/32 - sqrt(3)
*I*(13*d - 32*e)/288)**3 + 142606336*e**5 + 754974720*e**4*(9*d/3
2 - sqrt(3)*I*(13*d - 32*e)/288) + 3850371072*e**3*(9*d/32 - sqrt
(3)*I*(13*d - 32*e)/288)**2 + 20384317440*e**2*(9*d/32 - sqrt(3)*
I*(13*d - 32*e)/288)**3)/(217696167*d**5 - 1217128448*d**3*e**2 -
 617611264*d*e**4)) + (9*d/32 + sqrt(3)*I*(13*d - 32*e)/288)*log(
x + (-1025428432*d**4*e - 334752912*d**4*(9*d/32 + sqrt(3)*I*(13*
d - 32*e)/288) - 431308800*d**2*e**3 - 3143688192*d**2*e**2*(9*d/
32 + sqrt(3)*I*(13*d - 32*e)/288) + 9917005824*d**2*e*(9*d/32 + s
qrt(3)*I*(13*d - 32*e)/288)**2 + 11878244352*d**2*(9*d/32 + sqrt(
3)*I*(13*d - 32*e)/288)**3 + 142606336*e**5 + 754974720*e**4*(9*d
/32 + sqrt(3)*I*(13*d - 32*e)/288) + 3850371072*e**3*(9*d/32 + sq
rt(3)*I*(13*d - 32*e)/288)**2 + 20384317440*e**2*(9*d/32 + sqrt(3
)*I*(13*d - 32*e)/288)**3)/(217696167*d**5 - 1217128448*d**3*e**2
 - 617611264*d*e**4)) - (7*d*x**7 + 5*d*x**5 + 7*d*x**3 - 4*d*x -
 8*e*x**6 - 12*e*x**4 - 16*e*x**2 - 6*e)/(24*x**8 + 48*x**6 + 72*
x**4 + 48*x**2 + 24)
```

GIAC/XCAS [A] time = 0.265535, size = 177, normalized size = 0.96

$$\frac{1}{144}\sqrt{3}(13\,d - 32\,e)\arctan\left(\frac{1}{3}\sqrt{3}(2\,x + 1)\right) + \frac{1}{144}\sqrt{3}(13\,d + 32\,e)\arctan\left(\frac{1}{3}\sqrt{3}(2\,x - 1)\right) + \frac{9}{32}\,d\ln\left(x^2 + x + 1\right) - \frac{9}{32}\,d\ln\left(x^2 - x + 1\right) - \frac{7\,dx^7 - 8\,x^6 e + 5\,dx^5 - 12\,x^4 e + 7\,dx^3 - 16\,x^2 e - 4\,dx - 6\,e}{24\left(x^4 + x^2 + 1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x + d)/(x^4 + x^2 + 1)^3,x, algorithm="giac")
```

```
[Out] 1/144*sqrt(3)*(13*d - 32*e)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/144
*sqrt(3)*(13*d + 32*e)*arctan(1/3*sqrt(3)*(2*x - 1)) + 9/32*d*ln(
x^2 + x + 1) - 9/32*d*ln(x^2 - x + 1) - 1/24*(7*d*x^7 - 8*x^6*e +
 5*d*x^5 - 12*x^4*e + 7*d*x^3 - 16*x^2*e - 4*d*x - 6*e)/(x^4 + x^
2 + 1)^2
```

3.48 $\int \frac{d+ex+fx^2}{(1+x^2+x^4)^3} \, dx$

Optimal. Leaf size=223

$$-\frac{1}{32}(9d-4f)\log\left(x^2-x+1\right)+\frac{1}{32}(9d-4f)\log\left(x^2+x+1\right)+\frac{x\left(-7x^2(d-f)+2d+3f\right)}{24\left(x^4+x^2+1\right)}+\frac{x\left(x^2(-(d-2f))+d+f\right)}{12\left(x^4+x^2+1\right)^2}-\frac{(13d+2f)\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{48\sqrt{3}}+\frac{(13d+2f)\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{48\sqrt{3}}+\frac{2e\tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{e\left(2x^2+1\right)}{6\left(x^4+x^2+1\right)}+\frac{e\left(2x^2+1\right)}{12\left(x^4+x^2+1\right)^2}$$

```
[Out] (e*(1 + 2*x^2))/(12*(1 + x^2 + x^4)^2) + (x*(d + f - (d - 2*f)*x^
2))/(12*(1 + x^2 + x^4)^2) + (e*(1 + 2*x^2))/(6*(1 + x^2 + x^4))
+ (x*(2*d + 3*f - 7*(d - f)*x^2))/(24*(1 + x^2 + x^4)) - ((13*d +
 2*f)*ArcTan[(1 - 2*x)/Sqrt[3]])/(48*Sqrt[3]) + ((13*d + 2*f)*Arc
Tan[(1 + 2*x)/Sqrt[3]])/(48*Sqrt[3]) + (2*e*ArcTan[(1 + 2*x^2)/Sq
rt[3]])/(3*Sqrt[3]) - ((9*d - 4*f)*Log[1 - x + x^2])/32 + ((9*d -
 4*f)*Log[1 + x + x^2])/32
```

Rubi [A] time = 0.467998, antiderivative size = 223, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476

$$-\frac{1}{32}(9d-4f)\log\left(x^2-x+1\right)+\frac{1}{32}(9d-4f)\log\left(x^2+x+1\right)+\frac{x\left(-7x^2(d-f)+2d+3f\right)}{24\left(x^4+x^2+1\right)}+\frac{x\left(x^2(-(d-2f))+d+f\right)}{12\left(x^4+x^2+1\right)^2}-\frac{(13d+2f)\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{48\sqrt{3}}+\frac{(13d+2f)\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{48\sqrt{3}}+\frac{2e\tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{e\left(2x^2+1\right)}{6\left(x^4+x^2+1\right)}+\frac{e\left(2x^2+1\right)}{12\left(x^4+x^2+1\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x + f*x^2)/(1 + x^2 + x^4)^3,x]
```


```
[Out] (e*(1 + 2*x^2))/(12*(1 + x^2 + x^4)^2) + (x*(d + f - (d - 2*f)*x^
2))/(12*(1 + x^2 + x^4)^2) + (e*(1 + 2*x^2))/(6*(1 + x^2 + x^4))
+ (x*(2*d + 3*f - 7*(d - f)*x^2))/(24*(1 + x^2 + x^4)) - ((13*d +
 2*f)*ArcTan[(1 - 2*x)/Sqrt[3]])/(48*Sqrt[3]) + ((13*d + 2*f)*Arc
Tan[(1 + 2*x)/Sqrt[3]])/(48*Sqrt[3]) + (2*e*ArcTan[(1 + 2*x^2)/Sq
rt[3]])/(3*Sqrt[3]) - ((9*d - 4*f)*Log[1 - x + x^2])/32 + ((9*d -
 4*f)*Log[1 + x + x^2])/32
```

Rubi in Sympy [A] time = 75.6237, size = 194, normalized size = 0.87

$$\frac{2\sqrt{3}e\,\mathrm{atan}\left(\sqrt{3}\left(\frac{2x^2}{3}+\frac{1}{3}\right)\right)}{9}+\frac{x\left(6d-18ex^3+6ex+9f-x^2\left(21d-21f\right)\right)}{72\left(x^4+x^2+1\right)}+\frac{x\left(d-ex^3+ex+f-x^2\left(d-2f\right)\right)}{12\left(x^4+x^2+1\right)^2}-\left(\frac{9d}{32}-\frac{f}{8}\right)\log\left(x^2-x+1\right)+\left(\frac{9d}{32}-\frac{f}{8}\right)\log\left(x^2+x+1\right)+\frac{\sqrt{3}\left(\frac{13d}{2}+f\right)\mathrm{atan}\left(\sqrt{3}\left(\frac{2x}{3}-\frac{1}{3}\right)\right)}{72}+\frac{\sqrt{3}\left(\frac{13d}{2}+f\right)\mathrm{atan}\left(\sqrt{3}\left(\frac{2x}{3}+\frac{1}{3}\right)\right)}{72}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**2+e*x+d)/(x**4+x**2+1)**3,x)
```

```
[Out] 2*sqrt(3)*e*atan(sqrt(3)*(2*x**2/3 + 1/3))/9 + x*(6*d - 18*e*x**3
 + 6*e*x + 9*f - x**2*(21*d - 21*f))/(72*(x**4 + x**2 + 1)) + x*(
d - e*x**3 + e*x + f - x**2*(d - 2*f))/(12*(x**4 + x**2 + 1)**2)
- (9*d/32 - f/8)*log(x**2 - x + 1) + (9*d/32 - f/8)*log(x**2 + x
+ 1) + sqrt(3)*(13*d/2 + f)*atan(sqrt(3)*(2*x/3 - 1/3))/72 + sqrt
(3)*(13*d/2 + f)*atan(sqrt(3)*(2*x/3 + 1/3))/72
```

Mathematica [C] time = 1.14654, size = 235, normalized size = 1.05

$$\frac{1}{144}\left(\frac{12\left(x\left(-dx^2+d+2fx^2+f\right)+2ex^2+e\right)}{\left(x^4+x^2+1\right)^2}+\frac{6\left(-7dx^3+2dx+e\left(8x^2+4\right)+7fx^3+3fx\right)}{x^4+x^2+1}-\frac{\left(\left(7\sqrt{3}-47i\right)d+\left(-7\sqrt{3}+17i\right)f\right)\tan^{-1}\left(\frac{1}{2}\left(\sqrt{3}-i\right)x\right)}{\sqrt{\frac{1}{6}\left(1+i\sqrt{3}\right)}}-\frac{\left(\left(7\sqrt{3}+47i\right)d-\left(7\sqrt{3}+17i\right)f\right)\tan^{-1}\left(\frac{1}{2}\left(\sqrt{3}+i\right)x\right)}{\sqrt{\frac{1}{6}\left(1-i\sqrt{3}\right)}}-32\sqrt{3}e\tan^{-1}\left(\frac{\sqrt{3}}{2x^2+1}\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(d + e*x + f*x^2)/(1 + x^2 + x^4)^3,x]
```

```
[Out] ((6*(2*d*x + 3*f*x - 7*d*x^3 + 7*f*x^3 + e*(4 + 8*x^2)))/(1 + x^2
 + x^4) + (12*(e + 2*e*x^2 + x*(d + f - d*x^2 + 2*f*x^2)))/(1 + x
^2 + x^4)^2 - (((-47*I + 7*Sqrt[3])*d + (17*I - 7*Sqrt[3])*f)*Arc
```

```
Tan[((-I + Sqrt[3])*x)/2])/Sqrt[(1 + I*Sqrt[3])/6] - (((47*I + 7*
Sqrt[3])*d - (17*I + 7*Sqrt[3])*f)*ArcTan[((I + Sqrt[3])*x)/2])/S
qrt[(1 - I*Sqrt[3])/6] - 32*Sqrt[3]*e*ArcTan[Sqrt[3]/(1 + 2*x^2)]
)/144
```

Maple [A] time = 0.022, size = 264, normalized size = 1.2

$$\frac{1}{16\left(x^2+x+1\right)^2}\left(\left(-\frac{7d}{3}+\frac{7f}{3}-\frac{4e}{3}\right)x^3+\left(-6d+4f\right)x^2+\left(-\frac{20d}{3}+\frac{13f}{3}+\frac{e}{3}\right)x-4d+\frac{4f}{3}+2e\right)$$
$$+\frac{9d\ln\left(x^2+x+1\right)}{32}-\frac{\ln\left(x^2+x+1\right)f}{8}+\frac{13d\sqrt{3}}{144}\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)$$
$$-\frac{2\sqrt{3}e}{9}\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)+\frac{\sqrt{3}f}{72}\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)$$
$$-\frac{1}{16\left(x^2-x+1\right)^2}\left(\left(\frac{7d}{3}-\frac{7f}{3}-\frac{4e}{3}\right)x^3+\left(-6d+4f\right)x^2+\left(\frac{20d}{3}-\frac{13f}{3}+\frac{e}{3}\right)x-4d+\frac{4f}{3}-2e\right)$$
$$-\frac{9d\ln\left(x^2-x+1\right)}{32}+\frac{\ln\left(x^2-x+1\right)f}{8}+\frac{13d\sqrt{3}}{144}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)$$
$$+\frac{2\sqrt{3}e}{9}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)+\frac{\sqrt{3}f}{72}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int((f*x^2+e*x+d)/(x^4+x^2+1)^3,x)
```

```
[Out] 1/16*((-7/3*d+7/3*f-4/3*e)*x^3+(-6*d+4*f)*x^2+(-20/3*d+13/3*f+1/3
*e)*x-4*d+4/3*f+2*e)/(x^2+x+1)^2+9/32*d*ln(x^2+x+1)-1/8*ln(x^2+x+
1)*f+13/144*d*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)-2/9*3^(1/2)*arc
tan(1/3*(1+2*x)*3^(1/2))*e+1/72*3^(1/2)*arctan(1/3*(1+2*x)*3^(1/2
))*f-1/16*((7/3*d-7/3*f-4/3*e)*x^3+(-6*d+4*f)*x^2+(20/3*d-13/3*f+
1/3*e)*x-4*d+4/3*f-2*e)/(x^2-x+1)^2-9/32*d*ln(x^2-x+1)+1/8*ln(x^2
-x+1)*f+13/144*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))*d+2/9*3^(1/2)*
arctan(1/3*(2*x-1)*3^(1/2))*e+1/72*3^(1/2)*arctan(1/3*(2*x-1)*3^(
1/2))*f
```

Maxima [A] time = 0.783577, size = 234, normalized size = 1.05

$$\frac{1}{144}\sqrt{3}(13\,d-32\,e+2\,f)\arctan\left(\frac{1}{3}\sqrt{3}(2\,x+1)\right)+\frac{1}{144}\sqrt{3}(13\,d+32\,e+2\,f)\arctan\left(\frac{1}{3}\sqrt{3}(2\,x-1)\right)+\frac{1}{32}(9\,d-4\,f)\log\left(x^2+x+1\right)-\frac{1}{32}(9\,d-4\,f)\log\left(x^2-x+1\right)-\frac{7\,(d-f)x^7-8\,ex^6+5\,(d-2\,f)x^5-12\,ex^4+7\,(d-2\,f)x^3-16\,ex^2-(4\,d+5\,f)x-6\,e}{24\,(x^8+2\,x^6+3\,x^4+2\,x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^2 + e*x + d)/(x^4 + x^2 + 1)^3,x, algorithm="maxima")
```

```
[Out] 1/144*sqrt(3)*(13*d - 32*e + 2*f)*arctan(1/3*sqrt(3)*(2*x + 1)) +
 1/144*sqrt(3)*(13*d + 32*e + 2*f)*arctan(1/3*sqrt(3)*(2*x - 1))
+ 1/32*(9*d - 4*f)*log(x^2 + x + 1) - 1/32*(9*d - 4*f)*log(x^2 -
x + 1) - 1/24*(7*(d - f)*x^7 - 8*e*x^6 + 5*(d - 2*f)*x^5 - 12*e*x
^4 + 7*(d - 2*f)*x^3 - 16*e*x^2 - (4*d + 5*f)*x - 6*e)/(x^8 + 2*x
^6 + 3*x^4 + 2*x^2 + 1)
```

Fricas [A] time = 0.316786, size = 531, normalized size = 2.38

$$\frac{\sqrt{3}\Big(3\sqrt{3}\big((9\,d-4\,f)x^8+2\,(9\,d-4\,f)x^6+3\,(9\,d-4\,f)x^4+2\,(9\,d-4\,f)x^2+9\,d-4\,f\big)\log\left(x^2+x+1\right)-3\sqrt{3}\big((9\,d-4\,f)x^{8}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^2 + e*x + d)/(x^4 + x^2 + 1)^3,x, algorithm="fricas")
```

```
[Out] 1/288*sqrt(3)*(3*sqrt(3)*((9*d - 4*f)*x^8 + 2*(9*d - 4*f)*x^6 + 3
*(9*d - 4*f)*x^4 + 2*(9*d - 4*f)*x^2 + 9*d - 4*f)*log(x^2 + x + 1
) - 3*sqrt(3)*((9*d - 4*f)*x^8 + 2*(9*d - 4*f)*x^6 + 3*(9*d - 4*f
)*x^4 + 2*(9*d - 4*f)*x^2 + 9*d - 4*f)*log(x^2 - x + 1) + 2*((13*
d - 32*e + 2*f)*x^8 + 2*(13*d - 32*e + 2*f)*x^6 + 3*(13*d - 32*e
+ 2*f)*x^4 + 2*(13*d - 32*e + 2*f)*x^2 + 13*d - 32*e + 2*f)*arcta
n(1/3*sqrt(3)*(2*x + 1)) + 2*((13*d + 32*e + 2*f)*x^8 + 2*(13*d +
 32*e + 2*f)*x^6 + 3*(13*d + 32*e + 2*f)*x^4 + 2*(13*d + 32*e + 2
*f)*x^2 + 13*d + 32*e + 2*f)*arctan(1/3*sqrt(3)*(2*x - 1)) - 4*sq
rt(3)*(7*(d - f)*x^7 - 8*e*x^6 + 5*(d - 2*f)*x^5 - 12*e*x^4 + 7*(
d - 2*f)*x^3 - 16*e*x^2 - (4*d + 5*f)*x - 6*e))/(x^8 + 2*x^6 + 3*
x^4 + 2*x^2 + 1)
```

Sympy [A] time = 106.84, size = 4498, normalized size = 20.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x**2+e*x+d)/(x**4+x**2+1)**3,x)
```

```
[Out] (-9*d/32 + f/8 - sqrt(3)*I*(13*d + 32*e + 2*f)/288)*log(x + (-102
5428432*d**5*e - 334752912*d**5*(-9*d/32 + f/8 - sqrt(3)*I*(13*d
+ 32*e + 2*f)/288) + 2008961360*d**4*e*f + 1151575920*d**4*f*(-9*
d/32 + f/8 - sqrt(3)*I*(13*d + 32*e + 2*f)/288) - 431308800*d**3*
e**3 - 3143688192*d**3*e**2*(-9*d/32 + f/8 - sqrt(3)*I*(13*d + 32
*e + 2*f)/288) - 1598857120*d**3*e*f**2 + 9917005824*d**3*e*(-9*d
/32 + f/8 - sqrt(3)*I*(13*d + 32*e + 2*f)/288)**2 - 944300160*d**
3*f**2*(-9*d/32 + f/8 - sqrt(3)*I*(13*d + 32*e + 2*f)/288) + 1187
8244352*d**3*(-9*d/32 + f/8 - sqrt(3)*I*(13*d + 32*e + 2*f)/288)*
*3 + 233164800*d**2*e**3*f + 4409634816*d**2*e**2*f*(-9*d/32 + f/
8 - sqrt(3)*I*(13*d + 32*e + 2*f)/288) + 662937520*d**2*e*f**3 -
13004623872*d**2*e*f*(-9*d/32 + f/8 - sqrt(3)*I*(13*d + 32*e + 2*
f)/288)**2 + 231796080*d**2*f**3*(-9*d/32 + f/8 - sqrt(3)*I*(13*d
 + 32*e + 2*f)/288) - 10089639936*d**2*f*(-9*d/32 + f/8 - sqrt(3)
*I*(13*d + 32*e + 2*f)/288)**3 + 142606336*d*e**5 + 754974720*d*e
**4*(-9*d/32 + f/8 - sqrt(3)*I*(13*d + 32*e + 2*f)/288) - 1843200
*d*e**3*f**2 + 3850371072*d*e**3*(-9*d/32 + f/8 - sqrt(3)*I*(13*d
 + 32*e + 2*f)/288)**2 - 1926291456*d*e**2*f**2*(-9*d/32 + f/8 -
sqrt(3)*I*(13*d + 32*e + 2*f)/288) + 20384317440*d*e**2*(-9*d/32
+ f/8 - sqrt(3)*I*(13*d + 32*e + 2*f)/288)**3 - 146756960*d*e*f**
4 + 5813379072*d*e*f**2*(-9*d/32 + f/8 - sqrt(3)*I*(13*d + 32*e +
 2*f)/288)**2 + 12679200*d*f**4*(-9*d/32 + f/8 - sqrt(3)*I*(13*d
+ 32*e + 2*f)/288) + 1116758016*d*f**2*(-9*d/32 + f/8 - sqrt(3)*I
*(13*d + 32*e + 2*f)/288)**3 - 79691776*e**5*f - 188743680*e**4*f
*(-9*d/32 + f/8 - sqrt(3)*I*(13*d + 32*e + 2*f)/288) - 7372800*e*
*3*f**3 - 2151677952*e**3*f*(-9*d/32 + f/8 - sqrt(3)*I*(13*d + 32
*e + 2*f)/288)**2 + 287096832*e**2*f**3*(-9*d/32 + f/8 - sqrt(3)*
I*(13*d + 32*e + 2*f)/288) - 5096079360*e**2*f*(-9*d/32 + f/8 - s
qrt(3)*I*(13*d + 32*e + 2*f)/288)**3 + 14093632*e*f**5 - 85952102
4*e*f**3*(-9*d/32 + f/8 - sqrt(3)*I*(13*d + 32*e + 2*f)/288)**2 -
 7648128*f**5*(-9*d/32 + f/8 - sqrt(3)*I*(13*d + 32*e + 2*f)/288)
 + 453869568*f**3*(-9*d/32 + f/8 - sqrt(3)*I*(13*d + 32*e + 2*f)/
288)**3)/(217696167*d**6 - 301346487*d**5*f - 1217128448*d**4*e**
2 + 130506255*d**4*f**2 + 2181281792*d**3*e**2*f - 5619240*d**3*f
**3 - 617611264*d**2*e**4 - 1450149888*d**2*e**2*f**2 - 8036820*d
**2*f**4 + 495976448*d*e**4*f + 430088192*d*e**2*f**3 + 783648*d*
f**5 - 114294784*e**4*f**2 - 47771648*e**2*f**4 + 188352*f**6)) +
 (-9*d/32 + f/8 + sqrt(3)*I*(13*d + 32*e + 2*f)/288)*log(x + (-10
25428432*d**5*e - 334752912*d**5*(-9*d/32 + f/8 + sqrt(3)*I*(13*d
 + 32*e + 2*f)/288) + 2008961360*d**4*e*f + 1151575920*d**4*f*(-9
*d/32 + f/8 + sqrt(3)*I*(13*d + 32*e + 2*f)/288) - 431308800*d**3
*e**3 - 3143688192*d**3*e**2*(-9*d/32 + f/8 + sqrt(3)*I*(13*d + 3
2*e + 2*f)/288) - 1598857120*d**3*e*f**2 + 9917005824*d**3*e*(-9*
d/32 + f/8 + sqrt(3)*I*(13*d + 32*e + 2*f)/288)**2 - 944300160*d*
```

```
*3*f**2*(-9*d/32 + f/8 + sqrt(3)*I*(13*d + 32*e + 2*f)/288) + 118
78244352*d**3*(-9*d/32 + f/8 + sqrt(3)*I*(13*d + 32*e + 2*f)/288)
**3 + 233164800*d**2*e**3*f + 4409634816*d**2*e**2*f*(-9*d/32 + f
/8 + sqrt(3)*I*(13*d + 32*e + 2*f)/288) + 662937520*d**2*e*f**3 -
 13004623872*d**2*e*f*(-9*d/32 + f/8 + sqrt(3)*I*(13*d + 32*e + 2
*f)/288)**2 + 231796080*d**2*f**3*(-9*d/32 + f/8 + sqrt(3)*I*(13*
d + 32*e + 2*f)/288) - 10089639936*d**2*f*(-9*d/32 + f/8 + sqrt(3
)*I*(13*d + 32*e + 2*f)/288)**3 + 142606336*d*e**5 + 754974720*d*
e**4*(-9*d/32 + f/8 + sqrt(3)*I*(13*d + 32*e + 2*f)/288) - 184320
0*d*e**3*f**2 + 3850371072*d*e**3*(-9*d/32 + f/8 + sqrt(3)*I*(13*
d + 32*e + 2*f)/288)**2 - 1926291456*d*e**2*f**2*(-9*d/32 + f/8 +
 sqrt(3)*I*(13*d + 32*e + 2*f)/288) + 20384317440*d*e**2*(-9*d/32
 + f/8 + sqrt(3)*I*(13*d + 32*e + 2*f)/288)**3 - 146756960*d*e*f*
*4 + 5813379072*d*e*f**2*(-9*d/32 + f/8 + sqrt(3)*I*(13*d + 32*e
+ 2*f)/288)**2 + 12679200*d*f**4*(-9*d/32 + f/8 + sqrt(3)*I*(13*d
 + 32*e + 2*f)/288) + 1116758016*d*f**2*(-9*d/32 + f/8 + sqrt(3)*
I*(13*d + 32*e + 2*f)/288)**3 - 79691776*e**5*f - 188743680*e**4*
f*(-9*d/32 + f/8 + sqrt(3)*I*(13*d + 32*e + 2*f)/288) - 7372800*e
**3*f**3 - 2151677952*e**3*f*(-9*d/32 + f/8 + sqrt(3)*I*(13*d + 3
2*e + 2*f)/288)**2 + 287096832*e**2*f**3*(-9*d/32 + f/8 + sqrt(3)
*I*(13*d + 32*e + 2*f)/288) - 5096079360*e**2*f*(-9*d/32 + f/8 +
sqrt(3)*I*(13*d + 32*e + 2*f)/288)**3 + 14093632*e*f**5 - 8595210
24*e*f**3*(-9*d/32 + f/8 + sqrt(3)*I*(13*d + 32*e + 2*f)/288)**2
- 7648128*f**5*(-9*d/32 + f/8 + sqrt(3)*I*(13*d + 32*e + 2*f)/288
) + 453869568*f**3*(-9*d/32 + f/8 + sqrt(3)*I*(13*d + 32*e + 2*f)
/288)**3)/(217696167*d**6 - 301346487*d**5*f - 1217128448*d**4*e*
*2 + 130506255*d**4*f**2 + 2181281792*d**3*e**2*f - 5619240*d**3*
f**3 - 617611264*d**2*e**4 - 1450149888*d**2*e**2*f**2 - 8036820*
d**2*f**4 + 495976448*d*e**4*f + 430088192*d*e**2*f**3 + 783648*d
*f**5 - 114294784*e**4*f**2 - 47771648*e**2*f**4 + 188352*f**6))
+ (9*d/32 - f/8 - sqrt(3)*I*(13*d - 32*e + 2*f)/288)*log(x + (-10
25428432*d**5*e - 334752912*d**5*(9*d/32 - f/8 - sqrt(3)*I*(13*d
- 32*e + 2*f)/288) + 2008961360*d**4*e*f + 1151575920*d**4*f*(9*d
/32 - f/8 - sqrt(3)*I*(13*d - 32*e + 2*f)/288) - 431308800*d**3*e
**3 - 3143688192*d**3*e**2*(9*d/32 - f/8 - sqrt(3)*I*(13*d - 32*e
 + 2*f)/288) - 1598857120*d**3*e*f**2 + 9917005824*d**3*e*(9*d/32
 - f/8 - sqrt(3)*I*(13*d - 32*e + 2*f)/288)**2 - 944300160*d**3*f
**2*(9*d/32 - f/8 - sqrt(3)*I*(13*d - 32*e + 2*f)/288) + 11878244
352*d**3*(9*d/32 - f/8 - sqrt(3)*I*(13*d - 32*e + 2*f)/288)**3 +
233164800*d**2*e**3*f + 4409634816*d**2*e**2*f*(9*d/32 - f/8 - sq
rt(3)*I*(13*d - 32*e + 2*f)/288) + 662937520*d**2*e*f**3 - 130046
23872*d**2*e*f*(9*d/32 - f/8 - sqrt(3)*I*(13*d - 32*e + 2*f)/288)
**2 + 231796080*d**2*f**3*(9*d/32 - f/8 - sqrt(3)*I*(13*d - 32*e
+ 2*f)/288) - 10089639936*d**2*f*(9*d/32 - f/8 - sqrt(3)*I*(13*d
- 32*e + 2*f)/288)**3 + 142606336*d*e**5 + 754974720*d*e**4*(9*d/
32 - f/8 - sqrt(3)*I*(13*d - 32*e + 2*f)/288) - 1843200*d*e**3*f*
*2 + 3850371072*d*e**3*(9*d/32 - f/8 - sqrt(3)*I*(13*d - 32*e + 2
*f)/288)**2 - 1926291456*d*e**2*f**2*(9*d/32 - f/8 - sqrt(3)*I*(1
3*d - 32*e + 2*f)/288) + 20384317440*d*e**2*(9*d/32 - f/8 - sqrt(
3)*I*(13*d - 32*e + 2*f)/288)**3 - 146756960*d*e*f**4 + 581337907
2*d*e*f**2*(9*d/32 - f/8 - sqrt(3)*I*(13*d - 32*e + 2*f)/288)**2
+ 12679200*d*f**4*(9*d/32 - f/8 - sqrt(3)*I*(13*d - 32*e + 2*f)/2
88) + 1116758016*d*f**2*(9*d/32 - f/8 - sqrt(3)*I*(13*d - 32*e +
2*f)/288)**3 - 79691776*e**5*f - 188743680*e**4*f*(9*d/32 - f/8 -
```

```
 sqrt(3)*I*(13*d - 32*e + 2*f)/288) - 7372800*e**3*f**3 - 2151677
952*e**3*f*(9*d/32 - f/8 - sqrt(3)*I*(13*d - 32*e + 2*f)/288)**2
+ 287096832*e**2*f**3*(9*d/32 - f/8 - sqrt(3)*I*(13*d - 32*e + 2*
f)/288) - 5096079360*e**2*f*(9*d/32 - f/8 - sqrt(3)*I*(13*d - 32*
e + 2*f)/288)**3 + 14093632*e*f**5 - 859521024*e*f**3*(9*d/32 - f
/8 - sqrt(3)*I*(13*d - 32*e + 2*f)/288)**2 - 7648128*f**5*(9*d/32
 - f/8 - sqrt(3)*I*(13*d - 32*e + 2*f)/288) + 453869568*f**3*(9*d
/32 - f/8 - sqrt(3)*I*(13*d - 32*e + 2*f)/288)**3)/(217696167*d**
6 - 301346487*d**5*f - 1217128448*d**4*e**2 + 130506255*d**4*f**2
 + 2181281792*d**3*e**2*f - 5619240*d**3*f**3 - 617611264*d**2*e*
*4 - 1450149888*d**2*e**2*f**2 - 8036820*d**2*f**4 + 495976448*d*
e**4*f + 430088192*d*e**2*f**3 + 783648*d*f**5 - 114294784*e**4*f
**2 - 47771648*e**2*f**4 + 188352*f**6)) + (9*d/32 - f/8 + sqrt(3
)*I*(13*d - 32*e + 2*f)/288)*log(x + (-1025428432*d**5*e - 334752
912*d**5*(9*d/32 - f/8 + sqrt(3)*I*(13*d - 32*e + 2*f)/288) + 200
8961360*d**4*e*f + 1151575920*d**4*f*(9*d/32 - f/8 + sqrt(3)*I*(1
3*d - 32*e + 2*f)/288) - 431308800*d**3*e**3 - 3143688192*d**3*e*
*2*(9*d/32 - f/8 + sqrt(3)*I*(13*d - 32*e + 2*f)/288) - 159885712
0*d**3*e*f**2 + 9917005824*d**3*e*(9*d/32 - f/8 + sqrt(3)*I*(13*d
 - 32*e + 2*f)/288)**2 - 944300160*d**3*f**2*(9*d/32 - f/8 + sqrt
(3)*I*(13*d - 32*e + 2*f)/288) + 11878244352*d**3*(9*d/32 - f/8 +
 sqrt(3)*I*(13*d - 32*e + 2*f)/288)**3 + 233164800*d**2*e**3*f +
4409634816*d**2*e**2*f*(9*d/32 - f/8 + sqrt(3)*I*(13*d - 32*e + 2
*f)/288) + 662937520*d**2*e*f**3 - 13004623872*d**2*e*f*(9*d/32 -
 f/8 + sqrt(3)*I*(13*d - 32*e + 2*f)/288)**2 + 231796080*d**2*f**
3*(9*d/32 - f/8 + sqrt(3)*I*(13*d - 32*e + 2*f)/288) - 1008963993
6*d**2*f*(9*d/32 - f/8 + sqrt(3)*I*(13*d - 32*e + 2*f)/288)**3 +
142606336*d*e**5 + 754974720*d*e**4*(9*d/32 - f/8 + sqrt(3)*I*(13
*d - 32*e + 2*f)/288) - 1843200*d*e**3*f**2 + 3850371072*d*e**3*(
9*d/32 - f/8 + sqrt(3)*I*(13*d - 32*e + 2*f)/288)**2 - 1926291456
*d*e**2*f**2*(9*d/32 - f/8 + sqrt(3)*I*(13*d - 32*e + 2*f)/288) +
 20384317440*d*e**2*(9*d/32 - f/8 + sqrt(3)*I*(13*d - 32*e + 2*f)
/288)**3 - 146756960*d*e*f**4 + 5813379072*d*e*f**2*(9*d/32 - f/8
 + sqrt(3)*I*(13*d - 32*e + 2*f)/288)**2 + 12679200*d*f**4*(9*d/3
2 - f/8 + sqrt(3)*I*(13*d - 32*e + 2*f)/288) + 1116758016*d*f**2*
(9*d/32 - f/8 + sqrt(3)*I*(13*d - 32*e + 2*f)/288)**3 - 79691776*
e**5*f - 188743680*e**4*f*(9*d/32 - f/8 + sqrt(3)*I*(13*d - 32*e
+ 2*f)/288) - 7372800*e**3*f**3 - 2151677952*e**3*f*(9*d/32 - f/8
 + sqrt(3)*I*(13*d - 32*e + 2*f)/288)**2 + 287096832*e**2*f**3*(9
*d/32 - f/8 + sqrt(3)*I*(13*d - 32*e + 2*f)/288) - 5096079360*e**
2*f*(9*d/32 - f/8 + sqrt(3)*I*(13*d - 32*e + 2*f)/288)**3 + 14093
632*e*f**5 - 859521024*e*f**3*(9*d/32 - f/8 + sqrt(3)*I*(13*d - 3
2*e + 2*f)/288)**2 - 7648128*f**5*(9*d/32 - f/8 + sqrt(3)*I*(13*d
 - 32*e + 2*f)/288) + 453869568*f**3*(9*d/32 - f/8 + sqrt(3)*I*(1
3*d - 32*e + 2*f)/288)**3)/(217696167*d**6 - 301346487*d**5*f - 1
217128448*d**4*e**2 + 130506255*d**4*f**2 + 2181281792*d**3*e**2*
f - 5619240*d**3*f**3 - 617611264*d**2*e**4 - 1450149888*d**2*e**
2*f**2 - 8036820*d**2*f**4 + 495976448*d*e**4*f + 430088192*d*e**
2*f**3 + 783648*d*f**5 - 114294784*e**4*f**2 - 47771648*e**2*f**4
 + 188352*f**6)) - (-8*e*x**6 - 12*e*x**4 - 16*e*x**2 - 6*e + x**
7*(7*d - 7*f) + x**5*(5*d - 10*f) + x**3*(7*d - 14*f) + x*(-4*d -
 5*f))/(24*x**8 + 48*x**6 + 72*x**4 + 48*x**2 + 24)
```

GIAC/XCAS [A] time = 0.265839, size = 231, normalized size = 1.04

$$\frac{1}{144}\sqrt{3}(13\,d+2\,f-32\,e)\arctan\left(\frac{1}{3}\sqrt{3}(2\,x+1)\right)+\frac{1}{144}\sqrt{3}(13\,d+2\,f+32\,e)\arctan\left(\frac{1}{3}\sqrt{3}(2\,x-1)\right)+\frac{1}{32}(9\,d-4\,f)\ln\left(x^2+x+1\right)-\frac{1}{32}(9\,d-4\,f)\ln\left(x^2-x+1\right)-\frac{7\,dx^7-7\,fx^7-8\,x^6e+5\,dx^5-10\,fx^5-12\,x^4e+7\,dx^3-14\,fx^3-16\,x^2e-4\,dx-5\,fx-6\,e}{24\left(x^4+x^2+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^2 + e*x + d)/(x^4 + x^2 + 1)^3,x, algorithm="giac")
```

```
[Out] 1/144*sqrt(3)*(13*d + 2*f - 32*e)*arctan(1/3*sqrt(3)*(2*x + 1)) +
 1/144*sqrt(3)*(13*d + 2*f + 32*e)*arctan(1/3*sqrt(3)*(2*x - 1))
+ 1/32*(9*d - 4*f)*ln(x^2 + x + 1) - 1/32*(9*d - 4*f)*ln(x^2 - x
+ 1) - 1/24*(7*d*x^7 - 7*f*x^7 - 8*x^6*e + 5*d*x^5 - 10*f*x^5 - 1
2*x^4*e + 7*d*x^3 - 14*f*x^3 - 16*x^2*e - 4*d*x - 5*f*x - 6*e)/(x
^4 + x^2 + 1)^2
```

3.49 $\int \frac{d+ex+fx^2+gx^3}{(1+x^2+x^4)^3}\,dx$

Optimal. Leaf size=243

$$-\frac{1}{32}(9d-4f)\log\left(x^2-x+1\right)+\frac{1}{32}(9d-4f)\log\left(x^2+x+1\right)+\frac{x\left(-7x^2(d-f)+2d+3f\right)}{24\left(x^4+x^2+1\right)}+\frac{x\left(x^2(-(d-2f))+d+f\right)}{12\left(x^4+x^2+1\right)^2}-\frac{(13d+2f)\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{48\sqrt{3}}+\frac{(13d+2f)\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{48\sqrt{3}}+\frac{(2e-g)\tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{\left(2x^2+1\right)(2e-g)}{12\left(x^4+x^2+1\right)}+\frac{x^2(2e-g)+e-2g}{12\left(x^4+x^2+1\right)^2}$$

```
[Out] (x*(d + f - (d - 2*f)*x^2))/(12*(1 + x^2 + x^4)^2) + (e - 2*g + (
2*e - g)*x^2)/(12*(1 + x^2 + x^4)^2) + ((2*e - g)*(1 + 2*x^2))/(1
2*(1 + x^2 + x^4)) + (x*(2*d + 3*f - 7*(d - f)*x^2))/(24*(1 + x^2
 + x^4)) - ((13*d + 2*f)*ArcTan[(1 - 2*x)/Sqrt[3]])/(48*Sqrt[3])
+ ((13*d + 2*f)*ArcTan[(1 + 2*x)/Sqrt[3]])/(48*Sqrt[3]) + ((2*e -
 g)*ArcTan[(1 + 2*x^2)/Sqrt[3]])/(3*Sqrt[3]) - ((9*d - 4*f)*Log[1
 - x + x^2])/32 + ((9*d - 4*f)*Log[1 + x + x^2])/32
```

Rubi [A] time = 0.52391, antiderivative size = 243, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 10, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385

$$-\frac{1}{32}(9d-4f)\log\left(x^2-x+1\right)+\frac{1}{32}(9d-4f)\log\left(x^2+x+1\right)+\frac{x\left(-7x^2(d-f)+2d+3f\right)}{24\left(x^4+x^2+1\right)}+\frac{x\left(x^2(-(d-2f))+d+f\right)}{12\left(x^4+x^2+1\right)^2}-\frac{(13d+2f)\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{48\sqrt{3}}+\frac{(13d+2f)\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{48\sqrt{3}}+\frac{(2e-g)\tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{\left(2x^2+1\right)(2e-g)}{12\left(x^4+x^2+1\right)}+\frac{x^2(2e-g)+e-2g}{12\left(x^4+x^2+1\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x + f*x^2 + g*x^3)/(1 + x^2 + x^4)^3,x]
```


```
[Out] (x*(d + f - (d - 2*f)*x^2))/(12*(1 + x^2 + x^4)^2) + (e - 2*g + (
2*e - g)*x^2)/(12*(1 + x^2 + x^4)^2) + ((2*e - g)*(1 + 2*x^2))/(1
2*(1 + x^2 + x^4)) + (x*(2*d + 3*f - 7*(d - f)*x^2))/(24*(1 + x^2
 + x^4)) - ((13*d + 2*f)*ArcTan[(1 - 2*x)/Sqrt[3]])/(48*Sqrt[3])
+ ((13*d + 2*f)*ArcTan[(1 + 2*x)/Sqrt[3]])/(48*Sqrt[3]) + ((2*e -
 g)*ArcTan[(1 + 2*x^2)/Sqrt[3]])/(3*Sqrt[3]) - ((9*d - 4*f)*Log[1
 - x + x^2])/32 + ((9*d - 4*f)*Log[1 + x + x^2])/32
```

Rubi in Sympy [A] time = 83.973, size = 207, normalized size = 0.85

$$\frac{x\left(6d+9f-x^{3}\left(18e-18g\right)-x^{2}\left(21d-21f\right)+x\left(6e+6g\right)\right)}{72\left(x^{4}+x^{2}+1\right)}+\frac{x\left(d+f-x^{3}\left(e-2g\right)-x^{2}\left(d-2f\right)+x\left(e+g\right)\right)}{12\left(x^{4}+x^{2}+1\right)^{2}}-\left(\frac{9d}{32}-\frac{f}{8}\right)\log\left(x^{2}-x+1\right)+\left(\frac{9d}{32}-\frac{f}{8}\right)\log\left(x^{2}+x+1\right)+\frac{\sqrt{3}\left(\frac{13d}{2}+f\right)\operatorname{atan}\left(\sqrt{3}\left(\frac{2x}{3}-\frac{1}{3}\right)\right)}{72}+\frac{\sqrt{3}\left(\frac{13d}{2}+f\right)\operatorname{atan}\left(\sqrt{3}\left(\frac{2x}{3}+\frac{1}{3}\right)\right)}{72}+\frac{\sqrt{3}\left(2e-g\right)\operatorname{atan}\left(\sqrt{3}\left(\frac{2x^{2}}{3}+\frac{1}{3}\right)\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((g*x**3+f*x**2+e*x+d)/(x**4+x**2+1)**3,x)
```

```
[Out] x*(6*d + 9*f - x**3*(18*e - 18*g) - x**2*(21*d - 21*f) + x*(6*e +
 6*g))/(72*(x**4 + x**2 + 1)) + x*(d + f - x**3*(e - 2*g) - x**2*
(d - 2*f) + x*(e + g))/(12*(x**4 + x**2 + 1)**2) - (9*d/32 - f/8)
*log(x**2 - x + 1) + (9*d/32 - f/8)*log(x**2 + x + 1) + sqrt(3)*(
13*d/2 + f)*atan(sqrt(3)*(2*x/3 - 1/3))/72 + sqrt(3)*(13*d/2 + f)
*atan(sqrt(3)*(2*x/3 + 1/3))/72 + sqrt(3)*(2*e - g)*atan(sqrt(3)*
(2*x**2/3 + 1/3))/9
```

Mathematica [C] time = 1.31532, size = 259, normalized size = 1.07

$$\frac{1}{144}\left(\frac{12\left(x\left(-dx^{2}+d+2fx^{2}+f\right)+2ex^{2}+e-g\left(x^{2}+2\right)\right)}{\left(x^{4}+x^{2}+1\right)^{2}}+\frac{6\left(-7dx^{3}+2dx+e\left(8x^{2}+4\right)+7fx^{3}+3fx-2g\left(2x^{2}+1\right)\right)}{x^{4}+x^{2}+1}-\frac{\left(\left(7\sqrt{3}-47i\right)d+\left(-7\sqrt{3}+17i\right)f\right)\tan^{-1}\left(\frac{1}{2}\left(\sqrt{3}-i\right)x\right)}{\sqrt{\frac{1}{6}\left(1+i\sqrt{3}\right)}}-\frac{\left(\left(7\sqrt{3}+47i\right)d-\left(7\sqrt{3}+17i\right)f\right)\tan^{-1}\left(\frac{1}{2}\left(\sqrt{3}+i\right)x\right)}{\sqrt{\frac{1}{6}\left(1-i\sqrt{3}\right)}}-16\sqrt{3}(2e-g)\tan^{-1}\left(\frac{\sqrt{3}}{2x^{2}+1}\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(d + e*x + f*x^2 + g*x^3)/(1 + x^2 + x^4)^3,x]
```

```
[Out] ((6*(2*d*x + 3*f*x - 7*d*x^3 + 7*f*x^3 - 2*g*(1 + 2*x^2) + e*(4 +
 8*x^2)))/(1 + x^2 + x^4) + (12*(e + 2*e*x^2 - g*(2 + x^2) + x*(d
 + f - d*x^2 + 2*f*x^2)))/(1 + x^2 + x^4)^2 - (((-47*I + 7*Sqrt[3
])*d + (17*I - 7*Sqrt[3])*f)*ArcTan[((-I + Sqrt[3])*x)/2])/Sqrt[(
1 + I*Sqrt[3])/6] - (((47*I + 7*Sqrt[3])*d - (17*I + 7*Sqrt[3])*f
)*ArcTan[((I + Sqrt[3])*x)/2])/Sqrt[(1 - I*Sqrt[3])/6] - 16*Sqrt[
3]*(2*e - g)*ArcTan[Sqrt[3]/(1 + 2*x^2)])/144
```

Maple [A] time = 0.023, size = 322, normalized size = 1.3

$$\frac{1}{16\,(x^2+x+1)^2}\left(\left(-\frac{7\,d}{3}+\frac{7\,f}{3}-\frac{4\,e}{3}-\frac{g}{3}\right)x^3+(-6\,d+4\,f-2\,g)\,x^2+\left(-\frac{20\,d}{3}+\frac{13\,f}{3}+\frac{e}{3}-\frac{8\,g}{3}\right)x-4\,d+\frac{4\,f}{3}+2\,e-2\,g\right)$$
$$+\frac{9\,d\ln\left(x^2+x+1\right)}{32}-\frac{\ln\left(x^2+x+1\right)f}{8}+\frac{13\,d\sqrt{3}}{144}\arctan\left(\frac{(1+2\,x)\sqrt{3}}{3}\right)$$
$$-\frac{2\sqrt{3}e}{9}\arctan\left(\frac{(1+2\,x)\sqrt{3}}{3}\right)+\frac{\sqrt{3}f}{72}\arctan\left(\frac{(1+2\,x)\sqrt{3}}{3}\right)+\frac{\sqrt{3}g}{9}\arctan\left(\frac{(1+2\,x)\sqrt{3}}{3}\right)$$
$$-\frac{1}{16\,(x^2-x+1)^2}\left(\left(\frac{7\,d}{3}-\frac{7\,f}{3}-\frac{4\,e}{3}-\frac{g}{3}\right)x^3+(-6\,d+4\,f+2\,g)\,x^2+\left(\frac{20\,d}{3}-\frac{13\,f}{3}+\frac{e}{3}-\frac{8\,g}{3}\right)x-4\,d+\frac{4\,f}{3}-2\,e+2\,g\right)$$
$$-\frac{9\,d\ln\left(x^2-x+1\right)}{32}+\frac{\ln\left(x^2-x+1\right)f}{8}+\frac{13\,d\sqrt{3}}{144}\arctan\left(\frac{(2\,x-1)\sqrt{3}}{3}\right)$$
$$+\frac{2\sqrt{3}e}{9}\arctan\left(\frac{(2\,x-1)\sqrt{3}}{3}\right)+\frac{\sqrt{3}f}{72}\arctan\left(\frac{(2\,x-1)\sqrt{3}}{3}\right)-\frac{\sqrt{3}g}{9}\arctan\left(\frac{(2\,x-1)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((g*x^3+f*x^2+e*x+d)/(x^4+x^2+1)^3,x)
```

```
[Out] 1/16*((-7/3*d+7/3*f-4/3*e-1/3*g)*x^3+(-6*d+4*f-2*g)*x^2+(-20/3*d+
13/3*f+1/3*e-8/3*g)*x-4*d+4/3*f+2*e-2*g)/(x^2+x+1)^2+9/32*d*ln(x^
2+x+1)-1/8*ln(x^2+x+1)*f+13/144*d*arctan(1/3*(1+2*x)*3^(1/2))*3^(
1/2)-2/9*3^(1/2)*arctan(1/3*(1+2*x)*3^(1/2))*e+1/72*3^(1/2)*arcta
n(1/3*(1+2*x)*3^(1/2))*f+1/9*3^(1/2)*arctan(1/3*(1+2*x)*3^(1/2))*
g-1/16*((7/3*d-7/3*f-4/3*e-1/3*g)*x^3+(-6*d+4*f+2*g)*x^2+(20/3*d-
13/3*f+1/3*e-8/3*g)*x-4*d+4/3*f-2*e+2*g)/(x^2-x+1)^2-9/32*d*ln(x^
2-x+1)+1/8*ln(x^2-x+1)*f+13/144*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2
))*d+2/9*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))*e+1/72*3^(1/2)*arcta
n(1/3*(2*x-1)*3^(1/2))*f-1/9*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))*
g
```

Maxima [A] time = 0.785153, size = 270, normalized size = 1.11

$$\frac{1}{144}\sqrt{3}(13d - 32e + 2f + 16g)\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) + \frac{1}{144}\sqrt{3}(13d + 32e + 2f - 16g)\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) + \frac{1}{32}(9d - 4f)\log\left(x^2 + x + 1\right) - \frac{1}{32}(9d - 4f)\log\left(x^2 - x + 1\right) - \frac{7(d-f)x^7 - 4(2e-g)x^6 + 5(d-2f)x^5 - 6(2e-g)x^4 + 7(d-2f)x^3 - 8(2e-g)x^2 - (4d+5f)x - 6e + 6g}{24(x^8 + 2x^6 + 3x^4 + 2x^2 + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3 + f*x^2 + e*x + d)/(x^4 + x^2 + 1)^3,x, algorithm="maxima")
```

```
[Out] 1/144*sqrt(3)*(13*d - 32*e + 2*f + 16*g)*arctan(1/3*sqrt(3)*(2*x
+ 1)) + 1/144*sqrt(3)*(13*d + 32*e + 2*f - 16*g)*arctan(1/3*sqrt(
3)*(2*x - 1)) + 1/32*(9*d - 4*f)*log(x^2 + x + 1) - 1/32*(9*d - 4
*f)*log(x^2 - x + 1) - 1/24*(7*(d - f)*x^7 - 4*(2*e - g)*x^6 + 5*
(d - 2*f)*x^5 - 6*(2*e - g)*x^4 + 7*(d - 2*f)*x^3 - 8*(2*e - g)*x
^2 - (4*d + 5*f)*x - 6*e + 6*g)/(x^8 + 2*x^6 + 3*x^4 + 2*x^2 + 1)
```

Fricas [A] time = 0.503587, size = 599, normalized size = 2.47

$$\sqrt{3}\Big(3\sqrt{3}\big((9d - 4f)x^8 + 2(9d - 4f)x^6 + 3(9d - 4f)x^4 + 2(9d - 4f)x^2 + 9d - 4f\big)\log\left(x^2 + x + 1\right) - 3\sqrt{3}\big((9d - 4f)x^{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3 + f*x^2 + e*x + d)/(x^4 + x^2 + 1)^3,x, algorithm="fricas")
```

```
[Out] 1/288*sqrt(3)*(3*sqrt(3)*((9*d - 4*f)*x^8 + 2*(9*d - 4*f)*x^6 + 3
*(9*d - 4*f)*x^4 + 2*(9*d - 4*f)*x^2 + 9*d - 4*f)*log(x^2 + x + 1
) - 3*sqrt(3)*((9*d - 4*f)*x^8 + 2*(9*d - 4*f)*x^6 + 3*(9*d - 4*f
)*x^4 + 2*(9*d - 4*f)*x^2 + 9*d - 4*f)*log(x^2 - x + 1) + 2*((13*
d - 32*e + 2*f + 16*g)*x^8 + 2*(13*d - 32*e + 2*f + 16*g)*x^6 + 3
*(13*d - 32*e + 2*f + 16*g)*x^4 + 2*(13*d - 32*e + 2*f + 16*g)*x^
2 + 13*d - 32*e + 2*f + 16*g)*arctan(1/3*sqrt(3)*(2*x + 1)) + 2*(
(13*d + 32*e + 2*f - 16*g)*x^8 + 2*(13*d + 32*e + 2*f - 16*g)*x^6
 + 3*(13*d + 32*e + 2*f - 16*g)*x^4 + 2*(13*d + 32*e + 2*f - 16*g
)*x^2 + 13*d + 32*e + 2*f - 16*g)*arctan(1/3*sqrt(3)*(2*x - 1)) -
 4*sqrt(3)*(7*(d - f)*x^7 - 4*(2*e - g)*x^6 + 5*(d - 2*f)*x^5 - 6
*(2*e - g)*x^4 + 7*(d - 2*f)*x^3 - 8*(2*e - g)*x^2 - (4*d + 5*f)*
x - 6*e + 6*g))/(x^8 + 2*x^6 + 3*x^4 + 2*x^2 + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((g*x**3+f*x**2+e*x+d)/(x**4+x**2+1)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.273037, size = 267, normalized size = 1.1

$$\frac{1}{144}\sqrt{3}(13\,d + 2\,f + 16\,g - 32\,e)\arctan\left(\frac{1}{3}\sqrt{3}(2\,x + 1)\right) + \frac{1}{144}\sqrt{3}(13\,d + 2\,f - 16\,g + 32\,e)\arctan\left(\frac{1}{3}\sqrt{3}(2\,x - 1)\right) + \frac{1}{32}(9\,d - 4\,f)\ln\left(x^2 + x + 1\right) - \frac{1}{32}(9\,d - 4\,f)\ln\left(x^2 - x + 1\right) - \frac{7\,dx^7 - 7\,fx^7 + 4\,gx^6 - 8\,x^6e + 5\,dx^5 - 10\,fx^5 + 6\,gx^4 - 12\,x^4e + 7\,dx^3 - 14\,fx^3 + 8\,gx^2 - 16\,x^2e - 4\,dx - 5\,fx + 6\,g - 6}{24\,(x^4 + x^2 + 1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((g*x^3 + f*x^2 + e*x + d)/(x^4 + x^2 + 1)^3,x, algorithm="giac")
```

```
[Out] 1/144*sqrt(3)*(13*d + 2*f + 16*g - 32*e)*arctan(1/3*sqrt(3)*(2*x
+ 1)) + 1/144*sqrt(3)*(13*d + 2*f - 16*g + 32*e)*arctan(1/3*sqrt(
3)*(2*x - 1)) + 1/32*(9*d - 4*f)*ln(x^2 + x + 1) - 1/32*(9*d - 4*
f)*ln(x^2 - x + 1) - 1/24*(7*d*x^7 - 7*f*x^7 + 4*g*x^6 - 8*x^6*e
+ 5*d*x^5 - 10*f*x^5 + 6*g*x^4 - 12*x^4*e + 7*d*x^3 - 14*f*x^3 +
8*g*x^2 - 16*x^2*e - 4*d*x - 5*f*x + 6*g - 6*e)/(x^4 + x^2 + 1)^2
```

3.50 $\int \frac{d+ex+fx^2+gx^3+hx^4}{(1+x^2+x^4)^3} \, dx$

Optimal. Leaf size=263

$$-\frac{1}{32}\log\left(x^2-x+1\right)(9d-4f+3h)+\frac{1}{32}\log\left(x^2+x+1\right)(9d-4f+3h)+\frac{x\left(x^2(-(7d-7f+4h))+2d+3f-h\right)}{24\left(x^4+x^2+1\right)}+\frac{x\left(x^2(-(d-2f+h))+d+f-2h\right)}{12\left(x^4+x^2+1\right)^2}-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)(13d+2f+h)}{48\sqrt{3}}+\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)(13d+2f+h)}{48\sqrt{3}}+\frac{(2e-g)\tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{\left(2x^2+1\right)(2e-g)}{12\left(x^4+x^2+1\right)}+\frac{x^2(2e-g)+e-2g}{12\left(x^4+x^2+1\right)^2}$$

```
[Out] (e - 2*g + (2*e - g)*x^2)/(12*(1 + x^2 + x^4)^2) + (x*(d + f - 2*
h - (d - 2*f + h)*x^2))/(12*(1 + x^2 + x^4)^2) + ((2*e - g)*(1 +
2*x^2))/(12*(1 + x^2 + x^4)) + (x*(2*d + 3*f - h - (7*d - 7*f + 4
*h)*x^2))/(24*(1 + x^2 + x^4)) - ((13*d + 2*f + h)*ArcTan[(1 - 2*
x)/Sqrt[3]])/(48*Sqrt[3]) + ((13*d + 2*f + h)*ArcTan[(1 + 2*x)/Sq
rt[3]])/(48*Sqrt[3]) + ((2*e - g)*ArcTan[(1 + 2*x^2)/Sqrt[3]])/(3
*Sqrt[3]) - ((9*d - 4*f + 3*h)*Log[1 - x + x^2])/32 + ((9*d - 4*f
 + 3*h)*Log[1 + x + x^2])/32
```

Rubi [A] time = 0.614234, antiderivative size = 263, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 11, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.355

$$-\frac{1}{32}\log\left(x^2-x+1\right)(9d-4f+3h)+\frac{1}{32}\log\left(x^2+x+1\right)(9d-4f+3h)+\frac{x\left(x^2(-(7d-7f+4h))+2d+3f-h\right)}{24\left(x^4+x^2+1\right)}+\frac{x\left(x^2(-(d-2f+h))+d+f-2h\right)}{12\left(x^4+x^2+1\right)^2}-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)(13d+2f+h)}{48\sqrt{3}}+\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)(13d+2f+h)}{48\sqrt{3}}+\frac{(2e-g)\tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{\left(2x^2+1\right)(2e-g)}{12\left(x^4+x^2+1\right)}+\frac{x^2(2e-g)+e-2g}{12\left(x^4+x^2+1\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x + f*x^2 + g*x^3 + h*x^4)/(1 + x^2 + x^4)^3,x]
```

```
[Out] (e - 2*g + (2*e - g)*x^2)/(12*(1 + x^2 + x^4)^2) + (x*(d + f - 2*
h - (d - 2*f + h)*x^2))/(12*(1 + x^2 + x^4)^2) + ((2*e - g)*(1 +
2*x^2))/(12*(1 + x^2 + x^4)) + (x*(2*d + 3*f - h - (7*d - 7*f + 4
```

```
*h)*x^2))/(24*(1 + x^2 + x^4)) - ((13*d + 2*f + h)*ArcTan[(1 - 2*
x)/Sqrt[3]])/(48*Sqrt[3]) + ((13*d + 2*f + h)*ArcTan[(1 + 2*x)/Sq
rt[3]])/(48*Sqrt[3]) + ((2*e - g)*ArcTan[(1 + 2*x^2)/Sqrt[3]])/(3
*Sqrt[3]) - ((9*d - 4*f + 3*h)*Log[1 - x + x^2])/32 + ((9*d - 4*f
 + 3*h)*Log[1 + x + x^2])/32
```

Rubi in Sympy [A] time = 102.35, size = 233, normalized size = 0.89

$$\frac{x\left(6d+9f-3h-x^3(18e-18g)-x^2(21d-21f+12h)+x(6e+6g)\right)}{72(x^4+x^2+1)} + \frac{x\left(d+f-2h-x^3(e-2g)-x^2(d-2f+h)+x(e+g)\right)}{12(x^4+x^2+1)^2} + \frac{\sqrt{3}(2e-g)\,\mathrm{atan}\left(\sqrt{3}\left(\frac{2x^2}{3}+\frac{1}{3}\right)\right)}{9} - \left(\frac{9d}{32}-\frac{f}{8}+\frac{3h}{32}\right)\log\left(x^2-x+1\right) + \left(\frac{9d}{32}-\frac{f}{8}+\frac{3h}{32}\right)\log\left(x^2+x+1\right) + \frac{\sqrt{3}(13d+2f+h)\,\mathrm{atan}\left(\sqrt{3}\left(\frac{2x}{3}-\frac{1}{3}\right)\right)}{144} + \frac{\sqrt{3}(13d+2f+h)\,\mathrm{atan}\left(\sqrt{3}\left(\frac{2x}{3}+\frac{1}{3}\right)\right)}{144}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((h*x**4+g*x**3+f*x**2+e*x+d)/(x**4+x**2+1)**3,x)
```

```
[Out] x*(6*d + 9*f - 3*h - x**3*(18*e - 18*g) - x**2*(21*d - 21*f + 12*
h) + x*(6*e + 6*g))/(72*(x**4 + x**2 + 1)) + x*(d + f - 2*h - x**
3*(e - 2*g) - x**2*(d - 2*f + h) + x*(e + g))/(12*(x**4 + x**2 +
1)**2) + sqrt(3)*(2*e - g)*atan(sqrt(3)*(2*x**2/3 + 1/3))/9 - (9*
d/32 - f/8 + 3*h/32)*log(x**2 - x + 1) + (9*d/32 - f/8 + 3*h/32)*
log(x**2 + x + 1) + sqrt(3)*(13*d + 2*f + h)*atan(sqrt(3)*(2*x/3
- 1/3))/144 + sqrt(3)*(13*d + 2*f + h)*atan(sqrt(3)*(2*x/3 + 1/3)
)/144
```

Mathematica [C] time = 1.83384, size = 303, normalized size = 1.15

$$\frac{1}{144}\left(-\frac{6\left(x\left(7dx^2-2d-7fx^2-3f+4hx^2+h\right)-4e\left(2x^2+1\right)+g\left(4x^2+2\right)\right)}{x^4+x^2+1}+\frac{12\left(x\left(-dx^2+d+2fx^2+f-h\left(x^2+2\right)\right)+2ex^2+e-g\left(x^2+2\right)\right)}{\left(x^4+x^2+1\right)^2}-\frac{\tan^{-1}\left(\frac{1}{2}\left(\sqrt{3}-i\right)x\right)\left(\left(7\sqrt{3}-47i\right)d+\left(-7\sqrt{3}+17i\right)f+2\left(2\sqrt{3}-7i\right)h\right)}{\sqrt{\frac{1}{6}\left(1+i\sqrt{3}\right)}}-\frac{\tan^{-1}\left(\frac{1}{2}\left(\sqrt{3}+i\right)x\right)\left(\left(7\sqrt{3}+47i\right)d-\left(7\sqrt{3}+17i\right)f+2\left(2\sqrt{3}+7i\right)h\right)}{\sqrt{\frac{1}{6}\left(1-i\sqrt{3}\right)}}-16\sqrt{3}(2e-g)\tan^{-1}\left(\frac{\sqrt{3}}{2x^2+1}\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(d + e*x + f*x^2 + g*x^3 + h*x^4)/(1 + x^2 + x^4)^3,x]
```

```
[Out] ((-6*(-4*e*(1 + 2*x^2) + g*(2 + 4*x^2) + x*(-2*d - 3*f + h + 7*d*
x^2 - 7*f*x^2 + 4*h*x^2)))/(1 + x^2 + x^4) + (12*(e + 2*e*x^2 - g
*(2 + x^2) + x*(d + f - d*x^2 + 2*f*x^2 - h*(2 + x^2))))/(1 + x^2
 + x^4)^2 - (((-47*I + 7*Sqrt[3])*d + (17*I - 7*Sqrt[3])*f + 2*(-
7*I + 2*Sqrt[3])*h)*ArcTan[((-I + Sqrt[3])*x)/2])/Sqrt[(1 + I*Sqr
t[3])/6] - (((47*I + 7*Sqrt[3])*d - (17*I + 7*Sqrt[3])*f + 2*(7*I
 + 2*Sqrt[3])*h)*ArcTan[((I + Sqrt[3])*x)/2])/Sqrt[(1 - I*Sqrt[3]
)/6] - 16*Sqrt[3]*(2*e - g)*ArcTan[Sqrt[3]/(1 + 2*x^2)])/144
```

Maple [A] time = 0.023, size = 396, normalized size = 1.5

$$\frac{1}{16\left(x^{2}+x+1\right)^{2}}\left(\left(-\frac{7d}{3}+\frac{7f}{3}-\frac{4h}{3}-\frac{4e}{3}-\frac{g}{3}\right)x^{3}+\left(-6d+4f-2h-2g\right)x^{2}+\left(-\frac{20d}{3}+\frac{13f}{3}-\frac{5h}{3}+\frac{e}{3}-\frac{8g}{3}\right)x-4d\right.$$
$$+\frac{9d\ln\left(x^{2}+x+1\right)}{32}-\frac{\ln\left(x^{2}+x+1\right)f}{8}+\frac{3\ln\left(x^{2}+x+1\right)h}{32}$$
$$+\frac{13d\sqrt{3}}{144}\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)-\frac{2\sqrt{3}e}{9}\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)$$
$$+\frac{\sqrt{3}f}{72}\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)+\frac{\sqrt{3}g}{9}\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)+\frac{\sqrt{3}h}{144}\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)$$
$$-\frac{1}{16\left(x^{2}-x+1\right)^{2}}\left(\left(\frac{7d}{3}-\frac{7f}{3}+\frac{4h}{3}-\frac{4e}{3}-\frac{g}{3}\right)x^{3}+\left(-6d+4f-2h+2g\right)x^{2}+\left(\frac{20d}{3}-\frac{13f}{3}+\frac{5h}{3}+\frac{e}{3}-\frac{8g}{3}\right)x-4d+\right.$$
$$-\frac{9d\ln\left(x^{2}-x+1\right)}{32}+\frac{\ln\left(x^{2}-x+1\right)f}{8}-\frac{3\ln\left(x^{2}-x+1\right)h}{32}$$
$$+\frac{13d\sqrt{3}}{144}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)+\frac{2\sqrt{3}e}{9}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)$$
$$+\frac{\sqrt{3}f}{72}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)-\frac{\sqrt{3}g}{9}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)+\frac{\sqrt{3}h}{144}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int((h*x^4+g*x^3+f*x^2+e*x+d)/(x^4+x^2+1)^3,x)
```

```
[Out] 1/16*((-7/3*d+7/3*f-4/3*h-4/3*e-1/3*g)*x^3+(-6*d+4*f-2*h-2*g)*x^2
+(-20/3*d+13/3*f-5/3*h+1/3*e-8/3*g)*x-4*d+4/3*f+2*e-2*g)/(x^2+x+1
)^2+9/32*d*ln(x^2+x+1)-1/8*ln(x^2+x+1)*f+3/32*ln(x^2+x+1)*h+13/14
4*d*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)-2/9*3^(1/2)*arctan(1/3*(1
+2*x)*3^(1/2))*e+1/72*3^(1/2)*arctan(1/3*(1+2*x)*3^(1/2))*f+1/9*3
^(1/2)*arctan(1/3*(1+2*x)*3^(1/2))*g+1/144*3^(1/2)*arctan(1/3*(1+
2*x)*3^(1/2))*h-1/16*((7/3*d-7/3*f+4/3*h-4/3*e-1/3*g)*x^3+(-6*d+4
*f-2*h+2*g)*x^2+(20/3*d-13/3*f+5/3*h+1/3*e-8/3*g)*x-4*d+4/3*f-2*e
+2*g)/(x^2-x+1)^2-9/32*d*ln(x^2-x+1)+1/8*ln(x^2-x+1)*f-3/32*ln(x^
2-x+1)*h+13/144*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))*d+2/9*3^(1/2)
*arctan(1/3*(2*x-1)*3^(1/2))*e+1/72*3^(1/2)*arctan(1/3*(2*x-1)*3^
(1/2))*f-1/9*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))*g+1/144*3^(1/2)*
arctan(1/3*(2*x-1)*3^(1/2))*h
```

Maxima [A] time = 0.785515, size = 293, normalized size = 1.11

$$\frac{1}{144}\sqrt{3}(13\,d-32\,e+2\,f+16\,g+h)\arctan\left(\frac{1}{3}\sqrt{3}(2\,x+1)\right)+\frac{1}{144}\sqrt{3}(13\,d+32\,e+2\,f-16\,g+h)\arctan\left(\frac{1}{3}\sqrt{3}(2\,x-1)\right)+\frac{1}{32}(9\,d-4\,f+3\,h)\log\left(x^2+x+1\right)-\frac{1}{32}(9\,d-4\,f+3\,h)\log\left(x^2-x+1\right)-\frac{(7\,d-7\,f+4\,h)x^7-4\,(2\,e-g)x^6+5\,(d-2\,f+h)x^5-6\,(2\,e-g)x^4+7\,(d-2\,f+h)x^3-8\,(2\,e-g)x^2-(4\,d+5\,f-5\,h)x}{24\left(x^8+2\,x^6+3\,x^4+2\,x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^4 + g*x^3 + f*x^2 + e*x + d)/(x^4 + x^2 + 1)^3,x, algorithm="maxima
```

```
[Out] 1/144*sqrt(3)*(13*d - 32*e + 2*f + 16*g + h)*arctan(1/3*sqrt(3)*(
2*x + 1)) + 1/144*sqrt(3)*(13*d + 32*e + 2*f - 16*g + h)*arctan(1
/3*sqrt(3)*(2*x - 1)) + 1/32*(9*d - 4*f + 3*h)*log(x^2 + x + 1) -
 1/32*(9*d - 4*f + 3*h)*log(x^2 - x + 1) - 1/24*((7*d - 7*f + 4*h
)*x^7 - 4*(2*e - g)*x^6 + 5*(d - 2*f + h)*x^5 - 6*(2*e - g)*x^4 +
 7*(d - 2*f + h)*x^3 - 8*(2*e - g)*x^2 - (4*d + 5*f - 5*h)*x - 6*
e + 6*g)/(x^8 + 2*x^6 + 3*x^4 + 2*x^2 + 1)
```

Fricas [A] time = 1.51778, size = 666, normalized size = 2.53

$$\frac{\sqrt{3}\Big(3\sqrt{3}\big((9\,d-4\,f+3\,h)x^8+2\,(9\,d-4\,f+3\,h)x^6+3\,(9\,d-4\,f+3\,h)x^4+2\,(9\,d-4\,f+3\,h)x^2+9\,d-4\,f+3\,h\big)\log\left(x^2+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^4 + g*x^3 + f*x^2 + e*x + d)/(x^4 + x^2 + 1)^3,x, algorithm="fricas
```

```
[Out] 1/288*sqrt(3)*(3*sqrt(3)*((9*d - 4*f + 3*h)*x^8 + 2*(9*d - 4*f +
3*h)*x^6 + 3*(9*d - 4*f + 3*h)*x^4 + 2*(9*d - 4*f + 3*h)*x^2 + 9*
d - 4*f + 3*h)*log(x^2 + x + 1) - 3*sqrt(3)*((9*d - 4*f + 3*h)*x^
8 + 2*(9*d - 4*f + 3*h)*x^6 + 3*(9*d - 4*f + 3*h)*x^4 + 2*(9*d -
4*f + 3*h)*x^2 + 9*d - 4*f + 3*h)*log(x^2 - x + 1) + 2*((13*d - 3
2*e + 2*f + 16*g + h)*x^8 + 2*(13*d - 32*e + 2*f + 16*g + h)*x^6
+ 3*(13*d - 32*e + 2*f + 16*g + h)*x^4 + 2*(13*d - 32*e + 2*f + 1
6*g + h)*x^2 + 13*d - 32*e + 2*f + 16*g + h)*arctan(1/3*sqrt(3)*(
2*x + 1)) + 2*((13*d + 32*e + 2*f - 16*g + h)*x^8 + 2*(13*d + 32*
e + 2*f - 16*g + h)*x^6 + 3*(13*d + 32*e + 2*f - 16*g + h)*x^4 +
2*(13*d + 32*e + 2*f - 16*g + h)*x^2 + 13*d + 32*e + 2*f - 16*g +
 h)*arctan(1/3*sqrt(3)*(2*x - 1)) - 4*sqrt(3)*((7*d - 7*f + 4*h)*
x^7 - 4*(2*e - g)*x^6 + 5*(d - 2*f + h)*x^5 - 6*(2*e - g)*x^4 + 7
```

```
*(d - 2*f + h)*x^3 - 8*(2*e - g)*x^2 - (4*d + 5*f - 5*h)*x - 6*e
+ 6*g))/(x^8 + 2*x^6 + 3*x^4 + 2*x^2 + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((h*x**4+g*x**3+f*x**2+e*x+d)/(x**4+x**2+1)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.267982, size = 308, normalized size = 1.17

$$\frac{1}{144}\sqrt{3}(13\,d + 2\,f + 16\,g + h - 32\,e)\arctan\left(\frac{1}{3}\sqrt{3}(2\,x + 1)\right) + \frac{1}{144}\sqrt{3}(13\,d + 2\,f - 16\,g + h + 32\,e)\arctan\left(\frac{1}{3}\sqrt{3}(2\,x - 1)\right) + \frac{1}{32}(9\,d - 4\,f + 3\,h)\ln\left(x^2 + x + 1\right) - \frac{1}{32}(9\,d - 4\,f + 3\,h)\ln\left(x^2 - x + 1\right) - \frac{7\,dx^7 - 7\,fx^7 + 4\,hx^7 + 4\,gx^6 - 8\,x^6e + 5\,dx^5 - 10\,fx^5 + 5\,hx^5 + 6\,gx^4 - 12\,x^4e + 7\,dx^3 - 14\,fx^3 + 7\,hx^3 + 8\,gx^2 - 16\,x^2e}{24\,(x^4 + x^2 + 1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((h*x^4 + g*x^3 + f*x^2 + e*x + d)/(x^4 + x^2 + 1)^3,x, algorithm="giac")
```

```
[Out] 1/144*sqrt(3)*(13*d + 2*f + 16*g + h - 32*e)*arctan(1/3*sqrt(3)*(
2*x + 1)) + 1/144*sqrt(3)*(13*d + 2*f - 16*g + h + 32*e)*arctan(1
/3*sqrt(3)*(2*x - 1)) + 1/32*(9*d - 4*f + 3*h)*ln(x^2 + x + 1) -
1/32*(9*d - 4*f + 3*h)*ln(x^2 - x + 1) - 1/24*(7*d*x^7 - 7*f*x^7
+ 4*h*x^7 + 4*g*x^6 - 8*x^6*e + 5*d*x^5 - 10*f*x^5 + 5*h*x^5 + 6*
g*x^4 - 12*x^4*e + 7*d*x^3 - 14*f*x^3 + 7*h*x^3 + 8*g*x^2 - 16*x^
2*e - 4*d*x - 5*f*x + 5*h*x + 6*g - 6*e)/(x^4 + x^2 + 1)^2
```

3.51 $\int \frac{d+ex+fx^2+gx^3+hx^4+ix^5}{(1+x^2+x^4)^3}\,dx$

Optimal. Leaf size=269

$$-\frac{1}{32}\log\left(x^2-x+1\right)(9d-4f+3h)+\frac{1}{32}\log\left(x^2+x+1\right)(9d-4f+3h)+\frac{x\left(x^2(-(7d-7f+4h))+2d+3f-h\right)}{24\left(x^4+x^2+1\right)}+\frac{x\left(x^2(-(d-2f+h))+d+f-2h\right)}{12\left(x^4+x^2+1\right)^2}-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)(13d+2f+h)}{48\sqrt{3}}+\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)(13d+2f+h)}{48\sqrt{3}}+\frac{\tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right)(2e-g+i)}{3\sqrt{3}}+\frac{\left(2x^2+1\right)(2e-g+i)}{12\left(x^4+x^2+1\right)}+\frac{x^2(2e-g-i)+e-2g+i}{12\left(x^4+x^2+1\right)^2}$$

```
[Out] (x*(d + f - 2*h - (d - 2*f + h)*x^2))/(12*(1 + x^2 + x^4)^2) + (e
 - 2*g + i + (2*e - g - i)*x^2)/(12*(1 + x^2 + x^4)^2) + ((2*e -
g + i)*(1 + 2*x^2))/(12*(1 + x^2 + x^4)) + (x*(2*d + 3*f - h - (7
*d - 7*f + 4*h)*x^2))/(24*(1 + x^2 + x^4)) - ((13*d + 2*f + h)*Ar
cTan[(1 - 2*x)/Sqrt[3]])/(48*Sqrt[3]) + ((13*d + 2*f + h)*ArcTan[
(1 + 2*x)/Sqrt[3]])/(48*Sqrt[3]) + ((2*e - g + i)*ArcTan[(1 + 2*x
^2)/Sqrt[3]])/(3*Sqrt[3]) - ((9*d - 4*f + 3*h)*Log[1 - x + x^2])/
32 + ((9*d - 4*f + 3*h)*Log[1 + x + x^2])/32
```

Rubi [A] time = 0.629288, antiderivative size = 269, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 12, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$-\frac{1}{32}\log\left(x^2-x+1\right)(9d-4f+3h)+\frac{1}{32}\log\left(x^2+x+1\right)(9d-4f+3h)+\frac{x\left(x^2(-(7d-7f+4h))+2d+3f-h\right)}{24\left(x^4+x^2+1\right)}+\frac{x\left(x^2(-(d-2f+h))+d+f-2h\right)}{12\left(x^4+x^2+1\right)^2}-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)(13d+2f+h)}{48\sqrt{3}}+\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)(13d+2f+h)}{48\sqrt{3}}+\frac{\tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right)(2e-g+i)}{3\sqrt{3}}+\frac{\left(2x^2+1\right)(2e-g+i)}{12\left(x^4+x^2+1\right)}+\frac{x^2(2e-g-i)+e-2g+i}{12\left(x^4+x^2+1\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5)/(1 + x^2 + x^4)^3,x]
```

```
[Out] (x*(d + f - 2*h - (d - 2*f + h)*x^2))/(12*(1 + x^2 + x^4)^2) + (e
 - 2*g + i + (2*e - g - i)*x^2)/(12*(1 + x^2 + x^4)^2) + ((2*e -
g + i)*(1 + 2*x^2))/(12*(1 + x^2 + x^4)) + (x*(2*d + 3*f - h - (7
```

```
*d - 7*f + 4*h)*x^2))/(24*(1 + x^2 + x^4)) - ((13*d + 2*f + h)*Ar
cTan[(1 - 2*x)/Sqrt[3]])/(48*Sqrt[3]) + ((13*d + 2*f + h)*ArcTan[
(1 + 2*x)/Sqrt[3]])/(48*Sqrt[3]) + ((2*e - g + i)*ArcTan[(1 + 2*x
^2)/Sqrt[3]])/(3*Sqrt[3]) - ((9*d - 4*f + 3*h)*Log[1 - x + x^2])/
32 + ((9*d - 4*f + 3*h)*Log[1 + x + x^2])/32
```

Rubi in Sympy [A] time = 115.85, size = 240, normalized size = 0.89

$$\frac{x\left(6d+9f-3h-x^{3}\left(18e-18g+12\right)-x^{2}\left(21d-21f+12h\right)+x\left(6e+6g\right)\right)}{72\left(x^{4}+x^{2}+1\right)} + \frac{x\left(d+f-2h-x^{3}\left(e-2g+1\right)-x^{2}\left(d-2f+h\right)-x\left(-e-g+2\right)\right)}{12\left(x^{4}+x^{2}+1\right)^{2}} - \left(\frac{9d}{32}-\frac{f}{8}+\frac{3h}{32}\right)\log\left(x^{2}-x+1\right) + \left(\frac{9d}{32}-\frac{f}{8}+\frac{3h}{32}\right)\log\left(x^{2}+x+1\right) + \frac{\sqrt{3}\left(13d+2f+h\right)\operatorname{atan}\left(\sqrt{3}\left(\frac{2x}{3}-\frac{1}{3}\right)\right)}{144} + \frac{\sqrt{3}\left(13d+2f+h\right)\operatorname{atan}\left(\sqrt{3}\left(\frac{2x}{3}+\frac{1}{3}\right)\right)}{144} + \frac{\sqrt{3}\left(2e-g+1\right)\operatorname{atan}\left(\sqrt{3}\left(\frac{2x^{2}}{3}+\frac{1}{3}\right)\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((i*x**5+h*x**4+g*x**3+f*x**2+e*x+d)/(x**4+x**2+1)**3,x)
```

```
[Out] x*(6*d + 9*f - 3*h - x**3*(18*e - 18*g + 12) - x**2*(21*d - 21*f
+ 12*h) + x*(6*e + 6*g))/(72*(x**4 + x**2 + 1)) + x*(d + f - 2*h
- x**3*(e - 2*g + 1) - x**2*(d - 2*f + h) - x*(-e - g + 2))/(12*(
x**4 + x**2 + 1)**2) - (9*d/32 - f/8 + 3*h/32)*log(x**2 - x + 1)
+ (9*d/32 - f/8 + 3*h/32)*log(x**2 + x + 1) + sqrt(3)*(13*d + 2*f
 + h)*atan(sqrt(3)*(2*x/3 - 1/3))/144 + sqrt(3)*(13*d + 2*f + h)*
atan(sqrt(3)*(2*x/3 + 1/3))/144 + sqrt(3)*(2*e - g + 1)*atan(sqrt
(3)*(2*x**2/3 + 1/3))/9
```

Mathematica [C] time = 2.22492, size = 325, normalized size = 1.21

$$\frac{1}{144}\left(\frac{12\left(-dx^3+dx+2ex^2+e+2fx^3+fx-g\left(x^2+2\right)-hx^3-2hx-ix^2+i\right)}{\left(x^4+x^2+1\right)^2}+\frac{6\left(-7dx^3+2dx+e\left(8x^2+4\right)+7fx^3+3fx-2g\left(2x^2+1\right)-4hx^3-hx+4ix^2+2i\right)}{x^4+x^2+1}-\frac{\tan^{-1}\left(\frac{1}{2}\left(\sqrt{3}-i\right)x\right)\left(\left(7\sqrt{3}-47i\right)d+\left(-7\sqrt{3}+17i\right)f+2\left(2\sqrt{3}-7i\right)h\right)}{\sqrt{\frac{1}{6}\left(1+i\sqrt{3}\right)}}-\frac{\tan^{-1}\left(\frac{1}{2}\left(\sqrt{3}+i\right)x\right)\left(\left(7\sqrt{3}+47i\right)d-\left(7\sqrt{3}+17i\right)f+2\left(2\sqrt{3}+7i\right)h\right)}{\sqrt{\frac{1}{6}\left(1-i\sqrt{3}\right)}}-16\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}}{2x^2+1}\right)(2e-g+i)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5)/(1 + x^2 + x^4)^3,x]
```

```
[Out] ((12*(e + i + d*x + f*x - 2*h*x + 2*e*x^2 - i*x^2 - d*x^3 + 2*f*x
^3 - h*x^3 - g*(2 + x^2)))/(1 + x^2 + x^4)^2 + (6*(2*i + 2*d*x +
3*f*x - h*x + 4*i*x^2 - 7*d*x^3 + 7*f*x^3 - 4*h*x^3 - 2*g*(1 + 2*
x^2) + e*(4 + 8*x^2)))/(1 + x^2 + x^4) - (((-47*I + 7*Sqrt[3])*d
+ (17*I - 7*Sqrt[3])*f + 2*(-7*I + 2*Sqrt[3])*h)*ArcTan[((-I + Sq
rt[3])*x)/2])/Sqrt[(1 + I*Sqrt[3])/6] - (((47*I + 7*Sqrt[3])*d -
(17*I + 7*Sqrt[3])*f + 2*(7*I + 2*Sqrt[3])*h)*ArcTan[((I + Sqrt[3
])*x)/2])/Sqrt[(1 - I*Sqrt[3])/6] - 16*Sqrt[3]*(2*e - g + i)*ArcT
an[Sqrt[3]/(1 + 2*x^2)])/144
```

Maple [A] time = 0.025, size = 454, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4+x^2+1)^3,x)
```

```
[Out] 1/16*((-7/3*d+7/3*f-4/3*h-4/3*e-1/3*g+1/3*i)*x^3+(-6*d+4*f-2*h-2*
g+2*i)*x^2+(-20/3*d+13/3*f-5/3*h+1/3*e-8/3*g+7/3*i)*x-4*d+4/3*f+2
```

```
*e-2*g+4/3*i)/(x^2+x+1)^2+9/32*d*ln(x^2+x+1)-1/8*ln(x^2+x+1)*f+3/
32*ln(x^2+x+1)*h+13/144*d*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)-2/9
*3^(1/2)*arctan(1/3*(1+2*x)*3^(1/2))*e+1/72*3^(1/2)*arctan(1/3*(1
+2*x)*3^(1/2))*f+1/9*3^(1/2)*arctan(1/3*(1+2*x)*3^(1/2))*g+1/144*
3^(1/2)*arctan(1/3*(1+2*x)*3^(1/2))*h-1/9*3^(1/2)*arctan(1/3*(1+2
*x)*3^(1/2))*i-1/16*((7/3*d-7/3*f+4/3*h-4/3*e-1/3*g+1/3*i)*x^3+(-
6*d+4*f-2*h+2*g-2*i)*x^2+(20/3*d-13/3*f+5/3*h+1/3*e-8/3*g+7/3*i)*
x-4*d+4/3*f-2*e+2*g-4/3*i)/(x^2-x+1)^2-9/32*d*ln(x^2-x+1)+1/8*ln(
x^2-x+1)*f-3/32*ln(x^2-x+1)*h+13/144*3^(1/2)*arctan(1/3*(2*x-1)*3
^(1/2))*d+2/9*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))*e+1/72*3^(1/2)*
arctan(1/3*(2*x-1)*3^(1/2))*f-1/9*3^(1/2)*arctan(1/3*(2*x-1)*3^(1
/2))*g+1/144*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))*h+1/9*3^(1/2)*ar
ctan(1/3*(2*x-1)*3^(1/2))*i
```

Maxima [A] time = 0.778989, size = 309, normalized size = 1.15

$$\frac{1}{144}\sqrt{3}(13\,d-32\,e+2\,f+16\,g+h-16\,i)\arctan\left(\frac{1}{3}\sqrt{3}(2\,x+1)\right)+\frac{1}{144}\sqrt{3}(13\,d+32\,e+2\,f-16\,g+h+16\,i)\arctan\left(\frac{1}{3}\sqrt{3}(2\,x-1)\right)+\frac{1}{32}(9\,d-4\,f+3\,h)\log\left(x^2+x+1\right)-\frac{1}{32}(9\,d-4\,f+3\,h)\log\left(x^2-x+1\right)-\frac{(7\,d-7\,f+4\,h)x^7-4\,(2\,e-g+i)x^6+5\,(d-2\,f+h)x^5-6\,(2\,e-g+i)x^4+7\,(d-2\,f+h)x^3-4\,(4\,e-2\,g+i)x^2-(4\,d+}{24\left(x^8+2\,x^6+3\,x^4+2\,x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^5 + h*x^4 + g*x^3 + f*x^2 + e*x + d)/(x^4 + x^2 + 1)^3,x, algorithm=
```

```
[Out] 1/144*sqrt(3)*(13*d - 32*e + 2*f + 16*g + h - 16*i)*arctan(1/3*sq
rt(3)*(2*x + 1)) + 1/144*sqrt(3)*(13*d + 32*e + 2*f - 16*g + h +
16*i)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/32*(9*d - 4*f + 3*h)*log(
x^2 + x + 1) - 1/32*(9*d - 4*f + 3*h)*log(x^2 - x + 1) - 1/24*((7
*d - 7*f + 4*h)*x^7 - 4*(2*e - g + i)*x^6 + 5*(d - 2*f + h)*x^5 -
 6*(2*e - g + i)*x^4 + 7*(d - 2*f + h)*x^3 - 4*(4*e - 2*g + i)*x^
2 - (4*d + 5*f - 5*h)*x - 6*e + 6*g - 4*i)/(x^8 + 2*x^6 + 3*x^4 +
 2*x^2 + 1)
```

Fricas [A] time = 7.22812, size = 714, normalized size = 2.65

$$\frac{\sqrt{3}\left(3\sqrt{3}\left((9\,d-4\,f+3\,h)x^8+2\,(9\,d-4\,f+3\,h)x^6+3\,(9\,d-4\,f+3\,h)x^4+2\,(9\,d-4\,f+3\,h)x^2+9\,d-4\,f+3\,h\right)\log\left(x^2+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^5 + h*x^4 + g*x^3 + f*x^2 + e*x + d)/(x^4 + x^2 + 1)^3,x, algorithm=
```

```
[Out] 1/288*sqrt(3)*(3*sqrt(3)*((9*d - 4*f + 3*h)*x^8 + 2*(9*d - 4*f +
3*h)*x^6 + 3*(9*d - 4*f + 3*h)*x^4 + 2*(9*d - 4*f + 3*h)*x^2 + 9*
d - 4*f + 3*h)*log(x^2 + x + 1) - 3*sqrt(3)*((9*d - 4*f + 3*h)*x^
8 + 2*(9*d - 4*f + 3*h)*x^6 + 3*(9*d - 4*f + 3*h)*x^4 + 2*(9*d -
4*f + 3*h)*x^2 + 9*d - 4*f + 3*h)*log(x^2 - x + 1) + 2*((13*d - 3
2*e + 2*f + 16*g + h - 16*i)*x^8 + 2*(13*d - 32*e + 2*f + 16*g +
h - 16*i)*x^6 + 3*(13*d - 32*e + 2*f + 16*g + h - 16*i)*x^4 + 2*(
13*d - 32*e + 2*f + 16*g + h - 16*i)*x^2 + 13*d - 32*e + 2*f + 16
*g + h - 16*i)*arctan(1/3*sqrt(3)*(2*x + 1)) + 2*((13*d + 32*e +
2*f - 16*g + h + 16*i)*x^8 + 2*(13*d + 32*e + 2*f - 16*g + h + 16
*i)*x^6 + 3*(13*d + 32*e + 2*f - 16*g + h + 16*i)*x^4 + 2*(13*d +
 32*e + 2*f - 16*g + h + 16*i)*x^2 + 13*d + 32*e + 2*f - 16*g + h
 + 16*i)*arctan(1/3*sqrt(3)*(2*x - 1)) - 4*sqrt(3)*((7*d - 7*f +
4*h)*x^7 - 4*(2*e - g + i)*x^6 + 5*(d - 2*f + h)*x^5 - 6*(2*e - g
 + i)*x^4 + 7*(d - 2*f + h)*x^3 - 4*(4*e - 2*g + i)*x^2 - (4*d +
5*f - 5*h)*x - 6*e + 6*g - 4*i))/(x^8 + 2*x^6 + 3*x^4 + 2*x^2 + 1
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x**5+h*x**4+g*x**3+f*x**2+e*x+d)/(x**4+x**2+1)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.267752, size = 344, normalized size = 1.28

$$\frac{1}{144}\sqrt{3}(13\,d + 2\,f + 16\,g + h - 16\,i - 32\,e)\arctan\left(\frac{1}{3}\sqrt{3}(2\,x + 1)\right) + \frac{1}{144}\sqrt{3}(13\,d + 2\,f - 16\,g + h + 16\,i + 32\,e)\arctan\left(\frac{1}{3}\sqrt{3}(2\,x - 1)\right) + \frac{1}{32}(9\,d - 4\,f + 3\,h)\ln\left(x^2 + x + 1\right) - \frac{1}{32}(9\,d - 4\,f + 3\,h)\ln\left(x^2 - x + 1\right) - \frac{7\,dx^7 - 7\,fx^7 + 4\,hx^7 + 4\,gx^6 - 4\,ix^6 - 8\,x^6e + 5\,dx^5 - 10\,fx^5 + 5\,hx^5 + 6\,gx^4 - 6\,ix^4 - 12\,x^4e + 7\,dx^3 - 14\,fx^3 + 7\,hx^3 +}{24\left(x^4 + x^2 + 1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^5 + h*x^4 + g*x^3 + f*x^2 + e*x + d)/(x^4 + x^2 + 1)^3,x, algorithm=
```

```
[Out] 1/144*sqrt(3)*(13*d + 2*f + 16*g + h - 16*i - 32*e)*arctan(1/3*sq
rt(3)*(2*x + 1)) + 1/144*sqrt(3)*(13*d + 2*f - 16*g + h + 16*i +
32*e)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/32*(9*d - 4*f + 3*h)*ln(x
^2 + x + 1) - 1/32*(9*d - 4*f + 3*h)*ln(x^2 - x + 1) - 1/24*(7*d*
x^7 - 7*f*x^7 + 4*h*x^7 + 4*g*x^6 - 4*i*x^6 - 8*x^6*e + 5*d*x^5 -
 10*f*x^5 + 5*h*x^5 + 6*g*x^4 - 6*i*x^4 - 12*x^4*e + 7*d*x^3 - 14
*f*x^3 + 7*h*x^3 + 8*g*x^2 - 4*i*x^2 - 16*x^2*e - 4*d*x - 5*f*x +
 5*h*x + 6*g - 4*i - 6*e)/(x^4 + x^2 + 1)^2
```

3.52 $\int \frac{d+ex}{(a+bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=474

$$\frac{dx\left(3bcx^2\left(b^2-8ac\right)+\left(b^2-7ac\right)\left(3b^2-4ac\right)\right)}{8a^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{3\sqrt{c}d\left(56a^2c^2-10ab^2c+b\left(b^2-8ac\right)\sqrt{b^2-4ac}+b^4\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}a^2\left(b^2-4ac\right)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{3\sqrt{c}d\left(-\frac{56a^2c^2-10ab^2c+b^4}{\sqrt{b^2-4ac}}-8abc+b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{8\sqrt{2}a^2\left(b^2-4ac\right)^2\sqrt{\sqrt{b^2-4ac}+b}}-\frac{6c^2e\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/2}}+\frac{dx\left(-2ac+b^2+bcx^2\right)}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{3ce\left(b+2cx^2\right)}{2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}-\frac{e\left(b+2cx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}$$

```
[Out] -(e*(b + 2*c*x^2))/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (d*x
*(b^2 - 2*a*c + b*c*x^2))/(4*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^
2) + (3*c*e*(b + 2*c*x^2))/(2*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)
) + (d*x*((b^2 - 7*a*c)*(3*b^2 - 4*a*c) + 3*b*c*(b^2 - 8*a*c)*x^2
))/(8*a^2*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (3*Sqrt[c]*(b^4
- 10*a*b^2*c + 56*a^2*c^2 + b*(b^2 - 8*a*c)*Sqrt[b^2 - 4*a*c])*d*
ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[
2]*a^2*(b^2 - 4*a*c)^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (3*Sqrt
[c]*(b^3 - 8*a*b*c - (b^4 - 10*a*b^2*c + 56*a^2*c^2)/Sqrt[b^2 - 4
*a*c])*d*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])
/(8*Sqrt[2]*a^2*(b^2 - 4*a*c)^2*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - (6
*c^2*e*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(5
/2)
```

Rubi [A] time = 4.50264, antiderivative size = 474, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$

$$\frac{dx\left(3bcx^2\left(b^2-8ac\right)+\left(b^2-7ac\right)\left(3b^2-4ac\right)\right)}{8a^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{3\sqrt{c}d\left(56a^2c^2-10ab^2c+b\left(b^2-8ac\right)\sqrt{b^2-4ac}+b^4\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}a^2\left(b^2-4ac\right)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{3\sqrt{c}d\left(-\frac{56a^2c^2-10ab^2c+b^4}{\sqrt{b^2-4ac}}-8abc+b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{8\sqrt{2}a^2\left(b^2-4ac\right)^2\sqrt{\sqrt{b^2-4ac}+b}}-\frac{6c^2e\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/2}}+\frac{dx\left(-2ac+b^2+bcx^2\right)}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{3ce\left(b+2cx^2\right)}{2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}-\frac{e\left(b+2cx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x)/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] -(e*(b + 2*c*x^2))/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (d*x
*(b^2 - 2*a*c + b*c*x^2))/(4*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^
2) + (3*c*e*(b + 2*c*x^2))/(2*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)
) + (d*x*((b^2 - 7*a*c)*(3*b^2 - 4*a*c) + 3*b*c*(b^2 - 8*a*c)*x^2
))/(8*a^2*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (3*Sqrt[c]*(b^4
- 10*a*b^2*c + 56*a^2*c^2 + b*(b^2 - 8*a*c)*Sqrt[b^2 - 4*a*c])*d*
ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[
2]*a^2*(b^2 - 4*a*c)^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (3*Sqrt
[c]*(b^3 - 8*a*b*c - (b^4 - 10*a*b^2*c + 56*a^2*c^2)/Sqrt[b^2 - 4
*a*c])*d*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])
/(8*Sqrt[2]*a^2*(b^2 - 4*a*c)^2*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - (6
*c^2*e*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(5
/2)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x+d)/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 4.37103, size = 488, normalized size = 1.03

$$\frac{1}{16}\left(\frac{8a^2c(3be+cx(7d+6ex))-2abcdx\left(25b+24cx^2\right)+6b^3dx\left(b+cx^2\right)}{a^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{3\sqrt{2}\sqrt{c}d\left(56a^2c^2-10ab^2c-8abc\sqrt{b^2-4ac}+b^3\sqrt{b^2-4ac}+b^4\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{a^2\left(b^2-4ac\right)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{3\sqrt{2}\sqrt{c}d\left(56a^2c^2-10ab^2c+8abc\sqrt{b^2-4ac}-b^3\sqrt{b^2-4ac}+b^4\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{a^2\left(b^2-4ac\right)^{5/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{48c^2e\log\left(\sqrt{b^2-4ac}-b-2cx^2\right)}{\left(b^2-4ac\right)^{5/2}}-\frac{48c^2e\log\left(\sqrt{b^2-4ac}+b+2cx^2\right)}{\left(b^2-4ac\right)^{5/2}}+\frac{4abe+8acx(d+ex)-4bdx\left(b+cx^2\right)}{a\left(4ac-b^2\right)\left(a+bx^2+cx^4\right)^2}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x)/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] ((4*a*b*e + 8*a*c*x*(d + e*x) - 4*b*d*x*(b + c*x^2))/(a*(-b^2 + 4
*a*c)*(a + b*x^2 + c*x^4)^2) + (6*b^3*d*x*(b + c*x^2) - 2*a*b*c*d
*x*(25*b + 24*c*x^2) + 8*a^2*c*(3*b*e + c*x*(7*d + 6*e*x)))/(a^2*
(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (3*Sqrt[2]*Sqrt[c]*(b^4 -
10*a*b^2*c + 56*a^2*c^2 + b^3*Sqrt[b^2 - 4*a*c] - 8*a*b*c*Sqrt[b^
2 - 4*a*c])*d*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*
c]]])/(a^2*(b^2 - 4*a*c)^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (3*
Sqrt[2]*Sqrt[c]*(b^4 - 10*a*b^2*c + 56*a^2*c^2 - b^3*Sqrt[b^2 - 4
*a*c] + 8*a*b*c*Sqrt[b^2 - 4*a*c])*d*ArcTan[(Sqrt[2]*Sqrt[c]*x)/S
qrt[b + Sqrt[b^2 - 4*a*c]]])/(a^2*(b^2 - 4*a*c)^(5/2)*Sqrt[b + Sq
rt[b^2 - 4*a*c]]) + (48*c^2*e*Log[-b + Sqrt[b^2 - 4*a*c] - 2*c*x^
2])/(b^2 - 4*a*c)^(5/2) - (48*c^2*e*Log[b + Sqrt[b^2 - 4*a*c] + 2
*c*x^2])/(b^2 - 4*a*c)^(5/2))/16
```

Maple [B] time = 0.326, size = 3733, normalized size = 7.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x+d)/(c*x^4+b*x^2+a)^3,x)
```

```
[Out] 3/16/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)/(x^2+1/2/c*(-4*a*c+b^
2)^(1/2)+1/2*b/c)^2/a^2*d*x^3*(-4*a*c+b^2)^(1/2)*b^4+3*c^2/(16*a^
2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)*(-4*a*c+b^2)^(1/2)*e*ln(a^2*(2*c
*x^2+(-4*a*c+b^2)^(1/2)+b))+3/16*c/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*
a*c-b^2)*2^(1/2)/a^2/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*
2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*(-4*a*c+b^2)^(1/2)*b^4*
d-15/8*c^2/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)*2^(1/2)/a/((-b+
(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2
)^(1/2))*c)^(1/2))*(-4*a*c+b^2)^(1/2)*b^2*d-15/8*c^2/(16*a^2*c^2-
8*a*b^2*c+b^4)/(4*a*c-b^2)*2^(1/2)/a/((b+(-4*a*c+b^2)^(1/2))*c)^(
1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*(-4*a*c
+b^2)^(1/2)*b^2*d+3/16*c/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)*2
^(1/2)/a^2/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/
((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*(-4*a*c+b^2)^(1/2)*b^4*d-3/16*
c/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)*2^(1/2)/a^2/((b+(-4*a*c+
b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c
)^(1/2))*b^5*d+15/8*c/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)/(x^2
+1/2*b/c-1/2/c*(-4*a*c+b^2)^(1/2))^2/a*d*x^3*(-4*a*c+b^2)^(1/2)*b
^2+4*c/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)/(x^2+1/2/c*(-4*a*c+
b^2)^(1/2)+1/2*b/c)^2*e*(-4*a*c+b^2)^(1/2)*a-9/2*c^2/(16*a^2*c^2-
8*a*b^2*c+b^4)/(4*a*c-b^2)/(x^2+1/2*b/c-1/2/c*(-4*a*c+b^2)^(1/2))
^2*d*x^3*(-4*a*c+b^2)^(1/2)-6*c^2/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a
*c-b^2)/(x^2+1/2*b/c-1/2/c*(-4*a*c+b^2)^(1/2))^2*d*x^3*b+6*c^2/(1
6*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)/(x^2+1/2*b/c-1/2/c*(-4*a*c+b
^2)^(1/2))^2*e*x^2*a-3/2*c/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)
/(x^2+1/2*b/c-1/2/c*(-4*a*c+b^2)^(1/2))^2*e*x^2*b^2-11*c^2/(16*a^
2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)/(x^2+1/2*b/c-1/2/c*(-4*a*c+b^2)^
(1/2))^2*d*a*x+4*c/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)/(x^2+1/
2*b/c-1/2/c*(-4*a*c+b^2)^(1/2))^2*d*x*b^2-4*c/(16*a^2*c^2-8*a*b^2
*c+b^4)/(4*a*c-b^2)/(x^2+1/2*b/c-1/2/c*(-4*a*c+b^2)^(1/2))^2*e*(-
4*a*c+b^2)^(1/2)*a+3*c/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)/(x^
2+1/2*b/c-1/2/c*(-4*a*c+b^2)^(1/2))^2*e*a*b+9/2*c^2/(16*a^2*c^2-8
*a*b^2*c+b^4)/(4*a*c-b^2)/(x^2+1/2/c*(-4*a*c+b^2)^(1/2)+1/2*b/c)^
2*d*x^3*(-4*a*c+b^2)^(1/2)-6*c^2/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*
c-b^2)/(x^2+1/2/c*(-4*a*c+b^2)^(1/2)+1/2*b/c)^2*d*x^3*b-3/16/(16*
a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)/(x^2+1/2*b/c-1/2/c*(-4*a*c+b^2
)^(1/2))^2/a^2*d*x^3*b^5-5/16/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b
^2)/(x^2+1/2*b/c-1/2/c*(-4*a*c+b^2)^(1/2))^2*d/a*x*b^4-3/16/(16*a
^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)/(x^2+1/2/c*(-4*a*c+b^2)^(1/2)+1
/2*b/c)^2/a^2*d*x^3*b^5-5/16/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^
2)/(x^2+1/2/c*(-4*a*c+b^2)^(1/2)+1/2*b/c)^2*d/a*x*b^4+6*c^2/(16*a
^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)/(x^2+1/2/c*(-4*a*c+b^2)^(1/2)+1
/2*b/c)^2*e*x^2*a-3/2*c/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)/(x
^2+1/2/c*(-4*a*c+b^2)^(1/2)+1/2*b/c)^2*e*x^2*b^2+3*c/(16*a^2*c^2-
8*a*b^2*c+b^4)/(4*a*c-b^2)/(x^2+1/2/c*(-4*a*c+b^2)^(1/2)+1/2*b/c)
^2*e*a*b-11*c^2/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)/(x^2+1/2/c
*(-4*a*c+b^2)^(1/2)+1/2*b/c)^2*d*a*x+4*c/(16*a^2*c^2-8*a*b^2*c+b^
4)/(4*a*c-b^2)/(x^2+1/2/c*(-4*a*c+b^2)^(1/2)+1/2*b/c)^2*d*x*b^2-9
/4*c^2/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)*2^(1/2)/a/((-b+(-4*
a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1
/2))*c)^(1/2))*b^3*d+3/16*c/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2
```

```
)*2^(1/2)/a^2/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/
2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^5*d-15/8*c/(16*a^2*c^2-8*
a*b^2*c+b^4)/(4*a*c-b^2)/(x^2+1/2/c*(-4*a*c+b^2)^(1/2)+1/2*b/c)^2
/a*d*x^3*(-4*a*c+b^2)^(1/2)*b^2+9/4*c^2/(16*a^2*c^2-8*a*b^2*c+b^4
)/(4*a*c-b^2)*2^(1/2)/a/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c
*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^3*d-3/4/(16*a^2*c^
2-8*a*b^2*c+b^4)/(4*a*c-b^2)/(x^2+1/2/c*(-4*a*c+b^2)^(1/2)+1/2*b/
c)^2*e*b^3-1/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)/(x^2+1/2/c*(-
4*a*c+b^2)^(1/2)+1/2*b/c)^2*e*(-4*a*c+b^2)^(1/2)*b^2+1/(16*a^2*c^
2-8*a*b^2*c+b^4)/(4*a*c-b^2)/(x^2+1/2*b/c-1/2/c*(-4*a*c+b^2)^(1/2
))^2*e*(-4*a*c+b^2)^(1/2)*b^2-3/4/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a
*c-b^2)/(x^2+1/2*b/c-1/2/c*(-4*a*c+b^2)^(1/2))^2*e*b^3+5/16/(16*a
^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)/(x^2+1/2/c*(-4*a*c+b^2)^(1/2)+1
/2*b/c)^2*d/a*x*(-4*a*c+b^2)^(1/2)*b^3-3/16/(16*a^2*c^2-8*a*b^2*c
+b^4)/(4*a*c-b^2)/(x^2+1/2*b/c-1/2/c*(-4*a*c+b^2)^(1/2))^2/a^2*d*
x^3*(-4*a*c+b^2)^(1/2)*b^4-5/16/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c
-b^2)/(x^2+1/2*b/c-1/2/c*(-4*a*c+b^2)^(1/2))^2*d/a*x*(-4*a*c+b^2)
^(1/2)*b^3+6*c^3/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)*2^(1/2)/(
(-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c
+b^2)^(1/2))*c)^(1/2))*b*d+21/2*c^3/(16*a^2*c^2-8*a*b^2*c+b^4)/(4
*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2
^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*(-4*a*c+b^2)^(1/2)*d+9/
4*c/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)/(x^2+1/2*b/c-1/2/c*(-4
*a*c+b^2)^(1/2))^2/a*d*x^3*b^3+5/4*c/(16*a^2*c^2-8*a*b^2*c+b^4)/(
4*a*c-b^2)/(x^2+1/2*b/c-1/2/c*(-4*a*c+b^2)^(1/2))^2*d*x*(-4*a*c+b
^2)^(1/2)*b+9/4*c/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)/(x^2+1/2
/c*(-4*a*c+b^2)^(1/2)+1/2*b/c)^2/a*d*x^3*b^3-5/4*c/(16*a^2*c^2-8*
a*b^2*c+b^4)/(4*a*c-b^2)/(x^2+1/2/c*(-4*a*c+b^2)^(1/2)+1/2*b/c)^2
*d*x*(-4*a*c+b^2)^(1/2)*b+21/2*c^3/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*
a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1
/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*(-4*a*c+b^2)^(1/2)*d-6*c^3/
(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(
1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2
))*b*d-3*c^2/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)*(-4*a*c+b^2)^
(1/2)*e*ln(a^2*(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b))
```

<u>Maxima</u> [F] time = 0., size = 0, normalized size = 0.

$$\frac{24\,a^2c^3ex^6 + 36\,a^2bc^2ex^4 + 3\left(b^3c^2 - 8\,abc^3\right)dx^7 + \left(6\,b^4c - 49\,ab^2c^2 + 28\,a^2c^3\right)dx^5 + \left(3\,b^5 - 20\,ab^3c - 4\,a^2bc^2\right)dx^3 + 8\left(a^2b\right.}{8\left((a^2b^4c^2 - 8\,a^3b^2c^3 + 16\,a^4c^4)x^8 + a^4b^4 - 8\,a^5b^2c + 16\,a^6c^2 + 2\,(a^2b^5c - 8\,a^3b^3c^2 + 16\,a^4bc^3)x^6 + (a^2b^6 - \right.}$$

$$-\frac{-3\int\frac{16\,a^2c^2ex+\left(b^3c-8\,abc^2\right)dx^2+\left(b^4-9\,ab^2c+28\,a^2c^2\right)d}{cx^4+bx^2+a}\,dx}{8\,(a^2b^4 - 8\,a^3b^2c + 16\,a^4c^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x + d)/(c*x^4 + b*x^2 + a)^3,x, algorithm="maxima")
```

```
[Out] 1/8*(24*a^2*c^3*e*x^6 + 36*a^2*b*c^2*e*x^4 + 3*(b^3*c^2 - 8*a*b*c
^3)*d*x^7 + (6*b^4*c - 49*a*b^2*c^2 + 28*a^2*c^3)*d*x^5 + (3*b^5
```

```
- 20*a*b^3*c - 4*a^2*b*c^2)*d*x^3 + 8*(a^2*b^2*c + 5*a^3*c^2)*e*x
^2 + (5*a*b^4 - 37*a^2*b^2*c + 44*a^3*c^2)*d*x - 2*(a^2*b^3 - 10*
a^3*b*c)*e)/((a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*x^8 + a^4
*b^4 - 8*a^5*b^2*c + 16*a^6*c^2 + 2*(a^2*b^5*c - 8*a^3*b^3*c^2 +
16*a^4*b*c^3)*x^6 + (a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3)*x^4 + 2*
(a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*x^2) - 3/8*integrate(-(16*
a^2*c^2*e*x + (b^3*c - 8*a*b*c^2)*d*x^2 + (b^4 - 9*a*b^2*c + 28*a
^2*c^2)*d)/(c*x^4 + b*x^2 + a), x)/(a^2*b^4 - 8*a^3*b^2*c + 16*a^
4*c^2)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x + d)/(c*x^4 + b*x^2 + a)^3,x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x+d)/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 34.8359, size = 1, normalized size = 0.

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x + d)/(c*x^4 + b*x^2 + a)^3,x, algorithm="giac")
```

```
[Out] Done
```

3.53 $\int \frac{d+ex+fx^2}{(a+bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=621

$$\frac{x\left(cx^2\left(20a^2cf+ab^2f-24abcd+3b^3d\right)+8a^2bcf+28a^2c^2d+ab^3f-25ab^2cd+3b^4d\right)}{8a^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}$$
$$+\frac{\sqrt{c}\left(-\frac{-52a^2bcf+168a^2c^2d+ab^3f-30ab^2cd+3b^4d}{\sqrt{b^2-4ac}}+20a^2cf+ab^2f-24abcd+3b^3d\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{8\sqrt{2}a^2\left(b^2-4ac\right)^2\sqrt{\sqrt{b^2-4ac}+b}}$$
$$+\frac{\sqrt{c}\left(4a^2c\left(5f\sqrt{b^2-4ac}+42cd\right)-ab^2\left(30cd-f\sqrt{b^2-4ac}\right)-4abc\left(6d\sqrt{b^2-4ac}+13af\right)+b^3\left(3d\sqrt{b^2-4ac}+af\right)+3}{8\sqrt{2}a^2\left(b^2-4ac\right)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}$$
$$-\frac{6c^2e\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/2}}+\frac{x\left(cx^2(bd-2af)-abf-2acd+b^2d\right)}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}$$
$$+\frac{3ce\left(b+2cx^2\right)}{2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}-\frac{e\left(b+2cx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}$$

```
[Out] -(e*(b + 2*c*x^2))/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (x*(
b^2*d - 2*a*c*d - a*b*f + c*(b*d - 2*a*f)*x^2))/(4*a*(b^2 - 4*a*c
)*(a + b*x^2 + c*x^4)^2) + (3*c*e*(b + 2*c*x^2))/(2*(b^2 - 4*a*c)
^2*(a + b*x^2 + c*x^4)) + (x*(3*b^4*d - 25*a*b^2*c*d + 28*a^2*c^2
*d + a*b^3*f + 8*a^2*b*c*f + c*(3*b^3*d - 24*a*b*c*d + a*b^2*f +
20*a^2*c*f)*x^2))/(8*a^2*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (
Sqrt[c]*(3*b^4*d + b^3*(3*Sqrt[b^2 - 4*a*c]*d + a*f) - 4*a*b*c*(6
*Sqrt[b^2 - 4*a*c]*d + 13*a*f) - a*b^2*(30*c*d - Sqrt[b^2 - 4*a*c
]*f) + 4*a^2*c*(42*c*d + 5*Sqrt[b^2 - 4*a*c]*f))*ArcTan[(Sqrt[2]*
Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a^2*(b^2 - 4*
a*c)^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[c]*(3*b^3*d - 24*
a*b*c*d + a*b^2*f + 20*a^2*c*f - (3*b^4*d - 30*a*b^2*c*d + 168*a^
2*c^2*d + a*b^3*f - 52*a^2*b*c*f)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt
[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a^2*(b^2
- 4*a*c)^2*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - (6*c^2*e*ArcTanh[(b + 2
*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(5/2)
```

Rubi [A] time = 10.4248, antiderivative size = 621, normalized size of antiderivative = 1., number of

steps used = 12, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.36$

$$\frac{x\left(cx^2\left(20a^2cf+ab^2f-24abcd+3b^3d\right)+8a^2bcf+28a^2c^2d+ab^3f-25ab^2cd+3b^4d\right)}{8a^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}$$
$$+\frac{\sqrt{c}\left(-\frac{-52a^2bcf+168a^2c^2d+ab^3f-30ab^2cd+3b^4d}{\sqrt{b^2-4ac}}+20a^2cf+ab^2f-24abcd+3b^3d\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{8\sqrt{2}a^2\left(b^2-4ac\right)^2\sqrt{\sqrt{b^2-4ac}+b}}$$
$$+\frac{\sqrt{c}\left(4a^2c\left(5f\sqrt{b^2-4ac}+42cd\right)-ab^2\left(30cd-f\sqrt{b^2-4ac}\right)-4abc\left(6d\sqrt{b^2-4ac}+13af\right)+b^3\left(3d\sqrt{b^2-4ac}+af\right)+3}{8\sqrt{2}a^2\left(b^2-4ac\right)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}$$
$$-\frac{6c^2e\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/2}}+\frac{x\left(cx^2(bd-2af)-abf-2acd+b^2d\right)}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}$$
$$+\frac{3ce\left(b+2cx^2\right)}{2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}-\frac{e\left(b+2cx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x + f*x^2)/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] -(e*(b + 2*c*x^2))/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (x*(
b^2*d - 2*a*c*d - a*b*f + c*(b*d - 2*a*f)*x^2))/(4*a*(b^2 - 4*a*c
)*(a + b*x^2 + c*x^4)^2) + (3*c*e*(b + 2*c*x^2))/(2*(b^2 - 4*a*c)
^2*(a + b*x^2 + c*x^4)) + (x*(3*b^4*d - 25*a*b^2*c*d + 28*a^2*c^2
*d + a*b^3*f + 8*a^2*b*c*f + c*(3*b^3*d - 24*a*b*c*d + a*b^2*f +
20*a^2*c*f)*x^2))/(8*a^2*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (
Sqrt[c]*(3*b^4*d + b^3*(3*Sqrt[b^2 - 4*a*c]*d + a*f) - 4*a*b*c*(6
*Sqrt[b^2 - 4*a*c]*d + 13*a*f) - a*b^2*(30*c*d - Sqrt[b^2 - 4*a*c
]*f) + 4*a^2*c*(42*c*d + 5*Sqrt[b^2 - 4*a*c]*f))*ArcTan[(Sqrt[2]*
Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a^2*(b^2 - 4*
a*c)^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[c]*(3*b^3*d - 24*
a*b*c*d + a*b^2*f + 20*a^2*c*f - (3*b^4*d - 30*a*b^2*c*d + 168*a^
2*c^2*d + a*b^3*f - 52*a^2*b*c*f)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt
[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a^2*(b^2
- 4*a*c)^2*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - (6*c^2*e*ArcTanh[(b + 2
*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(5/2)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**2+e*x+d)/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 6.6145, size = 683, normalized size = 1.1

$$\frac{12a^2bce + 8a^2bcfx + 28a^2c^2dx + 24a^2c^2ex^2 + 20a^2c^2fx^3 + ab^3fx - 25ab^2cdx + ab^2cfx^3 - 24abc^2dx^3 + 3b^4dx + 3b^3cdx^3}{8a^2(4ac-b^2)^2(a+bx^2+cx^4)}$$
$$+\frac{\sqrt{c}\left(20a^2cf\sqrt{b^2-4ac} - 52a^2bcf + 168a^2c^2d + ab^3f - 30ab^2cd - 24abcd\sqrt{b^2-4ac} + ab^2f\sqrt{b^2-4ac} + 3b^3d\sqrt{b^2-4ac} + \right.}{8\sqrt{2}a^2(b^2-4ac)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}$$
$$+\frac{\sqrt{c}\left(20a^2cf\sqrt{b^2-4ac} + 52a^2bcf - 168a^2c^2d - ab^3f + 30ab^2cd - 24abcd\sqrt{b^2-4ac} + ab^2f\sqrt{b^2-4ac} + 3b^3d\sqrt{b^2-4ac} - \right.}{8\sqrt{2}a^2(b^2-4ac)^{5/2}\sqrt{\sqrt{b^2-4ac}+b}}$$
$$+\frac{3c^2e\log\left(\sqrt{b^2-4ac}-b-2cx^2\right)}{(b^2-4ac)^{5/2}} - \frac{3c^2e\log\left(\sqrt{b^2-4ac}+b+2cx^2\right)}{(b^2-4ac)^{5/2}}$$
$$+\frac{abe + abfx + 2acdx + 2acex^2 + 2acfx^3 - b^2dx - bcdx^3}{4a(4ac-b^2)(a+bx^2+cx^4)^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x + f*x^2)/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (a*b*e - b^2*d*x + 2*a*c*d*x + a*b*f*x + 2*a*c*e*x^2 - b*c*d*x^3
+ 2*a*c*f*x^3)/(4*a*(-b^2 + 4*a*c)*(a + b*x^2 + c*x^4)^2) + (12*a
^2*b*c*e + 3*b^4*d*x - 25*a*b^2*c*d*x + 28*a^2*c^2*d*x + a*b^3*f*
x + 8*a^2*b*c*f*x + 24*a^2*c^2*e*x^2 + 3*b^3*c*d*x^3 - 24*a*b*c^2
*d*x^3 + a*b^2*c*f*x^3 + 20*a^2*c^2*f*x^3)/(8*a^2*(-b^2 + 4*a*c)^
2*(a + b*x^2 + c*x^4)) + (Sqrt[c]*(3*b^4*d - 30*a*b^2*c*d + 168*a
^2*c^2*d + 3*b^3*Sqrt[b^2 - 4*a*c]*d - 24*a*b*c*Sqrt[b^2 - 4*a*c]
*d + a*b^3*f - 52*a^2*b*c*f + a*b^2*Sqrt[b^2 - 4*a*c]*f + 20*a^2*
c*Sqrt[b^2 - 4*a*c]*f)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b
^2 - 4*a*c]]])/(8*Sqrt[2]*a^2*(b^2 - 4*a*c)^(5/2)*Sqrt[b - Sqrt[b
^2 - 4*a*c]]) + (Sqrt[c]*(-3*b^4*d + 30*a*b^2*c*d - 168*a^2*c^2*d
 + 3*b^3*Sqrt[b^2 - 4*a*c]*d - 24*a*b*c*Sqrt[b^2 - 4*a*c]*d - a*b
^3*f + 52*a^2*b*c*f + a*b^2*Sqrt[b^2 - 4*a*c]*f + 20*a^2*c*Sqrt[b
^2 - 4*a*c]*f)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a
*c]]])/(8*Sqrt[2]*a^2*(b^2 - 4*a*c)^(5/2)*Sqrt[b + Sqrt[b^2 - 4*a
*c]]) + (3*c^2*e*Log[-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2])/(b^2 - 4*
a*c)^(5/2) - (3*c^2*e*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2
- 4*a*c)^(5/2)
```

Maple [B] time = 0.398, size = 10809, normalized size = 17.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x^2+e*x+d)/(c*x^4+b*x^2+a)^3,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{24\,a^2c^3ex^6 + 36\,a^2bc^2ex^4 + \left(3\left(b^3c^2 - 8\,abc^3\right)d + \left(ab^2c^2 + 20\,a^2c^3\right)f\right)x^7 + \left(\left(6\,b^4c - 49\,ab^2c^2 + 28\,a^2c^3\right)d + 2\left(ab^3c + 14\,a^2b \cdots\right.\right.}{8\left(\left(a^2b^4c^2 - 8\,a^3b^2c^3 + 16\,a^4c^4\right)x^8 + a^4b^4 - 8\,a^5b^2c + 1 \cdots\right.} + \frac{\int \frac{48\,a^2c^2ex + \left(3\left(b^3c - 8\,abc^2\right)d + \left(ab^2c + 20\,a^2c^2\right)f\right)x^2 + 3\left(b^4 - 9\,ab^2c + 28\,a^2c^2\right)d + \left(ab^3 - 16\,a^2bc\right)f}{cx^4 + bx^2 + a}\,dx}{8\left(a^2b^4 - 8\,a^3b^2c + 16\,a^4c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2 + e*x + d)/(c*x^4 + b*x^2 + a)^3,x, algorithm="maxima")
```

```
[Out] 1/8*(24*a^2*c^3*e*x^6 + 36*a^2*b*c^2*e*x^4 + (3*(b^3*c^2 - 8*a*b*
c^3)*d + (a*b^2*c^2 + 20*a^2*c^3)*f)*x^7 + ((6*b^4*c - 49*a*b^2*c
^2 + 28*a^2*c^3)*d + 2*(a*b^3*c + 14*a^2*b*c^2)*f)*x^5 + 8*(a^2*b
^2*c + 5*a^3*c^2)*e*x^2 + ((3*b^5 - 20*a*b^3*c - 4*a^2*b*c^2)*d +
 (a*b^4 + 5*a^2*b^2*c + 36*a^3*c^2)*f)*x^3 - 2*(a^2*b^3 - 10*a^3*
b*c)*e + ((5*a*b^4 - 37*a^2*b^2*c + 44*a^3*c^2)*d - (a^2*b^3 - 16
*a^3*b*c)*f)*x)/((a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*x^8 +
 a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2 + 2*(a^2*b^5*c - 8*a^3*b^3*c^
2 + 16*a^4*b*c^3)*x^6 + (a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3)*x^4
+ 2*(a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*x^2) + 1/8*integrate((
48*a^2*c^2*e*x + (3*(b^3*c - 8*a*b*c^2)*d + (a*b^2*c + 20*a^2*c^2
)*f)*x^2 + 3*(b^4 - 9*a*b^2*c + 28*a^2*c^2)*d + (a*b^3 - 16*a^2*b
*c)*f)/(c*x^4 + b*x^2 + a), x)/(a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^
2)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2 + e*x + d)/(c*x^4 + b*x^2 + a)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x**2+e*x+d)/(c*x**4+b*x**2+a)**3,x)

[Out] Timed out
```

GIAC/XCAS [A] time = 50.9725, size = 1, normalized size = 0.

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^2 + e*x + d)/(c*x^4 + b*x^2 + a)^3,x, algorithm="giac")

[Out] Done
```

3.54 $\int \frac{d+ex+fx^2+gx^3}{(a+bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=646

$$\frac{x\left(cx^2\left(20a^2cf+ab^2f-24abcd+3b^3d\right)+8a^2bcf+28a^2c^2d+ab^3f-25ab^2cd+3b^4d\right)}{8a^2(b^2-4ac)^2(a+bx^2+cx^4)}$$
$$+\frac{\sqrt{c}\left(-\frac{-52a^2bcf+168a^2c^2d+ab^3f-30ab^2cd+3b^4d}{\sqrt{b^2-4ac}}+20a^2cf+ab^2f-24abcd+3b^3d\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{8\sqrt{2}a^2(b^2-4ac)^2\sqrt{\sqrt{b^2-4ac}+b}}$$
$$+\frac{\sqrt{c}\left(4a^2c\left(5f\sqrt{b^2-4ac}+42cd\right)-ab^2\left(30cd-f\sqrt{b^2-4ac}\right)-4abc\left(6d\sqrt{b^2-4ac}+13af\right)+b^3\left(3d\sqrt{b^2-4ac}+af\right)+3}{8\sqrt{2}a^2(b^2-4ac)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}$$
$$+\frac{x\left(cx^2(bd-2af)-abf-2acd+b^2d\right)}{4a(b^2-4ac)(a+bx^2+cx^4)^2}-\frac{3c(2ce-bg)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{5/2}}$$
$$+\frac{3\left(b+2cx^2\right)(2ce-bg)}{4(b^2-4ac)^2(a+bx^2+cx^4)}-\frac{-2ag+x^2(2ce-bg)+be}{4(b^2-4ac)(a+bx^2+cx^4)^2}$$

```
[Out] (x*(b^2*d - 2*a*c*d - a*b*f + c*(b*d - 2*a*f)*x^2))/(4*a*(b^2 - 4
*a*c)*(a + b*x^2 + c*x^4)^2) - (b*e - 2*a*g + (2*c*e - b*g)*x^2)/
(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (3*(2*c*e - b*g)*(b + 2
*c*x^2))/(4*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (x*(3*b^4*d -
25*a*b^2*c*d + 28*a^2*c^2*d + a*b^3*f + 8*a^2*b*c*f + c*(3*b^3*d
- 24*a*b*c*d + a*b^2*f + 20*a^2*c*f)*x^2))/(8*a^2*(b^2 - 4*a*c)^2
*(a + b*x^2 + c*x^4)) + (Sqrt[c]*(3*b^4*d + b^3*(3*Sqrt[b^2 - 4*a
*c]*d + a*f) - 4*a*b*c*(6*Sqrt[b^2 - 4*a*c]*d + 13*a*f) - a*b^2*(
30*c*d - Sqrt[b^2 - 4*a*c]*f) + 4*a^2*c*(42*c*d + 5*Sqrt[b^2 - 4*
a*c]*f))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])
/(8*Sqrt[2]*a^2*(b^2 - 4*a*c)^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]])
+ (Sqrt[c]*(3*b^3*d - 24*a*b*c*d + a*b^2*f + 20*a^2*c*f - (3*b^4*
d - 30*a*b^2*c*d + 168*a^2*c^2*d + a*b^3*f - 52*a^2*b*c*f)/Sqrt[b
^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c
]]])/(8*Sqrt[2]*a^2*(b^2 - 4*a*c)^2*Sqrt[b + Sqrt[b^2 - 4*a*c]])
- (3*c*(2*c*e - b*g)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b
^2 - 4*a*c)^(5/2)
```

Rubi [A] time = 8.79238, antiderivative size = 646, normalized size of antiderivative = 1., number of

steps used = 11, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$

$$\frac{x\left(cx^2\left(20a^2cf+ab^2f-24abcd+3b^3d\right)+8a^2bcf+28a^2c^2d+ab^3f-25ab^2cd+3b^4d\right)}{8a^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}$$
$$+\frac{\sqrt{c}\left(-\frac{-52a^2bcf+168a^2c^2d+ab^3f-30ab^2cd+3b^4d}{\sqrt{b^2-4ac}}+20a^2cf+ab^2f-24abcd+3b^3d\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{8\sqrt{2}a^2\left(b^2-4ac\right)^2\sqrt{\sqrt{b^2-4ac}+b}}$$
$$+\frac{\sqrt{c}\left(4a^2c\left(5f\sqrt{b^2-4ac}+42cd\right)-ab^2\left(30cd-f\sqrt{b^2-4ac}\right)-4abc\left(6d\sqrt{b^2-4ac}+13af\right)+b^3\left(3d\sqrt{b^2-4ac}+af\right)+3}{8\sqrt{2}a^2\left(b^2-4ac\right)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}$$
$$+\frac{x\left(cx^2(bd-2af)-abf-2acd+b^2d\right)}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}-\frac{3c(2ce-bg)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/2}}$$
$$+\frac{3\left(b+2cx^2\right)(2ce-bg)}{4\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}-\frac{-2ag+x^2(2ce-bg)+be}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x + f*x^2 + g*x^3)/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (x*(b^2*d - 2*a*c*d - a*b*f + c*(b*d - 2*a*f)*x^2))/(4*a*(b^2 - 4
*a*c)*(a + b*x^2 + c*x^4)^2) - (b*e - 2*a*g + (2*c*e - b*g)*x^2)/
(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (3*(2*c*e - b*g)*(b + 2
*c*x^2))/(4*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (x*(3*b^4*d -
25*a*b^2*c*d + 28*a^2*c^2*d + a*b^3*f + 8*a^2*b*c*f + c*(3*b^3*d
- 24*a*b*c*d + a*b^2*f + 20*a^2*c*f)*x^2))/(8*a^2*(b^2 - 4*a*c)^2
*(a + b*x^2 + c*x^4)) + (Sqrt[c]*(3*b^4*d + b^3*(3*Sqrt[b^2 - 4*a
*c]*d + a*f) - 4*a*b*c*(6*Sqrt[b^2 - 4*a*c]*d + 13*a*f) - a*b^2*(
30*c*d - Sqrt[b^2 - 4*a*c]*f) + 4*a^2*c*(42*c*d + 5*Sqrt[b^2 - 4*
a*c]*f))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])
/(8*Sqrt[2]*a^2*(b^2 - 4*a*c)^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]])
+ (Sqrt[c]*(3*b^3*d - 24*a*b*c*d + a*b^2*f + 20*a^2*c*f - (3*b^4*
d - 30*a*b^2*c*d + 168*a^2*c^2*d + a*b^3*f - 52*a^2*b*c*f)/Sqrt[b
^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c
]]])/(8*Sqrt[2]*a^2*(b^2 - 4*a*c)^2*Sqrt[b + Sqrt[b^2 - 4*a*c]])
- (3*c*(2*c*e - b*g)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b
^2 - 4*a*c)^(5/2)
```

Rubi in Sympy **[F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((g*x**3+f*x**2+e*x+d)/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 6.75643, size = 733, normalized size = 1.13

$$\frac{-2a^2g + abe + abfx - abgx^2 + 2acdx + 2acex^2 + 2acfx^3 - b^2dx - bcdx^3}{4a\left(4ac - b^2\right)\left(a + bx^2 + cx^4\right)^2}$$
$$+ \frac{-6a^2b^2g + 12a^2bce + 8a^2bcfx - 12a^2bcgx^2 + 28a^2c^2dx + 24a^2c^2ex^2 + 20a^2c^2fx^3 + ab^3fx - 25ab^2cdx + ab^2cfx^3 - 24abc}{8a^2\left(4ac - b^2\right)^2\left(a + bx^2 + cx^4\right)}$$
$$+ \frac{\sqrt{c}\left(20a^2cf\sqrt{b^2 - 4ac} - 52a^2bcf + 168a^2c^2d + ab^3f - 30ab^2cd - 24abcd\sqrt{b^2 - 4ac} + ab^2f\sqrt{b^2 - 4ac} + 3b^3d\sqrt{b^2 - 4ac} + \right.}{8\sqrt{2}a^2\left(b^2 - 4ac\right)^{5/2}\sqrt{b - \sqrt{b^2 - 4ac}}}$$
$$+ \frac{\sqrt{c}\left(20a^2cf\sqrt{b^2 - 4ac} + 52a^2bcf - 168a^2c^2d - ab^3f + 30ab^2cd - 24abcd\sqrt{b^2 - 4ac} + ab^2f\sqrt{b^2 - 4ac} + 3b^3d\sqrt{b^2 - 4ac} - \right.}{8\sqrt{2}a^2\left(b^2 - 4ac\right)^{5/2}\sqrt{\sqrt{b^2 - 4ac} + b}}$$
$$+ \frac{3c(2ce - bg)\log\left(\sqrt{b^2 - 4ac} - b - 2cx^2\right)}{2\left(b^2 - 4ac\right)^{5/2}} - \frac{3c(2ce - bg)\log\left(\sqrt{b^2 - 4ac} + b + 2cx^2\right)}{2\left(b^2 - 4ac\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x + f*x^2 + g*x^3)/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (a*b*e - 2*a^2*g - b^2*d*x + 2*a*c*d*x + a*b*f*x + 2*a*c*e*x^2 -
a*b*g*x^2 - b*c*d*x^3 + 2*a*c*f*x^3)/(4*a*(-b^2 + 4*a*c)*(a + b*x
^2 + c*x^4)^2) + (12*a^2*b*c*e - 6*a^2*b^2*g + 3*b^4*d*x - 25*a*b
^2*c*d*x + 28*a^2*c^2*d*x + a*b^3*f*x + 8*a^2*b*c*f*x + 24*a^2*c^
2*e*x^2 - 12*a^2*b*c*g*x^2 + 3*b^3*c*d*x^3 - 24*a*b*c^2*d*x^3 + a
*b^2*c*f*x^3 + 20*a^2*c^2*f*x^3)/(8*a^2*(-b^2 + 4*a*c)^2*(a + b*x
^2 + c*x^4)) + (Sqrt[c]*(3*b^4*d - 30*a*b^2*c*d + 168*a^2*c^2*d +
 3*b^3*Sqrt[b^2 - 4*a*c]*d - 24*a*b*c*Sqrt[b^2 - 4*a*c]*d + a*b^3
*f - 52*a^2*b*c*f + a*b^2*Sqrt[b^2 - 4*a*c]*f + 20*a^2*c*Sqrt[b^2
 - 4*a*c]*f)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c
]]])/(8*Sqrt[2]*a^2*(b^2 - 4*a*c)^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c
]]) + (Sqrt[c]*(-3*b^4*d + 30*a*b^2*c*d - 168*a^2*c^2*d + 3*b^3*S
qrt[b^2 - 4*a*c]*d - 24*a*b*c*Sqrt[b^2 - 4*a*c]*d - a*b^3*f + 52*
a^2*b*c*f + a*b^2*Sqrt[b^2 - 4*a*c]*f + 20*a^2*c*Sqrt[b^2 - 4*a*c
]*f)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(8*
Sqrt[2]*a^2*(b^2 - 4*a*c)^(5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + (3
*c*(2*c*e - b*g)*Log[-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2])/(2*(b^2 -
 4*a*c)^(5/2)) - (3*c*(2*c*e - b*g)*Log[b + Sqrt[b^2 - 4*a*c] + 2
*c*x^2])/(2*(b^2 - 4*a*c)^(5/2))
```

Maple [B] time = 0.393, size = 13757, normalized size = 21.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)^3,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(3\left(b^{3}c^{2}-8\,abc^{3}\right)d+\left(ab^{2}c^{2}+20\,a^{2}c^{3}\right)f\right)x^{7}+12\left(2\,a^{2}c^{3}e-a^{2}bc^{2}g\right)x^{6}+\left(\left(6\,b^{4}c-49\,ab^{2}c^{2}+28\,a^{2}c^{3}\right)d+2\left(ab^{3}c+14\,a^{2}bc\right.\right.}{8\left(\left(a^{2}b^{4}c^{2}-8\,a^{3}b^{2}\right.\right.}$$
$$+\frac{\int\frac{\left(3\left(b^{3}c-8\,abc^{2}\right)d+\left(ab^{2}c+20\,a^{2}c^{2}\right)f\right)x^{2}+3\left(b^{4}-9\,ab^{2}c+28\,a^{2}c^{2}\right)d+\left(ab^{3}-16\,a^{2}bc\right)f+24\left(2\,a^{2}c^{2}e-a^{2}bcg\right)x}{cx^{4}+bx^{2}+a}\,dx}{8\left(a^{2}b^{4}-8\,a^{3}b^{2}c+16\,a^{4}c^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((g*x^3 + f*x^2 + e*x + d)/(c*x^4 + b*x^2 + a)^3,x, algorithm="maxima")
```

```
[Out] 1/8*((3*(b^3*c^2 - 8*a*b*c^3)*d + (a*b^2*c^2 + 20*a^2*c^3)*f)*x^7
 + 12*(2*a^2*c^3*e - a^2*b*c^2*g)*x^6 + ((6*b^4*c - 49*a*b^2*c^2
+ 28*a^2*c^3)*d + 2*(a*b^3*c + 14*a^2*b*c^2)*f)*x^5 + 18*(2*a^2*b
*c^2*e - a^2*b^2*c*g)*x^4 + ((3*b^5 - 20*a*b^3*c - 4*a^2*b*c^2)*d
 + (a*b^4 + 5*a^2*b^2*c + 36*a^3*c^2)*f)*x^3 + 4*(2*(a^2*b^2*c +
5*a^3*c^2)*e - (a^2*b^3 + 5*a^3*b*c)*g)*x^2 - 2*(a^2*b^3 - 10*a^3
*b*c)*e - 2*(a^3*b^2 + 8*a^4*c)*g + ((5*a*b^4 - 37*a^2*b^2*c + 44
*a^3*c^2)*d - (a^2*b^3 - 16*a^3*b*c)*f)*x)/((a^2*b^4*c^2 - 8*a^3*
b^2*c^3 + 16*a^4*c^4)*x^8 + a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2 +
2*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*x^6 + (a^2*b^6 - 6*a
^3*b^4*c + 32*a^5*c^3)*x^4 + 2*(a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*
c^2)*x^2) + 1/8*integrate(((3*(b^3*c - 8*a*b*c^2)*d + (a*b^2*c +
20*a^2*c^2)*f)*x^2 + 3*(b^4 - 9*a*b^2*c + 28*a^2*c^2)*d + (a*b^3
- 16*a^2*b*c)*f + 24*(2*a^2*c^2*e - a^2*b*c*g)*x)/(c*x^4 + b*x^2
+ a), x)/(a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((g*x^3 + f*x^2 + e*x + d)/(c*x^4 + b*x^2 + a)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((g*x**3+f*x**2+e*x+d)/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((g*x^3 + f*x^2 + e*x + d)/(c*x^4 + b*x^2 + a)^3,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.55 $\int \frac{d+ex+fx^2+gx^3+hx^4}{(a+bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=679

$$\frac{x\left(cx^2\left(20a^2cf+ab^2f-12ab(ah+2cd)+3b^3d\right)+8a^2bcf+4a^2c(ah+7cd)+ab^3f-ab^2(7ah+25cd)+3b^4d\right)}{8a^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}$$
$$+\frac{\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{-52a^2bcf+24a^2c(ah+7cd)+ab^3f-6ab^2(5cd-3ah)+3b^4d}{\sqrt{b^2-4ac}}+20a^2cf+ab^2f-12ab(ah+2cd)+3b^3d\right)}{8\sqrt{2}a^2\left(b^2-4ac\right)^2\sqrt{b-\sqrt{b^2-4ac}}}$$
$$+\frac{\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-\frac{-52a^2bcf+24a^2c(ah+7cd)+ab^3f-6ab^2(5cd-3ah)+3b^4d}{\sqrt{b^2-4ac}}+20a^2cf+ab^2f-12ab(ah+2cd)+3b^3d\right)}{8\sqrt{2}a^2\left(b^2-4ac\right)^2\sqrt{\sqrt{b^2-4ac}+b}}$$
$$+\frac{x\left(x^2(abh-2acf+bcd)-abf-2a(cd-ah)+b^2d\right)}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}-\frac{3c(2ce-bg)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/2}}$$
$$+\frac{3\left(b+2cx^2\right)(2ce-bg)}{4\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}-\frac{-2ag+x^2(2ce-bg)+be}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}$$

```
[Out] -(b*e - 2*a*g + (2*c*e - b*g)*x^2)/(4*(b^2 - 4*a*c)*(a + b*x^2 +
c*x^4)^2) + (x*(b^2*d - a*b*f - 2*a*(c*d - a*h) + (b*c*d - 2*a*c*
f + a*b*h)*x^2))/(4*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (3*(
2*c*e - b*g)*(b + 2*c*x^2))/(4*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4
)) + (x*(3*b^4*d + a*b^3*f + 8*a^2*b*c*f + 4*a^2*c*(7*c*d + a*h)
- a*b^2*(25*c*d + 7*a*h) + c*(3*b^3*d + a*b^2*f + 20*a^2*c*f - 12
*a*b*(2*c*d + a*h))*x^2))/(8*a^2*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x
^4)) + (Sqrt[c]*(3*b^3*d + a*b^2*f + 20*a^2*c*f - 12*a*b*(2*c*d +
 a*h) + (3*b^4*d + a*b^3*f - 52*a^2*b*c*f - 6*a*b^2*(5*c*d - 3*a*
h) + 24*a^2*c*(7*c*d + a*h))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*S
qrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a^2*(b^2 - 4*a
*c)^2*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[c]*(3*b^3*d + a*b^2*f
+ 20*a^2*c*f - 12*a*b*(2*c*d + a*h) - (3*b^4*d + a*b^3*f - 52*a^2
*b*c*f - 6*a*b^2*(5*c*d - 3*a*h) + 24*a^2*c*(7*c*d + a*h))/Sqrt[b
^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c
]]])/(8*Sqrt[2]*a^2*(b^2 - 4*a*c)^2*Sqrt[b + Sqrt[b^2 - 4*a*c]])
- (3*c*(2*c*e - b*g)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b
^2 - 4*a*c)^(5/2)
```

Rubi [A] time = 10.0953, antiderivative size = 679, normalized size of antiderivative = 1., number of

steps used = 11, number of rules used = 10, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$

$$\frac{x\left(cx^2\left(20a^2cf+ab^2f-12ab(ah+2cd)+3b^3d\right)+8a^2bcf+4a^2c(ah+7cd)+ab^3f-ab^2(7ah+25cd)+3b^4d\right)}{8a^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}$$
$$+\frac{\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{-52a^2bcf+24a^2c(ah+7cd)+ab^3f-6ab^2(5cd-3ah)+3b^4d}{\sqrt{b^2-4ac}}+20a^2cf+ab^2f-12ab(ah+2cd)+3b^3d\right)}{8\sqrt{2}a^2\left(b^2-4ac\right)^2\sqrt{b-\sqrt{b^2-4ac}}}$$
$$+\frac{\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-\frac{-52a^2bcf+24a^2c(ah+7cd)+ab^3f-6ab^2(5cd-3ah)+3b^4d}{\sqrt{b^2-4ac}}+20a^2cf+ab^2f-12ab(ah+2cd)+3b^3d\right)}{8\sqrt{2}a^2\left(b^2-4ac\right)^2\sqrt{\sqrt{b^2-4ac}+b}}$$
$$+\frac{x\left(x^2(abh-2acf+bcd)-abf-2a(cd-ah)+b^2d\right)}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}-\frac{3c(2ce-bg)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/2}}$$
$$+\frac{3\left(b+2cx^2\right)(2ce-bg)}{4\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}-\frac{-2ag+x^2(2ce-bg)+be}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x + f*x^2 + g*x^3 + h*x^4)/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] -(b*e - 2*a*g + (2*c*e - b*g)*x^2)/(4*(b^2 - 4*a*c)*(a + b*x^2 +
c*x^4)^2) + (x*(b^2*d - a*b*f - 2*a*(c*d - a*h) + (b*c*d - 2*a*c*
f + a*b*h)*x^2))/(4*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (3*(
2*c*e - b*g)*(b + 2*c*x^2))/(4*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4
)) + (x*(3*b^4*d + a*b^3*f + 8*a^2*b*c*f + 4*a^2*c*(7*c*d + a*h)
- a*b^2*(25*c*d + 7*a*h) + c*(3*b^3*d + a*b^2*f + 20*a^2*c*f - 12
*a*b*(2*c*d + a*h))*x^2))/(8*a^2*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x
^4)) + (Sqrt[c]*(3*b^3*d + a*b^2*f + 20*a^2*c*f - 12*a*b*(2*c*d +
 a*h) + (3*b^4*d + a*b^3*f - 52*a^2*b*c*f - 6*a*b^2*(5*c*d - 3*a*
h) + 24*a^2*c*(7*c*d + a*h))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*S
qrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a^2*(b^2 - 4*a
*c)^2*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[c]*(3*b^3*d + a*b^2*f
+ 20*a^2*c*f - 12*a*b*(2*c*d + a*h) - (3*b^4*d + a*b^3*f - 52*a^2
*b*c*f - 6*a*b^2*(5*c*d - 3*a*h) + 24*a^2*c*(7*c*d + a*h))/Sqrt[b
^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c
]]])/(8*Sqrt[2]*a^2*(b^2 - 4*a*c)^2*Sqrt[b + Sqrt[b^2 - 4*a*c]])
- (3*c*(2*c*e - b*g)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b
^2 - 4*a*c)^(5/2)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((h*x**4+g*x**3+f*x**2+e*x+d)/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 7.07484, size = 845, normalized size = 1.24

$$-\frac{bcdx^3-2acfx^3+abhx^3-2acex^2+abgx^2+b^2dx-2acdx-abfx+2a^2hx-abe+2a^2g}{4a\left(4ac-b^2\right)\left(cx^4+bx^2+a\right)^2}$$
$$+\frac{\sqrt{c}\left(3db^4+3\sqrt{b^2-4ac}db^3+afb^3-30acdb^2+a\sqrt{b^2-4ac}fb^2+18a^2hb^2-24ac\sqrt{b^2-4ac}db-52a^2cfb-12a^2\sqrt{b^2-4a\right.}{8\sqrt{2}a^2\left(b^2-4ac\right)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}$$
$$+\frac{\sqrt{c}\left(-3db^4+3\sqrt{b^2-4ac}db^3-afb^3+30acdb^2+a\sqrt{b^2-4ac}fb^2-18a^2hb^2-24ac\sqrt{b^2-4ac}db+52a^2cfb-12a^2\sqrt{b^2-\right.}{8\sqrt{2}a^2\left(b^2-4ac\right)^{5/2}\sqrt{b+\sqrt{b^2-4ac}}}$$
$$+\frac{3c(2ce-bg)\log\left(-2cx^2-b+\sqrt{b^2-4ac}\right)}{2\left(b^2-4ac\right)^{5/2}}-\frac{3c(2ce-bg)\log\left(2cx^2+b+\sqrt{b^2-4ac}\right)}{2\left(b^2-4ac\right)^{5/2}}$$
$$+\frac{3dxb^4+3cdx^3b^3+afxb^3+acfx^3b^2-6a^2gb^2-25acdxb^2-7a^2hxb^2-24ac^2dx^3b-12a^2chx^3b-12a^2cgx^2b+12a^2ceb+}{8a^2\left(4ac-b^2\right)^2\left(cx^4+bx^2+a\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2 + g*x^3 + h*x^4)/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] -(-(a*b*e) + 2*a^2*g + b^2*d*x - 2*a*c*d*x - a*b*f*x + 2*a^2*h*x
- 2*a*c*e*x^2 + a*b*g*x^2 + b*c*d*x^3 - 2*a*c*f*x^3 + a*b*h*x^3)/
(4*a*(-b^2 + 4*a*c)*(a + b*x^2 + c*x^4)^2) + (12*a^2*b*c*e - 6*a^
2*b^2*g + 3*b^4*d*x - 25*a*b^2*c*d*x + 28*a^2*c^2*d*x + a*b^3*f*x
 + 8*a^2*b*c*f*x - 7*a^2*b^2*h*x + 4*a^3*c*h*x + 24*a^2*c^2*e*x^2
 - 12*a^2*b*c*g*x^2 + 3*b^3*c*d*x^3 - 24*a*b*c^2*d*x^3 + a*b^2*c*
f*x^3 + 20*a^2*c^2*f*x^3 - 12*a^2*b*c*h*x^3)/(8*a^2*(-b^2 + 4*a*c
)^2*(a + b*x^2 + c*x^4)) + (Sqrt[c]*(3*b^4*d - 30*a*b^2*c*d + 168
*a^2*c^2*d + 3*b^3*Sqrt[b^2 - 4*a*c]*d - 24*a*b*c*Sqrt[b^2 - 4*a*
c]*d + a*b^3*f - 52*a^2*b*c*f + a*b^2*Sqrt[b^2 - 4*a*c]*f + 20*a^
2*c*Sqrt[b^2 - 4*a*c]*f + 18*a^2*b^2*h + 24*a^3*c*h - 12*a^2*b*Sq
rt[b^2 - 4*a*c]*h)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 -
 4*a*c]]])/(8*Sqrt[2]*a^2*(b^2 - 4*a*c)^(5/2)*Sqrt[b - Sqrt[b^2 -
 4*a*c]]) + (Sqrt[c]*(-3*b^4*d + 30*a*b^2*c*d - 168*a^2*c^2*d + 3
*b^3*Sqrt[b^2 - 4*a*c]*d - 24*a*b*c*Sqrt[b^2 - 4*a*c]*d - a*b^3*f
 + 52*a^2*b*c*f + a*b^2*Sqrt[b^2 - 4*a*c]*f + 20*a^2*c*Sqrt[b^2 -
 4*a*c]*f - 18*a^2*b^2*h - 24*a^3*c*h - 12*a^2*b*Sqrt[b^2 - 4*a*c
]*h)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(8*
Sqrt[2]*a^2*(b^2 - 4*a*c)^(5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + (3
*c*(2*c*e - b*g)*Log[-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2])/(2*(b^2 -
 4*a*c)^(5/2)) - (3*c*(2*c*e - b*g)*Log[b + Sqrt[b^2 - 4*a*c] + 2
```

```
*c*x^2])/(2*(b^2 - 4*a*c)^(5/2))
```

Maple [B] time = 0.314, size = 19742, normalized size = 29.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)^3,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((h*x^4 + g*x^3 + f*x^2 + e*x + d)/(c*x^4 + b*x^2 + a)^3,x, algorithm="ma
```

```
[Out] -1/8*((12*a^2*b*c^2*h - 3*(b^3*c^2 - 8*a*b*c^3)*d - (a*b^2*c^2 +
20*a^2*c^3)*f)*x^7 - 12*(2*a^2*c^3*e - a^2*b*c^2*g)*x^6 - ((6*b^4
*c - 49*a*b^2*c^2 + 28*a^2*c^3)*d + 2*(a*b^3*c + 14*a^2*b*c^2)*f
- (19*a^2*b^2*c - 4*a^3*c^2)*h)*x^5 - 18*(2*a^2*b*c^2*e - a^2*b^2
*c*g)*x^4 - ((3*b^5 - 20*a*b^3*c - 4*a^2*b*c^2)*d + (a*b^4 + 5*a^
2*b^2*c + 36*a^3*c^2)*f - (5*a^2*b^3 + 16*a^3*b*c)*h)*x^3 - 4*(2*
(a^2*b^2*c + 5*a^3*c^2)*e - (a^2*b^3 + 5*a^3*b*c)*g)*x^2 + 2*(a^2
*b^3 - 10*a^3*b*c)*e + 2*(a^3*b^2 + 8*a^4*c)*g - ((5*a*b^4 - 37*a
^2*b^2*c + 44*a^3*c^2)*d - (a^2*b^3 - 16*a^3*b*c)*f - 3*(a^3*b^2
+ 4*a^4*c)*h)*x)/((a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*x^8
+ a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2 + 2*(a^2*b^5*c - 8*a^3*b^3*c
^2 + 16*a^4*b*c^3)*x^6 + (a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3)*x^4
 + 2*(a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*x^2) - 1/8*integrate(
((12*a^2*b*c*h - 3*(b^3*c - 8*a*b*c^2)*d - (a*b^2*c + 20*a^2*c^2)
*f)*x^2 - 3*(b^4 - 9*a*b^2*c + 28*a^2*c^2)*d - (a*b^3 - 16*a^2*b*
c)*f - 3*(a^2*b^2 + 4*a^3*c)*h - 24*(2*a^2*c^2*e - a^2*b*c*g)*x)/
(c*x^4 + b*x^2 + a), x)/(a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((h*x^4 + g*x^3 + f*x^2 + e*x + d)/(c*x^4 + b*x^2 + a)^3,x, algorithm="fr
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((h*x**4+g*x**3+f*x**2+e*x+d)/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((h*x^4 + g*x^3 + f*x^2 + e*x + d)/(c*x^4 + b*x^2 + a)^3,x, algorithm="gi
```

```
[Out] Exception raised: NotImplementedError
```

3.56 $\int \frac{d+ex+fx^2+gx^3+hx^4+ix^5}{(a+bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=728

$$\frac{x\left(cx^2\left(20a^2cf+ab^2f-12ab(ah+2cd)+3b^3d\right)+8a^2bcf+4a^2c(ah+7cd)+ab^3f-ab^2(7ah+25cd)+3b^4d\right)}{8a^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}$$
$$+\frac{\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{-52a^2bcf+24a^2c(ah+7cd)+ab^3f-6ab^2(5cd-3ah)+3b^4d}{\sqrt{b^2-4ac}}+20a^2cf+ab^2f-12ab(ah+2cd)+3b^3d\right)}{8\sqrt{2}a^2\left(b^2-4ac\right)^2\sqrt{b-\sqrt{b^2-4ac}}}$$
$$+\frac{\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-\frac{-52a^2bcf+24a^2c(ah+7cd)+ab^3f-6ab^2(5cd-3ah)+3b^4d}{\sqrt{b^2-4ac}}+20a^2cf+ab^2f-12ab(ah+2cd)+3b^3d\right)}{8\sqrt{2}a^2\left(b^2-4ac\right)^2\sqrt{\sqrt{b^2-4ac}+b}}$$
$$-\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)\left(2aci+b^2i-3bcg+6c^2e\right)}{\left(b^2-4ac\right)^{5/2}}+\frac{x^2\left(-\left(-2aci+b^2i-bcg+2c^2e\right)\right)-b(ai+ce)+2acg}{4c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}$$
$$+\frac{x\left(x^2(abh-2acf+bcd)-abf-2a(cd-ah)+b^2d\right)}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{\left(b+2cx^2\right)\left(2ai+\frac{b^2i}{c}-3bg+6ce\right)}{4\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}$$

```
[Out] (x*(b^2*d - a*b*f - 2*a*(c*d - a*h) + (b*c*d - 2*a*c*f + a*b*h)*x
^2))/(4*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (2*a*c*g - b*(c*
e + a*i) - (2*c^2*e - b*c*g + b^2*i - 2*a*c*i)*x^2)/(4*c*(b^2 - 4
*a*c)*(a + b*x^2 + c*x^4)^2) + ((6*c*e - 3*b*g + 2*a*i + (b^2*i)/
c)*(b + 2*c*x^2))/(4*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (x*(3
*b^4*d + a*b^3*f + 8*a^2*b*c*f + 4*a^2*c*(7*c*d + a*h) - a*b^2*(2
5*c*d + 7*a*h) + c*(3*b^3*d + a*b^2*f + 20*a^2*c*f - 12*a*b*(2*c*
d + a*h))*x^2))/(8*a^2*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (Sq
rt[c]*(3*b^3*d + a*b^2*f + 20*a^2*c*f - 12*a*b*(2*c*d + a*h) + (3
*b^4*d + a*b^3*f - 52*a^2*b*c*f - 6*a*b^2*(5*c*d - 3*a*h) + 24*a^
2*c*(7*c*d + a*h))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/
Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a^2*(b^2 - 4*a*c)^2*Sqrt
[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[c]*(3*b^3*d + a*b^2*f + 20*a^2*c
*f - 12*a*b*(2*c*d + a*h) - (3*b^4*d + a*b^3*f - 52*a^2*b*c*f - 6
*a*b^2*(5*c*d - 3*a*h) + 24*a^2*c*(7*c*d + a*h))/Sqrt[b^2 - 4*a*c
])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(8*Sq
rt[2]*a^2*(b^2 - 4*a*c)^2*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - ((6*c^2*
e - 3*b*c*g + b^2*i + 2*a*c*i)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4
*a*c]])/(b^2 - 4*a*c)^(5/2)
```

Rubi [A] time = 8.18985, antiderivative size = 728, normalized size of antiderivative = 1., number of

steps used = 12, number of rules used = 11, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.275$

$$\frac{x\left(cx^2\left(20a^2cf+ab^2f-12ab(ah+2cd)+3b^3d\right)+8a^2bcf+4a^2c(ah+7cd)+ab^3f-ab^2(7ah+25cd)+3b^4d\right)}{8a^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}$$
$$+\frac{\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{-52a^2bcf+24a^2c(ah+7cd)+ab^3f-6ab^2(5cd-3ah)+3b^4d}{\sqrt{b^2-4ac}}+20a^2cf+ab^2f-12ab(ah+2cd)+3b^3d\right)}{8\sqrt{2}a^2\left(b^2-4ac\right)^2\sqrt{b-\sqrt{b^2-4ac}}}$$
$$+\frac{\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-\frac{-52a^2bcf+24a^2c(ah+7cd)+ab^3f-6ab^2(5cd-3ah)+3b^4d}{\sqrt{b^2-4ac}}+20a^2cf+ab^2f-12ab(ah+2cd)+3b^3d\right)}{8\sqrt{2}a^2\left(b^2-4ac\right)^2\sqrt{\sqrt{b^2-4ac}+b}}$$
$$-\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)\left(2aci+b^2i-3bcg+6c^2e\right)}{\left(b^2-4ac\right)^{5/2}}+\frac{x^2\left(-\left(-2aci+b^2i-bcg+2c^2e\right)\right)-b(ai+ce)+2acg}{4c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}$$
$$+\frac{x\left(x^2(abh-2acf+bcd)-abf-2a(cd-ah)+b^2d\right)}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{\left(b+2cx^2\right)\left(2ai+\frac{b^2i}{c}-3bg+6ce\right)}{4\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5)/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (x*(b^2*d - a*b*f - 2*a*(c*d - a*h) + (b*c*d - 2*a*c*f + a*b*h)*x
^2))/(4*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (2*a*c*g - b*(c*
e + a*i) - (2*c^2*e - b*c*g + b^2*i - 2*a*c*i)*x^2)/(4*c*(b^2 - 4
*a*c)*(a + b*x^2 + c*x^4)^2) + ((6*c*e - 3*b*g + 2*a*i + (b^2*i)/
c)*(b + 2*c*x^2))/(4*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (x*(3
*b^4*d + a*b^3*f + 8*a^2*b*c*f + 4*a^2*c*(7*c*d + a*h) - a*b^2*(2
5*c*d + 7*a*h) + c*(3*b^3*d + a*b^2*f + 20*a^2*c*f - 12*a*b*(2*c*
d + a*h))*x^2))/(8*a^2*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (Sq
rt[c]*(3*b^3*d + a*b^2*f + 20*a^2*c*f - 12*a*b*(2*c*d + a*h) + (3
*b^4*d + a*b^3*f - 52*a^2*b*c*f - 6*a*b^2*(5*c*d - 3*a*h) + 24*a^
2*c*(7*c*d + a*h))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/
Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a^2*(b^2 - 4*a*c)^2*Sqrt
[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[c]*(3*b^3*d + a*b^2*f + 20*a^2*c
*f - 12*a*b*(2*c*d + a*h) - (3*b^4*d + a*b^3*f - 52*a^2*b*c*f - 6
*a*b^2*(5*c*d - 3*a*h) + 24*a^2*c*(7*c*d + a*h))/Sqrt[b^2 - 4*a*c
])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(8*Sq
rt[2]*a^2*(b^2 - 4*a*c)^2*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - ((6*c^2*
e - 3*b*c*g + b^2*i + 2*a*c*i)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4
*a*c]])/(b^2 - 4*a*c)^(5/2)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((i*x**5+h*x**4+g*x**3+f*x**2+e*x+d)/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 7.39773, size = 980, normalized size = 1.35

$$\frac{-bc^2dx^3+2ac^2fx^3-abchx^3+2ac^2ex^2-abcgx^2+ab^2ix^2-2a^2cix^2+2ac^2dx-b^2cdx+abcfx-2a^2chx+abce-2a^2cg}{4ac\left(4ac-b^2\right)\left(cx^4+bx^2+a\right)^2}$$
$$+\frac{\sqrt{c}\left(3db^4+3\sqrt{b^2-4ac}db^3+afb^3-30acdb^2+a\sqrt{b^2-4ac}fb^2+18a^2hb^2-24ac\sqrt{b^2-4ac}db-52a^2cfb-12a^2\sqrt{b^2-4a}\right.}{8\sqrt{2}a^2\left(b^2-4ac\right)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}$$
$$+\frac{\sqrt{c}\left(-3db^4+3\sqrt{b^2-4ac}db^3-afb^3+30acdb^2+a\sqrt{b^2-4ac}fb^2-18a^2hb^2-24ac\sqrt{b^2-4ac}db+52a^2cfb-12a^2\sqrt{b^2-}\right.}{8\sqrt{2}a^2\left(b^2-4ac\right)^{5/2}\sqrt{b+\sqrt{b^2-4ac}}}$$
$$+\frac{\left(ib^2-3cgb+6c^2e+2aci\right)\log\left(-2cx^2-b+\sqrt{b^2-4ac}\right)}{2\left(b^2-4ac\right)^{5/2}}$$
$$+\frac{\left(-ib^2+3cgb-6c^2e-2aci\right)\log\left(2cx^2+b+\sqrt{b^2-4ac}\right)}{2\left(b^2-4ac\right)^{5/2}}$$
$$+\frac{3cdxb^4+3c^2dx^3b^3+2a^2ib^3+acfxb^3+ac^2fx^3b^2+4a^2cix^2b^2-6a^2cgb^2-25ac^2dxb^2-7a^2chxb^2-24ac^3dx^3b-12a^2c^2h}{8a^2c\left(4ac-b^2\right)^2\left(cx^4\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5)/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (a*b*c*e - 2*a^2*c*g + a^2*b*i - b^2*c*d*x + 2*a*c^2*d*x + a*b*c*
f*x - 2*a^2*c*h*x + 2*a*c^2*e*x^2 - a*b*c*g*x^2 + a*b^2*i*x^2 - 2
*a^2*c*i*x^2 - b*c^2*d*x^3 + 2*a*c^2*f*x^3 - a*b*c*h*x^3)/(4*a*c*
(-b^2 + 4*a*c)*(a + b*x^2 + c*x^4)^2) + (12*a^2*b*c^2*e - 6*a^2*b
^2*c*g + 2*a^2*b^3*i + 4*a^3*b*c*i + 3*b^4*c*d*x - 25*a*b^2*c^2*d
*x + 28*a^2*c^3*d*x + a*b^3*c*f*x + 8*a^2*b*c^2*f*x - 7*a^2*b^2*c
*h*x + 4*a^3*c^2*h*x + 24*a^2*c^3*e*x^2 - 12*a^2*b*c^2*g*x^2 + 4*
a^2*b^2*c*i*x^2 + 8*a^3*c^2*i*x^2 + 3*b^3*c^2*d*x^3 - 24*a*b*c^3*
d*x^3 + a*b^2*c^2*f*x^3 + 20*a^2*c^3*f*x^3 - 12*a^2*b*c^2*h*x^3)/
(8*a^2*c*(-b^2 + 4*a*c)^2*(a + b*x^2 + c*x^4)) + (Sqrt[c]*(3*b^4*
d - 30*a*b^2*c*d + 168*a^2*c^2*d + 3*b^3*Sqrt[b^2 - 4*a*c]*d - 24
*a*b*c*Sqrt[b^2 - 4*a*c]*d + a*b^3*f - 52*a^2*b*c*f + a*b^2*Sqrt[
b^2 - 4*a*c]*f + 20*a^2*c*Sqrt[b^2 - 4*a*c]*f + 18*a^2*b^2*h + 24
*a^3*c*h - 12*a^2*b*Sqrt[b^2 - 4*a*c]*h)*ArcTan[(Sqrt[2]*Sqrt[c]*
x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a^2*(b^2 - 4*a*c)^(5/
2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[c]*(-3*b^4*d + 30*a*b^2*c
*d - 168*a^2*c^2*d + 3*b^3*Sqrt[b^2 - 4*a*c]*d - 24*a*b*c*Sqrt[b^
2 - 4*a*c]*d - a*b^3*f + 52*a^2*b*c*f + a*b^2*Sqrt[b^2 - 4*a*c]*f
```

```
 + 20*a^2*c*Sqrt[b^2 - 4*a*c]*f - 18*a^2*b^2*h - 24*a^3*c*h - 12*
a^2*b*Sqrt[b^2 - 4*a*c]*h)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sq
rt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a^2*(b^2 - 4*a*c)^(5/2)*Sqrt[b + Sq
rt[b^2 - 4*a*c]]) + ((6*c^2*e - 3*b*c*g + b^2*i + 2*a*c*i)*Log[-b
 + Sqrt[b^2 - 4*a*c] - 2*c*x^2])/(2*(b^2 - 4*a*c)^(5/2)) + ((-6*c
^2*e + 3*b*c*g - b^2*i - 2*a*c*i)*Log[b + Sqrt[b^2 - 4*a*c] + 2*c
*x^2])/(2*(b^2 - 4*a*c)^(5/2))
```

Maple [B] time = 0.143, size = 21161, normalized size = 29.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)^3,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^5 + h*x^4 + g*x^3 + f*x^2 + e*x + d)/(c*x^4 + b*x^2 + a)^3,x, algori
```

```
[Out] -1/8*((12*a^2*b*c^2*h - 3*(b^3*c^2 - 8*a*b*c^3)*d - (a*b^2*c^2 +
20*a^2*c^3)*f)*x^7 - 4*(6*a^2*c^3*e - 3*a^2*b*c^2*g + (a^2*b^2*c
+ 2*a^3*c^2)*i)*x^6 - 12*a^4*b*i - ((6*b^4*c - 49*a*b^2*c^2 + 28*
a^2*c^3)*d + 2*(a*b^3*c + 14*a^2*b*c^2)*f - (19*a^2*b^2*c - 4*a^3
*c^2)*h)*x^5 - 6*(6*a^2*b*c^2*e - 3*a^2*b^2*c*g + (a^2*b^3 + 2*a^
3*b*c)*i)*x^4 - ((3*b^5 - 20*a*b^3*c - 4*a^2*b*c^2)*d + (a*b^4 +
5*a^2*b^2*c + 36*a^3*c^2)*f - (5*a^2*b^3 + 16*a^3*b*c)*h)*x^3 - 4
*(2*(a^2*b^2*c + 5*a^3*c^2)*e - (a^2*b^3 + 5*a^3*b*c)*g + (5*a^3*
b^2 - 2*a^4*c)*i)*x^2 + 2*(a^2*b^3 - 10*a^3*b*c)*e + 2*(a^3*b^2 +
 8*a^4*c)*g - ((5*a*b^4 - 37*a^2*b^2*c + 44*a^3*c^2)*d - (a^2*b^3
 - 16*a^3*b*c)*f - 3*(a^3*b^2 + 4*a^4*c)*h)*x)/((a^2*b^4*c^2 - 8*
a^3*b^2*c^3 + 16*a^4*c^4)*x^8 + a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^
2 + 2*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*x^6 + (a^2*b^6 -
 6*a^3*b^4*c + 32*a^5*c^3)*x^4 + 2*(a^3*b^5 - 8*a^4*b^3*c + 16*a^
5*b*c^2)*x^2) - 1/8*integrate((((12*a^2*b*c*h - 3*(b^3*c - 8*a*b*c
^2)*d - (a*b^2*c + 20*a^2*c^2)*f)*x^2 - 3*(b^4 - 9*a*b^2*c + 28*a
^2*c^2)*d - (a*b^3 - 16*a^2*b*c)*f - 3*(a^2*b^2 + 4*a^3*c)*h - 8*
(6*a^2*c^2*e - 3*a^2*b*c*g + (a^2*b^2 + 2*a^3*c)*i)*x)/(c*x^4 + b
```

```
*x^2 + a), x)/(a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^5 + h*x^4 + g*x^3 + f*x^2 + e*x + d)/(c*x^4 + b*x^2 + a)^3,x, algori
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x**5+h*x**4+g*x**3+f*x**2+e*x+d)/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 19.7113, size = 1, normalized size = 0.

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^5 + h*x^4 + g*x^3 + f*x^2 + e*x + d)/(c*x^4 + b*x^2 + a)^3,x, algori
```

```
[Out] Done
```

3.57 $\int \frac{d+ex+fx^2+gx^3+hx^4+jx^5+kx^6+lx^7+mx^8}{(a+bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=1150

result too large to display

```
[Out] -(b*c*(c*e + a*j) - a*b^2*l - 2*a*c*(c*g - a*l) + (2*c^3*e - c^2*
(b*g + 2*a*j) - b^3*l + b*c*(b*j + 3*a*l))*x^2)/(4*c^2*(b^2 - 4*a
*c)*(a + b*x^2 + c*x^4)^2) - (x*(a*b*c*(c*f + a*k) - b^2*(c^2*d +
 a^2*m) + 2*a*c*(c^2*d - a*c*h + a^2*m) + (a*b^2*c*k + 2*a*c^2*(c
*f - a*k) - a*b^3*m - b*c*(c^2*d + a*c*h - 3*a^2*m))*x^2))/(4*a*c
^2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + ((b^3*j)/c + 2*b*(3*c*e
 + a*j) - 16*a^2*l - (b^4*l)/c^2 - b^2*(3*g - (5*a*l)/c) + 2*(6*c
^2*e - 3*b*c*g + b^2*j + 2*a*c*j - 3*a*b*l)*x^2)/(4*(b^2 - 4*a*c)
^2*(a + b*x^2 + c*x^4)) + (x*(4*a^2*b*c^2*(2*c*f + a*k) + a*b^3*c
*(c*f + 2*a*k) - a*b^2*c*(25*c^2*d + 7*a*c*h - 11*a^2*m) + 4*a^2*
c^2*(7*c^2*d + a*c*h - 9*a^2*m) + b^4*(3*c^2*d - 2*a^2*m) + c*(a*
b^2*c*(c*f + 3*a*k) + 4*a^2*c^2*(5*c*f + 3*a*k) + b^3*(3*c^2*d +
a^2*m) - 4*a*b*c*(6*c^2*d + 3*a*c*h + 4*a^2*m))*x^2))/(8*a^2*c^2*
(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + ((a*b^2*c*(c*f + 3*a*k) +
4*a^2*c^2*(5*c*f + 3*a*k) + b^3*(3*c^2*d + a^2*m) - 4*a*b*c*(6*c^
2*d + 3*a*c*h + 4*a^2*m) + (a*b^3*c*(c*f - 3*a*k) - 4*a^2*b*c^2*(
13*c*f + 9*a*k) - 6*a*b^2*c*(5*c^2*d - 3*a*c*h - 3*a^2*m) + b^4*(
3*c^2*d - a^2*m) + 8*a^2*c^2*(21*c^2*d + 3*a*c*h + 5*a^2*m))/Sqrt
[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a
*c]]])/(8*Sqrt[2]*a^2*c^(3/2)*(b^2 - 4*a*c)^2*Sqrt[b - Sqrt[b^2 -
 4*a*c]]) + ((a*b^2*c*(c*f + 3*a*k) + 4*a^2*c^2*(5*c*f + 3*a*k) +
 b^3*(3*c^2*d + a^2*m) - 4*a*b*c*(6*c^2*d + 3*a*c*h + 4*a^2*m) -
(a*b^3*c*(c*f - 3*a*k) - 4*a^2*b*c^2*(13*c*f + 9*a*k) - 6*a*b^2*c
*(5*c^2*d - 3*a*c*h - 3*a^2*m) + b^4*(3*c^2*d - a^2*m) + 8*a^2*c^
2*(21*c^2*d + 3*a*c*h + 5*a^2*m))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt
[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a^2*c^(3/
2)*(b^2 - 4*a*c)^2*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - ((6*c^2*e - 3*b
*c*g + b^2*j + 2*a*c*j - 3*a*b*l)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2
- 4*a*c]])/(b^2 - 4*a*c)^(5/2)
```

Rubi [A] time = 24.6691, antiderivative size = 1144, normalized size of antiderivative = 0.99, number of steps used = 11, number of rules used = 9, integrand size = 55, $\frac{\text{number of rules}}{\text{integrand size}} = 0.164$

$$\frac{-\frac{lb^4}{c^2}+\frac{jb^3}{c}-\left(3g-\frac{5al}{c}\right)b^2+2(3ce+aj)b+2\left(jb^2-3cgb-3alb+6c^2e+2acj\right)x^2-16a^2l}{4\left(b^2-4ac\right)^2\left(cx^4+bx^2+a\right)}$$
$$+\frac{\left(\left(\frac{ma^2}{c}+3cd\right)b^3+a(cf+3ak)b^2-4a\left(4ma^2+3cha+6c^2d\right)b+4a^2c(5cf+3ak)+\frac{\left(3c^2d-a^2m\right)b^4+ac(cf-3ak)b^3-6ac\left(-3ma^2-3c\right.}{}\right.}{8\sqrt{2}a^2\sqrt{c}\left(b^2-4ac\right)^2\sqrt{b-\sqrt{b^2-4ac}}}$$
$$+\frac{\left(\left(\frac{ma^2}{c}+3cd\right)b^3+a(cf+3ak)b^2-4a\left(4ma^2+3cha+6c^2d\right)b+4a^2c(5cf+3ak)-\frac{\left(3c^2d-a^2m\right)b^4+ac(cf-3ak)b^3-6ac\left(-3ma^2-3c\right.}{}\right.}{8\sqrt{2}a^2\sqrt{c}\left(b^2-4ac\right)^2\sqrt{b+\sqrt{b^2-4ac}}}$$
$$-\frac{\left(jb^2-3cgb-3alb+6c^2e+2acj\right)\tanh^{-1}\left(\frac{2cx^2+b}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/2}}$$
$$+\frac{x\left(\left(3cd-\frac{2a^2m}{c}\right)b^4+a(cf+2ak)b^3-a\left(-11ma^2+7cha+25c^2d\right)b^2+4a^2c(2cf+ak)b+\left(\left(ma^2+3c^2d\right)b^3+ac(cf+3ak\right.\right.}{8a^2c\left(b^2-4ac\right)^2\left(cx^4+bx^2+a\right)}$$
$$-\frac{-alb^2+c(ce+aj)b+\left(-lb^3+c(bj+3al)b+2c^3e-c^2(bg+2aj)\right)x^2-2ac(cg-al)}{4c^2\left(b^2-4ac\right)\left(cx^4+bx^2+a\right)^2}$$
$$-\frac{x\left(-\left(ma^2+c^2d\right)b^2+ac(cf+ak)b+\left(-amb^3+ackb^2-c\left(-3ma^2+cha+c^2d\right)b+2ac^2(cf-ak)\right)x^2+2ac\left(ma^2-cha\right.\right.}{4ac^2\left(b^2-4ac\right)\left(cx^4+bx^2+a\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2 + g*x^3 + h*x^4 + j*x^5 + k*x^6 + l*x^7 + m*x^8)/(a + b*x^2 + c*
```

```
[Out] -(b*c*(c*e + a*j) - a*b^2*l - 2*a*c*(c*g - a*l) + (2*c^3*e - c^2*
(b*g + 2*a*j) - b^3*l + b*c*(b*j + 3*a*l))*x^2)/(4*c^2*(b^2 - 4*a
*c)*(a + b*x^2 + c*x^4)^2) - (x*(a*b*c*(c*f + a*k) - b^2*(c^2*d +
 a^2*m) + 2*a*c*(c^2*d - a*c*h + a^2*m) + (a*b^2*c*k + 2*a*c^2*(c
*f - a*k) - a*b^3*m - b*c*(c^2*d + a*c*h - 3*a^2*m))*x^2))/(4*a*c
^2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + ((b^3*j)/c + 2*b*(3*c*e
 + a*j) - 16*a^2*l - (b^4*l)/c^2 - b^2*(3*g - (5*a*l)/c) + 2*(6*c
^2*e - 3*b*c*g + b^2*j + 2*a*c*j - 3*a*b*l)*x^2)/(4*(b^2 - 4*a*c)
^2*(a + b*x^2 + c*x^4)) + (x*(4*a^2*b*c*(2*c*f + a*k) + a*b^3*(c*
f + 2*a*k) - a*b^2*(25*c^2*d + 7*a*c*h - 11*a^2*m) + 4*a^2*c*(7*c
^2*d + a*c*h - 9*a^2*m) + b^4*(3*c*d - (2*a^2*m)/c) + (a*b^2*c*(c
*f + 3*a*k) + 4*a^2*c^2*(5*c*f + 3*a*k) + b^3*(3*c^2*d + a^2*m) -
 4*a*b*c*(6*c^2*d + 3*a*c*h + 4*a^2*m))*x^2))/(8*a^2*c*(b^2 - 4*a
*c)^2*(a + b*x^2 + c*x^4)) + ((a*b^2*(c*f + 3*a*k) + 4*a^2*c*(5*c
*f + 3*a*k) - 4*a*b*(6*c^2*d + 3*a*c*h + 4*a^2*m) + b^3*(3*c*d +
(a^2*m)/c) + (a*b^3*c*(c*f - 3*a*k) - 4*a^2*b*c^2*(13*c*f + 9*a*k
) - 6*a*b^2*c*(5*c^2*d - 3*a*c*h - 3*a^2*m) + b^4*(3*c^2*d - a^2*
m) + 8*a^2*c^2*(21*c^2*d + 3*a*c*h + 5*a^2*m))/(c*Sqrt[b^2 - 4*a*
c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(8*
Sqrt[2]*a^2*Sqrt[c]*(b^2 - 4*a*c)^2*Sqrt[b - Sqrt[b^2 - 4*a*c]])
+ ((a*b^2*(c*f + 3*a*k) + 4*a^2*c*(5*c*f + 3*a*k) - 4*a*b*(6*c^2*
d + 3*a*c*h + 4*a^2*m) + b^3*(3*c*d + (a^2*m)/c) - (a*b^3*c*(c*f
```

```
- 3*a*k) - 4*a^2*b*c^2*(13*c*f + 9*a*k) - 6*a*b^2*c*(5*c^2*d - 3*
a*c*h - 3*a^2*m) + b^4*(3*c^2*d - a^2*m) + 8*a^2*c^2*(21*c^2*d +
3*a*c*h + 5*a^2*m))/(c*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c
]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a^2*Sqrt[c]*(b^2 -
4*a*c)^2*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - ((6*c^2*e - 3*b*c*g + b^2
*j + 2*a*c*j - 3*a*b*l)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])
/(b^2 - 4*a*c)^(5/2)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((m*x**8+l*x**7+k*x**6+j*x**5+h*x**4+g*x**3+f*x**2+e*x+d)/(c*x**
```

```
[Out] Timed out
```

Mathematica [A] time = 9.31706, size = 1590, normalized size = 1.38

$$\frac{2cla^3 + 2cmxa^3 - 2c^2kx^3a^2 + 3bcmx^3a^2 - 2c^2jx^2a^2 + 3bclx^2a^2 - 2c^2ga^2 + bcja^2 - b^2la^2 - 2c^2hxa^2 + bckxa^2 - b^2mxa^2 + 2}{4ac^2(4ac - b}$$
$$+ \frac{\left(40c^2ma^4 + 24c^3ha^3 - 36bc^2ka^3 + 12c^2\sqrt{b^2-4ac}ka^3 + 18b^2cma^3 - 16bc\sqrt{b^2-4ac}ma^3 + 168c^4da^2 - 52bc^3fa^2 + 20c^3\sqrt{b^2}\right.}{}$$
$$+ \frac{\left(-40c^2ma^4 - 24c^3ha^3 + 36bc^2ka^3 + 12c^2\sqrt{b^2-4ac}ka^3 - 18b^2cma^3 - 16bc\sqrt{b^2-4ac}ma^3 - 168c^4da^2 + 52bc^3fa^2 + 20c^3\sqrt{}\right.}{}$$
$$+ \frac{\left(jb^2 - 3cgb - 3alb + 6c^2e + 2acj\right)\log\left(-2cx^2 - b + \sqrt{b^2-4ac}\right)}{2\left(b^2-4ac\right)^{5/2}}$$
$$+ \frac{\left(-jb^2 + 3cgb + 3alb - 6c^2e - 2acj\right)\log\left(2cx^2 + b + \sqrt{b^2-4ac}\right)}{2\left(b^2-4ac\right)^{5/2}}$$
$$+ \frac{-32c^2la^4 - 36c^2mxa^4 + 12c^3kx^3a^3 - 16bc^2mx^3a^3 + 8c^3jx^2a^3 - 12bc^2lx^2a^3 + 4bc^2ja^3 + 10b^2cla^3 + 4c^3hxa^3 + 4bc^2kxa^3 + 1}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2 + g*x^3 + h*x^4 + j*x^5 + k*x^6 + l*x^7 + m*x^8)/(a + b*x
```

```
[Out] (a*b*c^2*e - 2*a^2*c^2*g + a^2*b*c*j - a^2*b^2*l + 2*a^3*c*l - b^
2*c^2*d*x + 2*a*c^3*d*x + a*b*c^2*f*x - 2*a^2*c^2*h*x + a^2*b*c*k
*x - a^2*b^2*m*x + 2*a^3*c*m*x + 2*a*c^3*e*x^2 - a*b*c^2*g*x^2 +
a*b^2*c*j*x^2 - 2*a^2*c^2*j*x^2 - a*b^3*l*x^2 + 3*a^2*b*c*l*x^2 -
 b*c^3*d*x^3 + 2*a*c^3*f*x^3 - a*b*c^2*h*x^3 + a*b^2*c*k*x^3 - 2*
a^2*c^2*k*x^3 - a*b^3*m*x^3 + 3*a^2*b*c*m*x^3)/(4*a*c^2*(-b^2 + 4
*a*c)*(a + b*x^2 + c*x^4)^2) + (12*a^2*b*c^3*e - 6*a^2*b^2*c^2*g
+ 2*a^2*b^3*c*j + 4*a^3*b*c^2*j - 2*a^2*b^4*l + 10*a^3*b^2*c*l -
32*a^4*c^2*l + 3*b^4*c^2*d*x - 25*a*b^2*c^3*d*x + 28*a^2*c^4*d*x
+ a*b^3*c^2*f*x + 8*a^2*b*c^3*f*x - 7*a^2*b^2*c^2*h*x + 4*a^3*c^3
*h*x + 2*a^2*b^3*c*k*x + 4*a^3*b*c^2*k*x - 2*a^2*b^4*m*x + 11*a^3
*b^2*c*m*x - 36*a^4*c^2*m*x + 24*a^2*c^4*e*x^2 - 12*a^2*b*c^3*g*x
^2 + 4*a^2*b^2*c^2*j*x^2 + 8*a^3*c^3*j*x^2 - 12*a^3*b*c^2*l*x^2 +
 3*b^3*c^3*d*x^3 - 24*a*b*c^4*d*x^3 + a*b^2*c^3*f*x^3 + 20*a^2*c^
4*f*x^3 - 12*a^2*b*c^3*h*x^3 + 3*a^2*b^2*c^2*k*x^3 + 12*a^3*c^3*k
*x^3 + a^2*b^3*c*m*x^3 - 16*a^3*b*c^2*m*x^3)/(8*a^2*c^2*(-b^2 + 4
*a*c)^2*(a + b*x^2 + c*x^4)) + ((3*b^4*c^2*d - 30*a*b^2*c^3*d + 1
68*a^2*c^4*d + 3*b^3*c^2*Sqrt[b^2 - 4*a*c]*d - 24*a*b*c^3*Sqrt[b^
2 - 4*a*c]*d + a*b^3*c^2*f - 52*a^2*b*c^3*f + a*b^2*c^2*Sqrt[b^2
- 4*a*c]*f + 20*a^2*c^3*Sqrt[b^2 - 4*a*c]*f + 18*a^2*b^2*c^2*h +
24*a^3*c^3*h - 12*a^2*b*c^2*Sqrt[b^2 - 4*a*c]*h - 3*a^2*b^3*c*k -
 36*a^3*b*c^2*k + 3*a^2*b^2*c*Sqrt[b^2 - 4*a*c]*k + 12*a^3*c^2*Sq
rt[b^2 - 4*a*c]*k - a^2*b^4*m + 18*a^3*b^2*c*m + 40*a^4*c^2*m + a
^2*b^3*Sqrt[b^2 - 4*a*c]*m - 16*a^3*b*c*Sqrt[b^2 - 4*a*c]*m)*ArcT
an[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a
^2*c^(3/2)*(b^2 - 4*a*c)^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((-
3*b^4*c^2*d + 30*a*b^2*c^3*d - 168*a^2*c^4*d + 3*b^3*c^2*Sqrt[b^2
 - 4*a*c]*d - 24*a*b*c^3*Sqrt[b^2 - 4*a*c]*d - a*b^3*c^2*f + 52*a
^2*b*c^3*f + a*b^2*c^2*Sqrt[b^2 - 4*a*c]*f + 20*a^2*c^3*Sqrt[b^2
- 4*a*c]*f - 18*a^2*b^2*c^2*h - 24*a^3*c^3*h - 12*a^2*b*c^2*Sqrt[
b^2 - 4*a*c]*h + 3*a^2*b^3*c*k + 36*a^3*b*c^2*k + 3*a^2*b^2*c*Sqr
t[b^2 - 4*a*c]*k + 12*a^3*c^2*Sqrt[b^2 - 4*a*c]*k + a^2*b^4*m - 1
8*a^3*b^2*c*m - 40*a^4*c^2*m + a^2*b^3*Sqrt[b^2 - 4*a*c]*m - 16*a
^3*b*c*Sqrt[b^2 - 4*a*c]*m)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + S
qrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a^2*c^(3/2)*(b^2 - 4*a*c)^(5/2)*Sq
rt[b + Sqrt[b^2 - 4*a*c]]) + ((6*c^2*e - 3*b*c*g + b^2*j + 2*a*c*
j - 3*a*b*l)*Log[-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2])/(2*(b^2 - 4*a
*c)^(5/2)) + ((-6*c^2*e + 3*b*c*g - b^2*j - 2*a*c*j + 3*a*b*l)*Lo
g[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(2*(b^2 - 4*a*c)^(5/2))
```

Maple [B] time = 0.344, size = 36326, normalized size = 31.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((m*x^8+l*x^7+k*x^6+j*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)^3,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((m*x^8 + l*x^7 + k*x^6 + j*x^5 + h*x^4 + g*x^3 + f*x^2 + e*x + d)/(c*x^4 +
```

```
[Out] -1/8*((12*a^2*b*c^3*h - 3*(b^3*c^3 - 8*a*b*c^4)*d - (a*b^2*c^3 +
20*a^2*c^4)*f - 3*(a^2*b^2*c^2 + 4*a^3*c^3)*k - (a^2*b^3*c - 16*a
^3*b*c^2)*m)*x^7 - 12*a^4*b*c*j - 4*(6*a^2*c^4*e - 3*a^2*b*c^3*g
- 3*a^3*b*c^2*l + (a^2*b^2*c^2 + 2*a^3*c^3)*j)*x^6 - ((6*b^4*c^2
- 49*a*b^2*c^3 + 28*a^2*c^4)*d + 2*(a*b^3*c^2 + 14*a^2*b*c^3)*f -
 (19*a^2*b^2*c^2 - 4*a^3*c^3)*h + (5*a^2*b^3*c + 16*a^3*b*c^2)*k
- (a^2*b^4 + 5*a^3*b^2*c + 36*a^4*c^2)*m)*x^5 - 2*(18*a^2*b*c^3*e
 - 9*a^2*b^2*c^2*g + 3*(a^2*b^3*c + 2*a^3*b*c^2)*j - (a^2*b^4 + a
^3*b^2*c + 16*a^4*c^2)*l)*x^4 - ((3*b^5*c - 20*a*b^3*c^2 - 4*a^2*
b*c^3)*d + (a*b^4*c + 5*a^2*b^2*c^2 + 36*a^3*c^3)*f - (5*a^2*b^3*
c + 16*a^3*b*c^2)*h + (19*a^3*b^2*c - 4*a^4*c^2)*k - 2*(a^3*b^3 +
 14*a^4*b*c)*m)*x^3 - 4*(2*(a^2*b^2*c^2 + 5*a^3*c^3)*e - (a^2*b^3
*c + 5*a^3*b*c^2)*g + (5*a^3*b^2*c - 2*a^4*c^2)*j - (a^3*b^3 + 5*
a^4*b*c)*l)*x^2 + 2*(a^2*b^3*c - 10*a^3*b*c^2)*e + 2*(a^3*b^2*c +
 8*a^4*c^2)*g + 2*(a^4*b^2 + 8*a^5*c)*l - (12*a^4*b*c*k + (5*a*b^
4*c - 37*a^2*b^2*c^2 + 44*a^3*c^3)*d - (a^2*b^3*c - 16*a^3*b*c^2)
*f - 3*(a^3*b^2*c + 4*a^4*c^2)*h - (a^4*b^2 + 20*a^5*c)*m)*x)/(a^
4*b^4*c - 8*a^5*b^2*c^2 + 16*a^6*c^3 + (a^2*b^4*c^3 - 8*a^3*b^2*c
^4 + 16*a^4*c^5)*x^8 + 2*(a^2*b^5*c^2 - 8*a^3*b^3*c^3 + 16*a^4*b*
c^4)*x^6 + (a^2*b^6*c - 6*a^3*b^4*c^2 + 32*a^5*c^4)*x^4 + 2*(a^3*
b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*x^2) - 1/8*integrate((12*a^
3*b*c*k + (12*a^2*b*c^2*h - 3*(b^3*c^2 - 8*a*b*c^3)*d - (a*b^2*c^
2 + 20*a^2*c^3)*f - 3*(a^2*b^2*c + 4*a^3*c^2)*k - (a^2*b^3 - 16*a
^3*b*c)*m)*x^2 - 3*(b^4*c - 9*a*b^2*c^2 + 28*a^2*c^3)*d - (a*b^3*
c - 16*a^2*b*c^2)*f - 3*(a^2*b^2*c + 4*a^3*c^2)*h - (a^3*b^2 + 20
*a^4*c)*m - 8*(6*a^2*c^3*e - 3*a^2*b*c^2*g - 3*a^3*b*c*l + (a^2*b
^2*c + 2*a^3*c^2)*j)*x)/(c*x^4 + b*x^2 + a), x)/(a^2*b^4*c - 8*a^
3*b^2*c^2 + 16*a^4*c^3)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((m*x^8 + l*x^7 + k*x^6 + j*x^5 + h*x^4 + g*x^3 + f*x^2 + e*x + d)/(c*x^4 +
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((m*x**8+l*x**7+k*x**6+j*x**5+h*x**4+g*x**3+f*x**2+e*x+d)/(c*x**4+b*x
```

```
[Out] Timed out
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((m*x^8 + l*x^7 + k*x^6 + j*x^5 + h*x^4 + g*x^3 + f*x^2 + e*x + d)/(c*x^4 +
```

```
[Out] Exception raised: TypeError
```

3.58 $\int \frac{d+ex+fx^2+gx^3+hx^4+ix^5+jx^6+kx^7}{(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=645

$$\frac{x\left(x^2\left(-ab^2j+bc(ah+cd)-2ac(cf-aj)\right)+c\left(-\frac{ab(aj+cf)}{c}-2a(cd-ah)+b^2d\right)\right)}{2ac\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{ab^2j}{c}+\frac{-ab^3j+b^2c(cd-ah)+4abc(2aj+cf)-4ac^2(ah+3cd)}{c\sqrt{b^2-4ac}}+b(ah+cd)-2a(3aj+cf)\right)}{2\sqrt{2}a\sqrt{c}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(\frac{ab^2j}{c}-\frac{-ab^3j+b^2c(cd-ah)+4abc(2aj+cf)-4ac^2(ah+3cd)}{c\sqrt{b^2-4ac}}+b(ah+cd)-2a(3aj+cf)\right)}{2\sqrt{2}a\sqrt{c}\left(b^2-4ac\right)\sqrt{\sqrt{b^2-4ac}+b}}+\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)\left(-c^2(2bg-4ai)-6abck+b^3k+4c^3e\right)}{2c^2\left(b^2-4ac\right)^{3/2}}-\frac{x^2\left(-c^2(2ai+bg)+bc(3ak+bi)+b^3(-k)+2c^3e\right)-ab^2k+bc(ai+ce)-2ac(cg-ak)}{2c^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{k\log\left(a+bx^2+cx^4\right)}{4c^2}$$

```
[Out] (x*(c*(b^2*d - 2*a*(c*d - a*h) - (a*b*(c*f + a*j))/c) + (b*c*(c*d
 + a*h) - a*b^2*j - 2*a*c*(c*f - a*j))*x^2))/(2*a*c*(b^2 - 4*a*c)
*(a + b*x^2 + c*x^4)) - (b*c*(c*e + a*i) - a*b^2*k - 2*a*c*(c*g -
 a*k) + (2*c^3*e - c^2*(b*g + 2*a*i) - b^3*k + b*c*(b*i + 3*a*k))
*x^2)/(2*c^2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + ((b*(c*d + a*h)
 + (a*b^2*j)/c - 2*a*(c*f + 3*a*j) + (b^2*c*(c*d - a*h) - 4*a*c^2
*(3*c*d + a*h) - a*b^3*j + 4*a*b*c*(c*f + 2*a*j))/(c*Sqrt[b^2 - 4
*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/
(2*Sqrt[2]*a*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) +
 ((b*(c*d + a*h) + (a*b^2*j)/c - 2*a*(c*f + 3*a*j) - (b^2*c*(c*d
- a*h) - 4*a*c^2*(3*c*d + a*h) - a*b^3*j + 4*a*b*c*(c*f + 2*a*j))
/(c*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[
b^2 - 4*a*c]]])/(2*Sqrt[2]*a*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b + Sqrt[
b^2 - 4*a*c]]) + ((4*c^3*e - c^2*(2*b*g - 4*a*i) + b^3*k - 6*a*b*
c*k)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c^2*(b^2 - 4*a*
c)^(3/2)) + (k*Log[a + b*x^2 + c*x^4])/(4*c^2)
```

Rubi [A] time = 8.50723, antiderivative size = 645, normalized size of antiderivative = 1., number of

steps used = 11, number of rules used = 10, integrand size = 50, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$\frac{x\left(x^2\left(-ab^2j+bc(ah+cd)-2ac(cf-aj)\right)-ab(aj+cf)-2ac(cd-ah)+b^2cd\right)}{2ac\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{ab^2j}{c}+\frac{-ab^3j+b^2c(cd-ah)+4abc(2aj+cf)-4ac^2(ah+3cd)}{c\sqrt{b^2-4ac}}+b(ah+cd)-2a(3aj+cf)\right)}{2\sqrt{2}a\sqrt{c}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(\frac{ab^2j}{c}-\frac{-ab^3j+b^2c(cd-ah)+4abc(2aj+cf)-4ac^2(ah+3cd)}{c\sqrt{b^2-4ac}}+b(ah+cd)-2a(3aj+cf)\right)}{2\sqrt{2}a\sqrt{c}\left(b^2-4ac\right)\sqrt{\sqrt{b^2-4ac}+b}}+\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)\left(-c^2(2bg-4ai)-6abck+b^3k+4c^3e\right)}{2c^2\left(b^2-4ac\right)^{3/2}}-\frac{x^2\left(-c^2(2ai+bg)+bc(3ak+bi)+b^3(-k)+2c^3e\right)-ab^2k+bc(ai+ce)-2ac(cg-ak)}{2c^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{k\log\left(a+bx^2+cx^4\right)}{4c^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5 + j*x^6 + k*x^7)/(a + b*x^2 + c*x^4)^2,
```

```
[Out] (x*(b^2*c*d - 2*a*c*(c*d - a*h) - a*b*(c*f + a*j) + (b*c*(c*d + a
*h) - a*b^2*j - 2*a*c*(c*f - a*j))*x^2))/(2*a*c*(b^2 - 4*a*c)*(a
+ b*x^2 + c*x^4)) - (b*c*(c*e + a*i) - a*b^2*k - 2*a*c*(c*g - a*k
) + (2*c^3*e - c^2*(b*g + 2*a*i) - b^3*k + b*c*(b*i + 3*a*k))*x^2
)/(2*c^2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + ((b*(c*d + a*h) + (
a*b^2*j)/c - 2*a*(c*f + 3*a*j) + (b^2*c*(c*d - a*h) - 4*a*c^2*(3*
c*d + a*h) - a*b^3*j + 4*a*b*c*(c*f + 2*a*j))/(c*Sqrt[b^2 - 4*a*c
]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*S
qrt[2]*a*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((b
*(c*d + a*h) + (a*b^2*j)/c - 2*a*(c*f + 3*a*j) - (b^2*c*(c*d - a*
h) - 4*a*c^2*(3*c*d + a*h) - a*b^3*j + 4*a*b*c*(c*f + 2*a*j))/(c*
Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2
- 4*a*c]]])/(2*Sqrt[2]*a*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2
- 4*a*c]]) + ((4*c^3*e - c^2*(2*b*g - 4*a*i) + b^3*k - 6*a*b*c*k)
*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c^2*(b^2 - 4*a*c)^(
3/2)) + (k*Log[a + b*x^2 + c*x^4])/(4*c^2)
```

Rubi in Sympy [F(-1)]   time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((k*x**7+j*x**6+i*x**5+h*x**4+g*x**3+f*x**2+e*x+d)/(c*x**4+b*x**
```

```
[Out] Timed out
```

Mathematica [A] time = 7.88598, size = 908, normalized size = 1.41

$$\frac{2cka^3 - 2c^2jx^3a^2 - 2c^2ix^2a^2 + 3bckx^2a^2 - 2c^2ga^2 + bcia^2 - b^2ka^2 - 2c^2hxa^2 + bcjxa^2 + 2c^3fx^3a - bc^2hx^3a + b^2cjx^3a + 2c}{2ac^2(4ac - b^2)(cx^4 + bx^2 + a)}$$

$$+ \frac{\left(-ajb^3 + c^2db^2 - achb^2 + a\sqrt{b^2-4ac}jb^2 + c^2\sqrt{b^2-4ac}db + 4ac^2fb + ac\sqrt{b^2-4ac}hb + 8a^2cjb - 12ac^3d - 2ac^2\sqrt{b^2-4ac}\right.}{2\sqrt{2}ac^{3/2}(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}$$

$$+ \frac{\left(ajb^3 - c^2db^2 + achb^2 + a\sqrt{b^2-4ac}jb^2 + c^2\sqrt{b^2-4ac}db - 4ac^2fb + ac\sqrt{b^2-4ac}hb - 8a^2cjb + 12ac^3d - 2ac^2\sqrt{b^2-4ac}\right.}{2\sqrt{2}ac^{3/2}(b^2-4ac)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}$$

$$- \frac{\left(kb^3 - \sqrt{b^2-4ac}kb^2 - 2c^2gb - 6ackb + 4c^3e + 4ac^2i + 4ac\sqrt{b^2-4ac}k\right)\log\left(-2cx^2 - b + \sqrt{b^2-4ac}\right)}{4c^2(b^2-4ac)^{3/2}}$$

$$- \frac{\left(-kb^3 - \sqrt{b^2-4ac}kb^2 + 2c^2gb + 6ackb - 4c^3e - 4ac^2i + 4ac\sqrt{b^2-4ac}k\right)\log\left(2cx^2 + b + \sqrt{b^2-4ac}\right)}{4c^2(b^2-4ac)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5 + j*x^6 + k*x^7)/(a + b*x^2 + c*x
```

```
[Out] (a*b*c^2*e - 2*a^2*c^2*g + a^2*b*c*i - a^2*b^2*k + 2*a^3*c*k - b^
2*c^2*d*x + 2*a*c^3*d*x + a*b*c^2*f*x - 2*a^2*c^2*h*x + a^2*b*c*j
*x + 2*a*c^3*e*x^2 - a*b*c^2*g*x^2 + a*b^2*c*i*x^2 - 2*a^2*c^2*i*
x^2 - a*b^3*k*x^2 + 3*a^2*b*c*k*x^2 - b*c^3*d*x^3 + 2*a*c^3*f*x^3
 - a*b*c^2*h*x^3 + a*b^2*c*j*x^3 - 2*a^2*c^2*j*x^3)/(2*a*c^2*(-b^
2 + 4*a*c)*(a + b*x^2 + c*x^4)) + ((b^2*c^2*d - 12*a*c^3*d + b*c^
2*Sqrt[b^2 - 4*a*c]*d + 4*a*b*c^2*f - 2*a*c^2*Sqrt[b^2 - 4*a*c]*f
 - a*b^2*c*h - 4*a^2*c^2*h + a*b*c*Sqrt[b^2 - 4*a*c]*h - a*b^3*j
+ 8*a^2*b*c*j + a*b^2*Sqrt[b^2 - 4*a*c]*j - 6*a^2*c*Sqrt[b^2 - 4*
a*c]*j)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/
(2*Sqrt[2]*a*c^(3/2)*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*
c]]) + ((-(b^2*c^2*d) + 12*a*c^3*d + b*c^2*Sqrt[b^2 - 4*a*c]*d -
4*a*b*c^2*f - 2*a*c^2*Sqrt[b^2 - 4*a*c]*f + a*b^2*c*h + 4*a^2*c^2
*h + a*b*c*Sqrt[b^2 - 4*a*c]*h + a*b^3*j - 8*a^2*b*c*j + a*b^2*Sq
rt[b^2 - 4*a*c]*j - 6*a^2*c*Sqrt[b^2 - 4*a*c]*j)*ArcTan[(Sqrt[2]*
Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*c^(3/2)*(b^
2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - ((4*c^3*e - 2*b*c
^2*g + 4*a*c^2*i + b^3*k - 6*a*b*c*k - b^2*Sqrt[b^2 - 4*a*c]*k +
4*a*c*Sqrt[b^2 - 4*a*c]*k)*Log[-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2])
/(4*c^2*(b^2 - 4*a*c)^(3/2)) - ((-4*c^3*e + 2*b*c^2*g - 4*a*c^2*i
 - b^3*k + 6*a*b*c*k - b^2*Sqrt[b^2 - 4*a*c]*k + 4*a*c*Sqrt[b^2 -
```

```
 4*a*c]*k)*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(4*c^2*(b^2 - 4*
a*c)^(3/2))
```

Maple [B] time = 0.186, size = 14103, normalized size = 21.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((k*x^7+j*x^6+i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{abc^2e - 2a^2c^2g + a^2bci - \left(bc^3d - 2ac^3f + abc^2h - \left(ab^2c - 2a^2c^2\right)j\right)x^3 + \left(2ac^3e - abc^2g + \left(ab^2c - 2a^2c^2\right)i - \left(ab^3 - 3a\right.\right.}{2\left(a^2b^2c^2 - 4a^3c^3 + \left(ab^2c^3 - 4a^2c^4\right)x^4 + \left(ab^3c^2 - 4a\right.\right.}$$

$$-\frac{-\int \frac{2\left(ab^2-4a^2c\right)kx^3+abcf-2a^2ch+a^2bj+\left(bc^2d-2ac^2f+abch+\left(ab^2-6a^2c\right)j\right)x^2+\left(b^2c-6ac^2\right)d-2\left(2ac^2e-abcg+2a^2ci-a^2bk\right)x}{cx^4+bx^2+a}\,dx}{2\left(ab^2c-4a^2c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((k*x^7 + j*x^6 + i*x^5 + h*x^4 + g*x^3 + f*x^2 + e*x + d)/(c*x^4 + b*x^2 +
```

```
[Out] -1/2*(a*b*c^2*e - 2*a^2*c^2*g + a^2*b*c*i - (b*c^3*d - 2*a*c^3*f
+ a*b*c^2*h - (a*b^2*c - 2*a^2*c^2)*j)*x^3 + (2*a*c^3*e - a*b*c^2
*g + (a*b^2*c - 2*a^2*c^2)*i - (a*b^3 - 3*a^2*b*c)*k)*x^2 - (a^2*
b^2 - 2*a^3*c)*k + (a*b*c^2*f - 2*a^2*c^2*h + a^2*b*c*j - (b^2*c^
2 - 2*a*c^3)*d)*x)/(a^2*b^2*c^2 - 4*a^3*c^3 + (a*b^2*c^3 - 4*a^2*
c^4)*x^4 + (a*b^3*c^2 - 4*a^2*b*c^3)*x^2) - 1/2*integrate(-(2*(a*
b^2 - 4*a^2*c)*k*x^3 + a*b*c*f - 2*a^2*c*h + a^2*b*j + (b*c^2*d -
 2*a*c^2*f + a*b*c*h + (a*b^2 - 6*a^2*c)*j)*x^2 + (b^2*c - 6*a*c^
2)*d - 2*(2*a*c^2*e - a*b*c*g + 2*a^2*c*i - a^2*b*k)*x)/(c*x^4 +
b*x^2 + a), x)/(a*b^2*c - 4*a^2*c^2)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((k*x^7 + j*x^6 + i*x^5 + h*x^4 + g*x^3 + f*x^2 + e*x + d)/(c*x^4 + b*x^2 +
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((k*x**7+j*x**6+i*x**5+h*x**4+g*x**3+f*x**2+e*x+d)/(c*x**4+b*x**2+a)*
```

```
[Out] Timed out
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((k*x^7 + j*x^6 + i*x^5 + h*x^4 + g*x^3 + f*x^2 + e*x + d)/(c*x^4 + b*x^2 +
```

```
[Out] Exception raised: TypeError
```

3.59 $\int \frac{d+ex+fx^2+gx^3+hx^4+ix^5+jx^8+kx^{11}}{(a+bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=1177

result too large to display

```
[Out] -(x*(c^2*(a*b*f - b^2*(d + (a^2*j)/c^2) + 2*a*(c*d - a*h + (a^2*j
)/c)) + (2*a*c^3*f - a*b^3*j - b*c*(c^2*d + a*c*h - 3*a^2*j))*x^2
))/(4*a*c^2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) - (b*c^3*(c*e +
a*i) - a*b^4*k + 4*a^2*b^2*c*k - 2*a*c^2*(c^2*g + a^2*k) + (2*c^5
*e + b^2*c^3*i - c^4*(b*g + 2*a*i) - b^5*k + 5*a*b^3*c*k - 5*a^2*
b*c^2*k)*x^2)/(4*c^4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (x*(c
*(a*b^3*f + 8*a^2*b*c*f + 4*a^2*(7*c^2*d + a*c*h - 9*a^2*j) + b^4
*(3*d - (2*a^2*j)/c^2) - a*b^2*(25*c*d + 7*a*h - (11*a^2*j)/c)) +
 (a*b^2*c^2*f + 20*a^2*c^3*f + b^3*(3*c^2*d + a^2*j) - 4*a*b*c*(6
*c^2*d + 3*a*c*h + 4*a^2*j))*x^2))/(8*a^2*c*(b^2 - 4*a*c)^2*(a +
b*x^2 + c*x^4)) + (b^3*c^2*i + 2*b*c^3*(3*c*e + a*i) + 11*a*b^4*k
 - (b^6*k)/c + 32*a^3*c^2*k - 3*b^2*(c^3*g + 13*a^2*c*k) + 2*(6*c
^5*e + b^2*c^3*i - c^4*(3*b*g - 2*a*i) + 2*b^5*k - 15*a*b^3*c*k +
 25*a^2*b*c^2*k)*x^2)/(4*c^3*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4))
 + ((a*b^2*c^2*f + 20*a^2*c^3*f + b^3*(3*c^2*d + a^2*j) - 4*a*b*c
*(6*c^2*d + 3*a*c*h + 4*a^2*j) + (a*b^3*c^2*f - 52*a^2*b*c^3*f -
6*a*b^2*c*(5*c^2*d - 3*a*c*h - 3*a^2*j) + b^4*(3*c^2*d - a^2*j) +
 8*a^2*c^2*(21*c^2*d + 3*a*c*h + 5*a^2*j))/Sqrt[b^2 - 4*a*c])*Arc
Tan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*
a^2*c^(3/2)*(b^2 - 4*a*c)^2*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((a*b^
2*c^2*f + 20*a^2*c^3*f + b^3*(3*c^2*d + a^2*j) - 4*a*b*c*(6*c^2*d
 + 3*a*c*h + 4*a^2*j) - (a*b^3*c^2*f - 52*a^2*b*c^3*f - 6*a*b^2*c
*(5*c^2*d - 3*a*c*h - 3*a^2*j) + b^4*(3*c^2*d - a^2*j) + 8*a^2*c^
2*(21*c^2*d + 3*a*c*h + 5*a^2*j))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt
[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a^2*c^(3/
2)*(b^2 - 4*a*c)^2*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - ((12*c^5*e + 2*
b^2*c^3*i - c^4*(6*b*g - 4*a*i) - b^5*k + 10*a*b^3*c*k - 30*a^2*b
*c^2*k)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c^3*(b^2 - 4
*a*c)^(5/2)) + (k*Log[a + b*x^2 + c*x^4])/(4*c^3)
```

Rubi [A] time = 21.295, antiderivative size = 1179, normalized size of antiderivative = 1., number of

steps used = 13, number of rules used = 10, integrand size = 50, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$-\frac{x\left(\left(-\left(\frac{ja^2}{c^2}+d\right)b^2+afb+2a\left(\frac{ja^2}{c}-ha+cd\right)\right)c^2+\left(-ajb^3-c\left(-3ja^2+cha+c^2d\right)b+2ac^3f\right)x^2\right)}{4ac^2\left(b^2-4ac\right)\left(cx^4+bx^2+a\right)^2}$$

$$+\frac{\Big(\Big(\frac{ja^2}{c}+3cd\Big)b^3+acfb^2-4a\left(4ja^2+3cha+6c^2d\right)b+20a^2c^2f+\frac{\left(3c^2d-a^2j\right)b^4+ac^2fb^3-6ac\left(-3ja^2-3cha+5c^2d\right)b^2-52a^2c^3fb+8a^2c^2(}{c\sqrt{b^2-4ac}}}{8\sqrt{2}a^2\sqrt{c}\left(b^2-4ac\right)^2\sqrt{b-\sqrt{b^2-4ac}}}$$

$$+\frac{\Big(\Big(\frac{ja^2}{c}+3cd\Big)b^3+acfb^2-4a\left(4ja^2+3cha+6c^2d\right)b+20a^2c^2f-\frac{\left(3c^2d-a^2j\right)b^4+ac^2fb^3-6ac\left(-3ja^2-3cha+5c^2d\right)b^2-52a^2c^3fb+8a^2c^2(}{c\sqrt{b^2-4ac}}}{8\sqrt{2}a^2\sqrt{c}\left(b^2-4ac\right)^2\sqrt{b+\sqrt{b^2-4ac}}}$$

$$-\frac{\left(-kb^5+10ackb^3+2c^3ib^2-30a^2c^2kb+12c^5e-c^4(6bg-4ai)\right)\tanh^{-1}\left(\frac{2cx^2+b}{\sqrt{b^2-4ac}}\right)}{2c^3\left(b^2-4ac\right)^{5/2}}$$

$$+\frac{k\log\left(cx^4+bx^2+a\right)}{4c^3}$$

$$+\frac{x\Big(\Big(\left(ja^2+3c^2d\right)b^3+ac^2fb^2-4ac\left(4ja^2+3cha+6c^2d\right)b+20a^2c^3f\Big)x^2+c\Big(\Big(3d-\frac{2a^2j}{c^2}\Big)b^4+afb^3-a\Big(-\frac{11ja^2}{c}+7ha+}{8a^2c\left(b^2-4ac\right)^2\left(cx^4+bx^2+a\right)}$$

$$+\frac{-\frac{kb^6}{c}+11akb^4+c^2ib^3-3\left(gc^3+13a^2kc\right)b^2+2c^3(3ce+ai)b+2\Big(2kb^5-15ackb^3+c^3ib^2+25a^2c^2kb+6c^5e-c^4(3bg-2}{4c^3\left(b^2-4ac\right)^2\left(cx^4+bx^2+a\right)}$$

$$-\frac{-akb^4+4a^2ckb^2+c^3(ce+ai)b+\left(-kb^5+5ackb^3+c^3ib^2-5a^2c^2kb+2c^5e-c^4(bg+2ai)\right)x^2-2ac^2\left(ka^2+c^2g\right)}{4c^4\left(b^2-4ac\right)\left(cx^4+bx^2+a\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5 + j*x^8 + k*x^11)/(a + b*x^2 + c*x^4)^3
```

```
[Out] -(x*(c^2*(a*b*f - b^2*(d + (a^2*j)/c^2) + 2*a*(c*d - a*h + (a^2*j
)/c)) + (2*a*c^3*f - a*b^3*j - b*c*(c^2*d + a*c*h - 3*a^2*j))*x^2
))/(4*a*c^2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) - (b*c^3*(c*e +
a*i) - a*b^4*k + 4*a^2*b^2*c*k - 2*a*c^2*(c^2*g + a^2*k) + (2*c^5
*e + b^2*c^3*i - c^4*(b*g + 2*a*i) - b^5*k + 5*a*b^3*c*k - 5*a^2*
b*c^2*k)*x^2)/(4*c^4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (x*(c
*(a*b^3*f + 8*a^2*b*c*f + 4*a^2*(7*c^2*d + a*c*h - 9*a^2*j) + b^4
*(3*d - (2*a^2*j)/c^2) - a*b^2*(25*c*d + 7*a*h - (11*a^2*j)/c)) +
 (a*b^2*c^2*f + 20*a^2*c^3*f + b^3*(3*c^2*d + a^2*j) - 4*a*b*c*(6
*c^2*d + 3*a*c*h + 4*a^2*j))*x^2))/(8*a^2*c*(b^2 - 4*a*c)^2*(a +
b*x^2 + c*x^4)) + (b^3*c^2*i + 2*b*c^3*(3*c*e + a*i) + 11*a*b^4*k
 - (b^6*k)/c + 32*a^3*c^2*k - 3*b^2*(c^3*g + 13*a^2*c*k) + 2*(6*c
^5*e + b^2*c^3*i - c^4*(3*b*g - 2*a*i) + 2*b^5*k - 15*a*b^3*c*k +
 25*a^2*b*c^2*k)*x^2)/(4*c^3*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4))
 + ((a*b^2*c*f + 20*a^2*c^2*f - 4*a*b*(6*c^2*d + 3*a*c*h + 4*a^2*
j) + b^3*(3*c*d + (a^2*j)/c) + (a*b^3*c^2*f - 52*a^2*b*c^3*f - 6*
a*b^2*c*(5*c^2*d - 3*a*c*h - 3*a^2*j) + b^4*(3*c^2*d - a^2*j) + 8
*a^2*c^2*(21*c^2*d + 3*a*c*h + 5*a^2*j))/(c*Sqrt[b^2 - 4*a*c]))*A
rcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2
```

```
]*a^2*Sqrt[c]*(b^2 - 4*a*c)^2*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((a*
b^2*c*f + 20*a^2*c^2*f - 4*a*b*(6*c^2*d + 3*a*c*h + 4*a^2*j) + b^
3*(3*c*d + (a^2*j)/c) - (a*b^3*c^2*f - 52*a^2*b*c^3*f - 6*a*b^2*c
*(5*c^2*d - 3*a*c*h - 3*a^2*j) + b^4*(3*c^2*d - a^2*j) + 8*a^2*c^
2*(21*c^2*d + 3*a*c*h + 5*a^2*j))/(c*Sqrt[b^2 - 4*a*c]))*ArcTan[(
Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a^2*S
qrt[c]*(b^2 - 4*a*c)^2*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - ((12*c^5*e
+ 2*b^2*c^3*i - c^4*(6*b*g - 4*a*i) - b^5*k + 10*a*b^3*c*k - 30*a
^2*b*c^2*k)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c^3*(b^2
 - 4*a*c)^(5/2)) + (k*Log[a + b*x^2 + c*x^4])/(4*c^3)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((k*x**11+j*x**8+i*x**5+h*x**4+g*x**3+f*x**2+e*x+d)/(c*x**4+b*x*
```

```
[Out] Timed out
```

Mathematica [A] time = 8.93561, size = 1649, normalized size = 1.4

$$\frac{-akx^2b^5 - a^2kb^4 - ac^2jx^3b^3 + 5a^2ckx^2b^3 + ac^3ix^2b^2 + 4a^3ckb^2 - c^4dxb^2 - a^2c^2jxb^2 - c^5dx^3b - ac^4hx^3b + 3a^2c^3jx^3b - ac}{4ac^4\left(4ac - b\right.}$$
$$+\frac{\left(40c^2ja^4 + 24c^3ha^3 + 18b^2cja^3 - 16bc\sqrt{b^2-4ac}ja^3 + 168c^4da^2 - 52bc^3fa^2 + 20c^3\sqrt{b^2-4ac}fa^2 + 18b^2c^2ha^2 - 12bc^2\sqrt{b^2}\right.}{8\sqrt{2}a^2}$$
$$+\frac{\left(-40c^2ja^4 - 24c^3ha^3 - 18b^2cja^3 - 16bc\sqrt{b^2-4ac}ja^3 - 168c^4da^2 + 52bc^3fa^2 + 20c^3\sqrt{b^2-4ac}fa^2 - 18b^2c^2ha^2 - 12bc^2\sqrt{}\right.}{8\sqrt{2}a}$$
$$+\frac{\left(-kb^5 + \sqrt{b^2-4ac}kb^4 + 10ackb^3 + 2c^3ib^2 - 8ac\sqrt{b^2-4ac}kb^2 - 6c^4gb - 30a^2c^2kb + 12c^5e + 4ac^4i + 16a^2c^2\sqrt{b^2-4ac}k\right) \mathrm{l}}{4c^3\left(b^2-4ac\right)^{5/2}}$$
$$+\frac{\left(kb^5 + \sqrt{b^2-4ac}kb^4 - 10ackb^3 - 2c^3ib^2 - 8ac\sqrt{b^2-4ac}kb^2 + 6c^4gb + 30a^2c^2kb - 12c^5e - 4ac^4i + 16a^2c^2\sqrt{b^2-4ac}k\right) \mathrm{lo}}{4c^3\left(b^2-4ac\right)^{5/2}}$$
$$+\frac{-2a^2kb^6 + 8a^2ckx^2b^5 + 22a^3ckb^4 + 3c^4dxb^4 - 2a^2c^2jxb^4 + 3c^5dx^3b^3 + a^2c^3jx^3b^3 - 60a^3c^2kx^2b^3 + 2a^2c^3ib^3 + ac^4fxb^3 + a}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5 + j*x^8 + k*x^11)/(a + b*x^2 + c*
```

```
[Out] (a*b*c^4*e - 2*a^2*c^4*g + a^2*b*c^3*i - a^2*b^4*k + 4*a^3*b^2*c*
k - 2*a^4*c^2*k - b^2*c^4*d*x + 2*a*c^5*d*x + a*b*c^4*f*x - 2*a^2
*c^4*h*x - a^2*b^2*c^2*j*x + 2*a^3*c^3*j*x + 2*a*c^5*e*x^2 - a*b*
c^4*g*x^2 + a*b^2*c^3*i*x^2 - 2*a^2*c^4*i*x^2 - a*b^5*k*x^2 + 5*a
^2*b^3*c*k*x^2 - 5*a^3*b*c^2*k*x^2 - b*c^5*d*x^3 + 2*a*c^5*f*x^3
- a*b*c^4*h*x^3 - a*b^3*c^2*j*x^3 + 3*a^2*b*c^3*j*x^3)/(4*a*c^4*(
-b^2 + 4*a*c)*(a + b*x^2 + c*x^4)^2) + (12*a^2*b*c^5*e - 6*a^2*b^
2*c^4*g + 2*a^2*b^3*c^3*i + 4*a^3*b*c^4*i - 2*a^2*b^6*k + 22*a^3*
b^4*c*k - 78*a^4*b^2*c^2*k + 64*a^5*c^3*k + 3*b^4*c^4*d*x - 25*a*
b^2*c^5*d*x + 28*a^2*c^6*d*x + a*b^3*c^4*f*x + 8*a^2*b*c^5*f*x -
7*a^2*b^2*c^4*h*x + 4*a^3*c^5*h*x - 2*a^2*b^4*c^2*j*x + 11*a^3*b^
2*c^3*j*x - 36*a^4*c^4*j*x + 24*a^2*c^6*e*x^2 - 12*a^2*b*c^5*g*x^
2 + 4*a^2*b^2*c^4*i*x^2 + 8*a^3*c^5*i*x^2 + 8*a^2*b^5*c*k*x^2 - 6
0*a^3*b^3*c^2*k*x^2 + 100*a^4*b*c^3*k*x^2 + 3*b^3*c^5*d*x^3 - 24*
a*b*c^6*d*x^3 + a*b^2*c^5*f*x^3 + 20*a^2*c^6*f*x^3 - 12*a^2*b*c^5
*h*x^3 + a^2*b^3*c^3*j*x^3 - 16*a^3*b*c^4*j*x^3)/(8*a^2*c^4*(-b^2
 + 4*a*c)^2*(a + b*x^2 + c*x^4)) + ((3*b^4*c^2*d - 30*a*b^2*c^3*d
 + 168*a^2*c^4*d + 3*b^3*c^2*Sqrt[b^2 - 4*a*c]*d - 24*a*b*c^3*Sqr
t[b^2 - 4*a*c]*d + a*b^3*c^2*f - 52*a^2*b*c^3*f + a*b^2*c^2*Sqrt[
b^2 - 4*a*c]*f + 20*a^2*c^3*Sqrt[b^2 - 4*a*c]*f + 18*a^2*b^2*c^2*
h + 24*a^3*c^3*h - 12*a^2*b*c^2*Sqrt[b^2 - 4*a*c]*h - a^2*b^4*j +
 18*a^3*b^2*c*j + 40*a^4*c^2*j + a^2*b^3*Sqrt[b^2 - 4*a*c]*j - 16
*a^3*b*c*Sqrt[b^2 - 4*a*c]*j)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b -
 Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a^2*c^(3/2)*(b^2 - 4*a*c)^(5/2)*
Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((-3*b^4*c^2*d + 30*a*b^2*c^3*d -
168*a^2*c^4*d + 3*b^3*c^2*Sqrt[b^2 - 4*a*c]*d - 24*a*b*c^3*Sqrt[b
^2 - 4*a*c]*d - a*b^3*c^2*f + 52*a^2*b*c^3*f + a*b^2*c^2*Sqrt[b^2
 - 4*a*c]*f + 20*a^2*c^3*Sqrt[b^2 - 4*a*c]*f - 18*a^2*b^2*c^2*h -
 24*a^3*c^3*h - 12*a^2*b*c^2*Sqrt[b^2 - 4*a*c]*h + a^2*b^4*j - 18
*a^3*b^2*c*j - 40*a^4*c^2*j + a^2*b^3*Sqrt[b^2 - 4*a*c]*j - 16*a^
3*b*c*Sqrt[b^2 - 4*a*c]*j)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sq
rt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a^2*c^(3/2)*(b^2 - 4*a*c)^(5/2)*Sqr
t[b + Sqrt[b^2 - 4*a*c]]) + ((12*c^5*e - 6*b*c^4*g + 2*b^2*c^3*i
+ 4*a*c^4*i - b^5*k + 10*a*b^3*c*k - 30*a^2*b*c^2*k + b^4*Sqrt[b^
2 - 4*a*c]*k - 8*a*b^2*c*Sqrt[b^2 - 4*a*c]*k + 16*a^2*c^2*Sqrt[b^
2 - 4*a*c]*k)*Log[-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2])/(4*c^3*(b^2
- 4*a*c)^(5/2)) + ((-12*c^5*e + 6*b*c^4*g - 2*b^2*c^3*i - 4*a*c^4
*i + b^5*k - 10*a*b^3*c*k + 30*a^2*b*c^2*k + b^4*Sqrt[b^2 - 4*a*c
]*k - 8*a*b^2*c*Sqrt[b^2 - 4*a*c]*k + 16*a^2*c^2*Sqrt[b^2 - 4*a*c
]*k)*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(4*c^3*(b^2 - 4*a*c)^(
5/2))
```

Maple [B] time = 0.361, size = 35336, normalized size = 30.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((k*x^11+j*x^8+i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)^3,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((k*x^11 + j*x^8 + i*x^5 + h*x^4 + g*x^3 + f*x^2 + e*x + d)/(c*x^4 + b*x^2
```

```
[Out] 1/8*(12*a^4*b*c^3*i - (12*a^2*b*c^5*h - 3*(b^3*c^5 - 8*a*b*c^6)*d
 - (a*b^2*c^5 + 20*a^2*c^6)*f - (a^2*b^3*c^3 - 16*a^3*b*c^4)*j)*x
^7 + 4*(6*a^2*c^6*e - 3*a^2*b*c^5*g + (a^2*b^2*c^4 + 2*a^3*c^5)*i
 + (2*a^2*b^5*c - 15*a^3*b^3*c^2 + 25*a^4*b*c^3)*k)*x^6 + ((6*b^4
*c^4 - 49*a*b^2*c^5 + 28*a^2*c^6)*d + 2*(a*b^3*c^4 + 14*a^2*b*c^5
)*f - (19*a^2*b^2*c^4 - 4*a^3*c^5)*h - (a^2*b^4*c^2 + 5*a^3*b^2*c
^3 + 36*a^4*c^4)*j)*x^5 + 2*(18*a^2*b*c^5*e - 9*a^2*b^2*c^4*g + 3
*(a^2*b^3*c^3 + 2*a^3*b*c^4)*i + (3*a^2*b^6 - 19*a^3*b^4*c + 11*a
^4*b^2*c^2 + 32*a^5*c^3)*k)*x^4 + ((3*b^5*c^3 - 20*a*b^3*c^4 - 4*
a^2*b*c^5)*d + (a*b^4*c^3 + 5*a^2*b^2*c^4 + 36*a^3*c^5)*f - (5*a^
2*b^3*c^3 + 16*a^3*b*c^4)*h - 2*(a^3*b^3*c^2 + 14*a^4*b*c^3)*j)*x
^3 + 4*(2*(a^2*b^2*c^4 + 5*a^3*c^5)*e - (a^2*b^3*c^3 + 5*a^3*b*c^
4)*g + (5*a^3*b^2*c^3 - 2*a^4*c^4)*i + (3*a^3*b^5 - 22*a^4*b^3*c
+ 31*a^5*b*c^2)*k)*x^2 - 2*(a^2*b^3*c^3 - 10*a^3*b*c^4)*e - 2*(a^
3*b^2*c^3 + 8*a^4*c^4)*g + 6*(a^4*b^4 - 7*a^5*b^2*c + 8*a^6*c^2)*
k + ((5*a*b^4*c^3 - 37*a^2*b^2*c^4 + 44*a^3*c^5)*d - (a^2*b^3*c^3
 - 16*a^3*b*c^4)*f - 3*(a^3*b^2*c^3 + 4*a^4*c^4)*h - (a^4*b^2*c^2
 + 20*a^5*c^3)*j)*x)/(a^4*b^4*c^3 - 8*a^5*b^2*c^4 + 16*a^6*c^5 +
(a^2*b^4*c^5 - 8*a^3*b^2*c^6 + 16*a^4*c^7)*x^8 + 2*(a^2*b^5*c^4 -
 8*a^3*b^3*c^5 + 16*a^4*b*c^6)*x^6 + (a^2*b^6*c^3 - 6*a^3*b^4*c^4
 + 32*a^5*c^6)*x^4 + 2*(a^3*b^5*c^3 - 8*a^4*b^3*c^4 + 16*a^5*b*c^
5)*x^2) + 1/8*integrate((8*(a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2)*k
*x^3 - (12*a^2*b*c^3*h - 3*(b^3*c^3 - 8*a*b*c^4)*d - (a*b^2*c^3 +
 20*a^2*c^4)*f - (a^2*b^3*c - 16*a^3*b*c^2)*j)*x^2 + 3*(b^4*c^2 -
 9*a*b^2*c^3 + 28*a^2*c^4)*d + (a*b^3*c^2 - 16*a^2*b*c^3)*f + 3*(
a^2*b^2*c^2 + 4*a^3*c^3)*h + (a^3*b^2*c + 20*a^4*c^2)*j + 8*(6*a^
2*c^4*e - 3*a^2*b*c^3*g + (a^2*b^2*c^2 + 2*a^3*c^3)*i + (a^3*b^3
- 7*a^4*b*c)*k)*x)/(c*x^4 + b*x^2 + a), x)/(a^2*b^4*c^2 - 8*a^3*b
^2*c^3 + 16*a^4*c^4)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((k*x^11 + j*x^8 + i*x^5 + h*x^4 + g*x^3 + f*x^2 + e*x + d)/(c*x^4 + b*x^2
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((k*x**11+j*x**8+i*x**5+h*x**4+g*x**3+f*x**2+e*x+d)/(c*x**4+b*x**2+a)
```

```
[Out] Timed out
```

GIAC/XCAS [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((k*x^11 + j*x^8 + i*x^5 + h*x^4 + g*x^3 + f*x^2 + e*x + d)/(c*x^4 + b*x^2
```

```
[Out] Timed out
```

3.60 $\int \left(a+bx^2+cx^4\right)^3 \left(ad+aex+(bd+af)x^2+bex^3+(cd+bf)x^4+ce\right.$

Optimal. Leaf size=416

$$a^4dx+\frac{1}{2}a^4ex^2+\frac{1}{3}a^3x^3(af+4bd)+a^3bex^4+\frac{2}{5}a^2x^5\left(2abf+2acd+3b^2d\right)+\frac{1}{3}a^2ex^6\left(2ac+3b^2\right)+\frac{1}{10}ex^{10}\left(6a^2c^2+12ab^2c+b^4\right)+\frac{2}{7}ax^7\left(2a^2cf+3ab^2f+6abcd+2b^3d\right)+\frac{1}{11}x^{11}\left(6a^2c^2f+12ab^2cf+12abc^2d+b^4f+4b^3cd\right)+\frac{1}{9}x^9\left(12a^2bcf+6a^2c^2d+4ab^3f+12ab^2cd+b^4d\right)+\frac{2}{15}c^2x^{15}\left(2acf+3b^2f+2bcd\right)+\frac{1}{7}c^2ex^{14}\left(2ac+3b^2\right)+\frac{1}{3}bcex^{12}\left(3ac+b^2\right)+\frac{1}{2}abex^8\left(3ac+b^2\right)+\frac{2}{13}cx^{13}\left(6abcf+2ac^2d+2b^3f+3b^2cd\right)+\frac{1}{17}c^3x^{17}(4bf+cd)+\frac{1}{4}bc^3ex^{16}+\frac{1}{18}c^4ex^{18}+\frac{1}{19}c^4fx^{19}$$

```
[Out] a^4*d*x + (a^4*e*x^2)/2 + (a^3*(4*b*d + a*f)*x^3)/3 + a^3*b*e*x^4
 + (2*a^2*(3*b^2*d + 2*a*c*d + 2*a*b*f)*x^5)/5 + (a^2*(3*b^2 + 2*
a*c)*e*x^6)/3 + (2*a*(2*b^3*d + 6*a*b*c*d + 3*a*b^2*f + 2*a^2*c*f
)*x^7)/7 + (a*b*(b^2 + 3*a*c)*e*x^8)/2 + ((b^4*d + 12*a*b^2*c*d +
 6*a^2*c^2*d + 4*a*b^3*f + 12*a^2*b*c*f)*x^9)/9 + ((b^4 + 12*a*b^
2*c + 6*a^2*c^2)*e*x^10)/10 + ((4*b^3*c*d + 12*a*b*c^2*d + b^4*f
+ 12*a*b^2*c*f + 6*a^2*c^2*f)*x^11)/11 + (b*c*(b^2 + 3*a*c)*e*x^1
2)/3 + (2*c*(3*b^2*c*d + 2*a*c^2*d + 2*b^3*f + 6*a*b*c*f)*x^13)/1
3 + (c^2*(3*b^2 + 2*a*c)*e*x^14)/7 + (2*c^2*(2*b*c*d + 3*b^2*f +
2*a*c*f)*x^15)/15 + (b*c^3*e*x^16)/4 + (c^3*(c*d + 4*b*f)*x^17)/1
7 + (c^4*e*x^18)/18 + (c^4*f*x^19)/19
```

Rubi [A] time = 1.22087, antiderivative size = 416, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 63, $\frac{\text{number of rules}}{\text{integrand size}} = 0.016$

$$a^4dx+\frac{1}{2}a^4ex^2+\frac{1}{3}a^3x^3(af+4bd)+a^3bex^4+\frac{2}{5}a^2x^5\left(2abf+2acd+3b^2d\right)+\frac{1}{3}a^2ex^6\left(2ac+3b^2\right)+\frac{1}{10}ex^{10}\left(6a^2c^2+12ab^2c+b^4\right)+\frac{2}{7}ax^7\left(2a^2cf+3ab^2f+6abcd+2b^3d\right)+\frac{1}{11}x^{11}\left(6a^2c^2f+12ab^2cf+12abc^2d+b^4f+4b^3cd\right)+\frac{1}{9}x^9\left(12a^2bcf+6a^2c^2d+4ab^3f+12ab^2cd+b^4d\right)+\frac{2}{15}c^2x^{15}\left(2acf+3b^2f+2bcd\right)+\frac{1}{7}c^2ex^{14}\left(2ac+3b^2\right)+\frac{1}{3}bcex^{12}\left(3ac+b^2\right)+\frac{1}{2}abex^8\left(3ac+b^2\right)+\frac{2}{13}cx^{13}\left(6abcf+2ac^2d+2b^3f+3b^2cd\right)+\frac{1}{17}c^3x^{17}(4bf+cd)+\frac{1}{4}bc^3ex^{16}+\frac{1}{18}c^4ex^{18}+\frac{1}{19}c^4fx^{19}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^3*(a*d + a*e*x + (b*d + a*f)*x^2 + b*e*x^3 + (c*d + b*f)*x^
```

```
[Out] a^4*d*x + (a^4*e*x^2)/2 + (a^3*(4*b*d + a*f)*x^3)/3 + a^3*b*e*x^4
 + (2*a^2*(3*b^2*d + 2*a*c*d + 2*a*b*f)*x^5)/5 + (a^2*(3*b^2 + 2*
a*c)*e*x^6)/3 + (2*a*(2*b^3*d + 6*a*b*c*d + 3*a*b^2*f + 2*a^2*c*f
)*x^7)/7 + (a*b*(b^2 + 3*a*c)*e*x^8)/2 + ((b^4*d + 12*a*b^2*c*d +
 6*a^2*c^2*d + 4*a*b^3*f + 12*a^2*b*c*f)*x^9)/9 + ((b^4 + 12*a*b^
2*c + 6*a^2*c^2)*e*x^10)/10 + ((4*b^3*c*d + 12*a*b*c^2*d + b^4*f
+ 12*a*b^2*c*f + 6*a^2*c^2*f)*x^11)/11 + (b*c*(b^2 + 3*a*c)*e*x^1
2)/3 + (2*c*(3*b^2*c*d + 2*a*c^2*d + 2*b^3*f + 6*a*b*c*f)*x^13)/1
3 + (c^2*(3*b^2 + 2*a*c)*e*x^14)/7 + (2*c^2*(2*b*c*d + 3*b^2*f +
2*a*c*f)*x^15)/15 + (b*c^3*e*x^16)/4 + (c^3*(c*d + 4*b*f)*x^17)/1
7 + (c^4*e*x^18)/18 + (c^4*f*x^19)/19
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^4e\int x\,dx+a^4\int d\,dx+a^3bex^4+\frac{a^3x^3\left(af+4bd\right)}{3}+\frac{a^2ex^6\left(2ac+3b^2\right)}{3}+\frac{2a^2x^5\left(2abf+2acd+3b^2d\right)}{5}+\frac{abex^8\left(3ac+b^2\right)}{2}+\frac{2ax^7\left(2a^2cf+3ab^2f+6abcd+2b^3d\right)}{7}+\frac{bc^3ex^{16}}{4}+\frac{bcex^{12}\left(3ac+b^2\right)}{3}+\frac{c^4ex^{18}}{18}+\frac{c^4fx^{19}}{19}+\frac{c^3x^{17}\left(4bf+cd\right)}{17}+\frac{c^2ex^{14}\left(2ac+3b^2\right)}{7}+\frac{2c^2x^{15}\left(2acf+3b^2f+2bcd\right)}{15}+\frac{2cx^{13}\left(6abcf+2ac^2d+2b^3f+3b^2cd\right)}{13}+\frac{ex^{10}\left(6a^2c^2+12ab^2c+b^4\right)}{10}+x^{11}\left(\frac{6a^2c^2f}{11}+\frac{12ab^2cf}{11}+\frac{12abc^2d}{11}+\frac{b^4f}{11}+\frac{4b^3cd}{11}\right)+x^9\left(\frac{4a^2bcf}{3}+\frac{2a^2c^2d}{3}+\frac{4ab^3f}{9}+\frac{4ab^2cd}{3}+\frac{b^4d}{9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((c*x**4+b*x**2+a)**3*(a*d+a*e*x+(a*f+b*d)*x**2+b*e*x**3+(b*f+c*
```

```
[Out] a**4*e*Integral(x, x) + a**4*Integral(d, x) + a**3*b*e*x**4 + a**
3*x**3*(a*f + 4*b*d)/3 + a**2*e*x**6*(2*a*c + 3*b**2)/3 + 2*a**2*
x**5*(2*a*b*f + 2*a*c*d + 3*b**2*d)/5 + a*b*e*x**8*(3*a*c + b**2)
/2 + 2*a*x**7*(2*a**2*c*f + 3*a*b**2*f + 6*a*b*c*d + 2*b**3*d)/7
+ b*c**3*e*x**16/4 + b*c*e*x**12*(3*a*c + b**2)/3 + c**4*e*x**18/
18 + c**4*f*x**19/19 + c**3*x**17*(4*b*f + c*d)/17 + c**2*e*x**14
*(2*a*c + 3*b**2)/7 + 2*c**2*x**15*(2*a*c*f + 3*b**2*f + 2*b*c*d)
/15 + 2*c*x**13*(6*a*b*c*f + 2*a*c**2*d + 2*b**3*f + 3*b**2*c*d)/
13 + e*x**10*(6*a**2*c**2 + 12*a*b**2*c + b**4)/10 + x**11*(6*a**
2*c**2*f/11 + 12*a*b**2*c*f/11 + 12*a*b*c**2*d/11 + b**4*f/11 + 4
*b**3*c*d/11) + x**9*(4*a**2*b*c*f/3 + 2*a**2*c**2*d/3 + 4*a*b**3
*f/9 + 4*a*b**2*c*d/3 + b**4*d/9)
```

Mathematica [A] time = 0.208483, size = 416, normalized size = 1.

$$a^4dx+\frac{1}{2}a^4ex^2+\frac{1}{3}a^3x^3(af+4bd)+a^3bex^4+\frac{2}{5}a^2x^5\left(2abf+2acd+3b^2d\right)+\frac{1}{3}a^2ex^6\left(2ac+3b^2\right)+\frac{1}{10}ex^{10}\left(6a^2c^2+12ab^2c+b^4\right)+\frac{2}{7}ax^7\left(2a^2cf+3ab^2f+6abcd+2b^3d\right)+\frac{1}{11}x^{11}\left(6a^2c^2f+12ab^2cf+12abc^2d+b^4f+4b^3cd\right)+\frac{1}{9}x^9\left(12a^2bcf+6a^2c^2d+4ab^3f+12ab^2cd+b^4d\right)+\frac{2}{15}c^2x^{15}\left(2acf+3b^2f+2bcd\right)+\frac{1}{7}c^2ex^{14}\left(2ac+3b^2\right)+\frac{1}{3}bcex^{12}\left(3ac+b^2\right)+\frac{1}{2}abex^8\left(3ac+b^2\right)+\frac{2}{13}cx^{13}\left(6abcf+2ac^2d+2b^3f+3b^2cd\right)+\frac{1}{17}c^3x^{17}(4bf+cd)+\frac{1}{4}bc^3ex^{16}+\frac{1}{18}c^4ex^{18}+\frac{1}{19}c^4fx^{19}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^3*(a*d + a*e*x + (b*d + a*f)*x^2 + b*e*x^3 + (c*d + b
```

```
[Out] a^4*d*x + (a^4*e*x^2)/2 + (a^3*(4*b*d + a*f)*x^3)/3 + a^3*b*e*x^4
 + (2*a^2*(3*b^2*d + 2*a*c*d + 2*a*b*f)*x^5)/5 + (a^2*(3*b^2 + 2*
a*c)*e*x^6)/3 + (2*a*(2*b^3*d + 6*a*b*c*d + 3*a*b^2*f + 2*a^2*c*f
)*x^7)/7 + (a*b*(b^2 + 3*a*c)*e*x^8)/2 + ((b^4*d + 12*a*b^2*c*d +
 6*a^2*c^2*d + 4*a*b^3*f + 12*a^2*b*c*f)*x^9)/9 + ((b^4 + 12*a*b^
2*c + 6*a^2*c^2)*e*x^10)/10 + ((4*b^3*c*d + 12*a*b*c^2*d + b^4*f
+ 12*a*b^2*c*f + 6*a^2*c^2*f)*x^11)/11 + (b*c*(b^2 + 3*a*c)*e*x^1
2)/3 + (2*c*(3*b^2*c*d + 2*a*c^2*d + 2*b^3*f + 6*a*b*c*f)*x^13)/1
3 + (c^2*(3*b^2 + 2*a*c)*e*x^14)/7 + (2*c^2*(2*b*c*d + 3*b^2*f +
2*a*c*f)*x^15)/15 + (b*c^3*e*x^16)/4 + (c^3*(c*d + 4*b*f)*x^17)/1
7 + (c^4*e*x^18)/18 + (c^4*f*x^19)/19
```

Maple [B] time = 0.002, size = 829, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^3*(a*d+a*e*x+(a*f+b*d)*x^2+b*e*x^3+(b*f+c*d)*x^4+c*e*x^5+c
```

```
[Out] 1/19*c^4*f*x^19+1/18*c^4*e*x^18+1/17*(3*b*c^3*f+c^3*(b*f+c*d))*x^
17+1/4*b*c^3*e*x^16+1/15*((a*c^2+2*b^2*c+c*(2*a*c+b^2))*c*f+3*c^2
*b*(b*f+c*d)+c^3*(a*f+b*d))*x^15+1/14*((a*c^2+2*b^2*c+c*(2*a*c+b^
2))*c*e+3*c^2*b^2*e+a*c^3*e)*x^14+1/13*((4*a*b*c+b*(2*a*c+b^2))*c
*f+(a*c^2+2*b^2*c+c*(2*a*c+b^2))*(b*f+c*d)+3*c^2*b*(a*f+b*d)+c^3*
a*d)*x^13+1/12*((4*a*b*c+b*(2*a*c+b^2))*c*e+(a*c^2+2*b^2*c+c*(2*a
```

```
*c+b^2))*b*e+3*a*b*c^2*e)*x^12+1/11*((a*(2*a*c+b^2)+2*a*b^2+a^2*c
)*c*f+(4*a*b*c+b*(2*a*c+b^2))*(b*f+c*d)+(a*c^2+2*b^2*c+c*(2*a*c+b
^2))*(a*f+b*d)+3*a*b*c^2*d)*x^11+1/10*((a*(2*a*c+b^2)+2*a*b^2+a^2
*c)*c*e+(4*a*b*c+b*(2*a*c+b^2))*b*e+(a*c^2+2*b^2*c+c*(2*a*c+b^2))
*a*e)*x^10+1/9*(3*a^2*b*c*f+(a*(2*a*c+b^2)+2*a*b^2+a^2*c)*(b*f+c*
d)+(4*a*b*c+b*(2*a*c+b^2))*(a*f+b*d)+(a*c^2+2*b^2*c+c*(2*a*c+b^2)
)*a*d)*x^9+1/8*(3*a^2*b*c*e+(a*(2*a*c+b^2)+2*a*b^2+a^2*c)*b*e+(4*
a*b*c+b*(2*a*c+b^2))*a*e)*x^8+1/7*(a^3*c*f+3*a^2*b*(b*f+c*d)+(a*(
2*a*c+b^2)+2*a*b^2+a^2*c)*(a*f+b*d)+(4*a*b*c+b*(2*a*c+b^2))*a*d)*
x^7+1/6*(a^3*c*e+3*a^2*b^2*e+(a*(2*a*c+b^2)+2*a*b^2+a^2*c)*a*e)*x
^6+1/5*(a^3*(b*f+c*d)+3*a^2*b*(a*f+b*d)+(a*(2*a*c+b^2)+2*a*b^2+a^
2*c)*a*d)*x^5+a^3*b*e*x^4+1/3*(a^3*(a*f+b*d)+3*a^3*b*d)*x^3+1/2*a
^4*e*x^2+a^4*d*x
```

Maxima [A] time = 0.710354, size = 564, normalized size = 1.36

$$\frac{1}{19}c^4fx^{19}+\frac{1}{18}c^4ex^{18}+\frac{1}{4}bc^3ex^{16}+\frac{1}{17}\left(c^4d+4\,bc^3f\right)x^{17}+\frac{1}{7}\left(3\,b^2c^2+2\,ac^3\right)ex^{14}+\frac{2}{15}\left(2\,bc^3d+\left(3\,b^2c^2+2\,ac^3\right)f\right)x^{15}+\frac{1}{3}\left(b^3c+3\,abc^2\right)ex^{12}+\frac{2}{13}\left(\left(3\,b^2c^2+2\,ac^3\right)d+2\left(b^3c+3\,abc^2\right)f\right)x^{13}+\frac{1}{10}\left(b^4+12\,ab^2c+6\,a^2c^2\right)ex^{10}+\frac{1}{11}\left(4\left(b^3c+3\,abc^2\right)d+\left(b^4+12\,ab^2c+6\,a^2c^2\right)f\right)x^{11}+\frac{1}{2}\left(ab^3+3\,a^2bc\right)ex^8+\frac{1}{9}\left(\left(b^4+12\,ab^2c+6\,a^2c^2\right)d+4\left(ab^3+3\,a^2bc\right)f\right)x^9+a^3bex^4+\frac{1}{3}\left(3\,a^2b^2+2\,a^3c\right)ex^6+\frac{2}{7}\left(2\left(ab^3+3\,a^2bc\right)d+\left(3\,a^2b^2+2\,a^3c\right)f\right)x^7+\frac{1}{2}a^4ex^2+a^4dx+\frac{2}{5}\left(2\,a^3bf+\left(3\,a^2b^2+2\,a^3c\right)d\right)x^5+\frac{1}{3}\left(4\,a^3bd+a^4f\right)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*f*x^6 + c*e*x^5 + b*e*x^3 + (c*d + b*f)*x^4 + a*e*x + (b*d + a*f)*x^2
```

```
[Out] 1/19*c^4*f*x^19 + 1/18*c^4*e*x^18 + 1/4*b*c^3*e*x^16 + 1/17*(c^4*
d + 4*b*c^3*f)*x^17 + 1/7*(3*b^2*c^2 + 2*a*c^3)*e*x^14 + 2/15*(2*
b*c^3*d + (3*b^2*c^2 + 2*a*c^3)*f)*x^15 + 1/3*(b^3*c + 3*a*b*c^2)
*e*x^12 + 2/13*((3*b^2*c^2 + 2*a*c^3)*d + 2*(b^3*c + 3*a*b*c^2)*f
)*x^13 + 1/10*(b^4 + 12*a*b^2*c + 6*a^2*c^2)*e*x^10 + 1/11*(4*(b^
3*c + 3*a*b*c^2)*d + (b^4 + 12*a*b^2*c + 6*a^2*c^2)*f)*x^11 + 1/2
*(a*b^3 + 3*a^2*b*c)*e*x^8 + 1/9*((b^4 + 12*a*b^2*c + 6*a^2*c^2)*
d + 4*(a*b^3 + 3*a^2*b*c)*f)*x^9 + a^3*b*e*x^4 + 1/3*(3*a^2*b^2 +
 2*a^3*c)*e*x^6 + 2/7*(2*(a*b^3 + 3*a^2*b*c)*d + (3*a^2*b^2 + 2*a
^3*c)*f)*x^7 + 1/2*a^4*e*x^2 + a^4*d*x + 2/5*(2*a^3*b*f + (3*a^2*
b^2 + 2*a^3*c)*d)*x^5 + 1/3*(4*a^3*b*d + a^4*f)*x^3
```

<u>Fricas</u> [A] time = 0.248385, size = 1, normalized size = 0.

$$\frac{1}{19}x^{19}fc^4 + \frac{1}{18}x^{18}ec^4 + \frac{1}{17}x^{17}dc^4 + \frac{4}{17}x^{17}fc^3b + \frac{1}{4}x^{16}ec^3b + \frac{4}{15}x^{15}dc^3b + \frac{2}{5}x^{15}fc^2b^2 + \frac{4}{15}x^{15}fc^3a + \frac{3}{7}x^{14}ec^2b^2 + \frac{2}{7}x^{14}ec^3a + \frac{6}{13}x^{13}dc^2b^2 + \frac{4}{13}x^{13}fcb^3 + \frac{4}{13}x^{13}dc^3a + \frac{12}{13}x^{13}fc^2ba + \frac{1}{3}x^{12}ecb^3 + x^{12}ec^2ba + \frac{4}{11}x^{11}dcb^3 + \frac{1}{11}x^{11}fb^4 + \frac{12}{11}x^{11}dc^2ba + \frac{12}{11}x^{11}fcb^2a + \frac{6}{11}x^{11}fc^2a^2 + \frac{1}{10}x^{10}eb^4 + \frac{6}{5}x^{10}ecb^2a + \frac{3}{5}x^{10}ec^2a^2 + \frac{1}{9}x^9db^4 + \frac{4}{3}x^9dcb^2a + \frac{4}{9}x^9fb^3a + \frac{2}{3}x^9dc^2a^2 + \frac{4}{3}x^9fcba^2 + \frac{1}{2}x^8eb^3a + \frac{3}{2}x^8ecba^2 + \frac{4}{7}x^7db^3a + \frac{12}{7}x^7dcba^2 + \frac{6}{7}x^7fb^2a^2 + \frac{4}{7}x^7fca^3 + x^6eb^2a^2 + \frac{2}{3}x^6eca^3 + \frac{6}{5}x^5db^2a^2 + \frac{4}{5}x^5dca^3 + \frac{4}{5}x^5fba^3 + x^4eba^3 + \frac{4}{3}x^3dba^3 + \frac{1}{3}x^3fa^4 + \frac{1}{2}x^2ea^4 + xda^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*f*x^6 + c*e*x^5 + b*e*x^3 + (c*d + b*f)*x^4 + a*e*x + (b*d + a*f)*x^2
```

```
[Out] 1/19*x^19*f*c^4 + 1/18*x^18*e*c^4 + 1/17*x^17*d*c^4 + 4/17*x^17*f
*c^3*b + 1/4*x^16*e*c^3*b + 4/15*x^15*d*c^3*b + 2/5*x^15*f*c^2*b^
2 + 4/15*x^15*f*c^3*a + 3/7*x^14*e*c^2*b^2 + 2/7*x^14*e*c^3*a + 6
/13*x^13*d*c^2*b^2 + 4/13*x^13*f*c*b^3 + 4/13*x^13*d*c^3*a + 12/1
3*x^13*f*c^2*b*a + 1/3*x^12*e*c*b^3 + x^12*e*c^2*b*a + 4/11*x^11*
d*c*b^3 + 1/11*x^11*f*b^4 + 12/11*x^11*d*c^2*b*a + 12/11*x^11*f*c
*b^2*a + 6/11*x^11*f*c^2*a^2 + 1/10*x^10*e*b^4 + 6/5*x^10*e*c*b^2
*a + 3/5*x^10*e*c^2*a^2 + 1/9*x^9*d*b^4 + 4/3*x^9*d*c*b^2*a + 4/9
*x^9*f*b^3*a + 2/3*x^9*d*c^2*a^2 + 4/3*x^9*f*c*b*a^2 + 1/2*x^8*e*
b^3*a + 3/2*x^8*e*c*b*a^2 + 4/7*x^7*d*b^3*a + 12/7*x^7*d*c*b*a^2
+ 6/7*x^7*f*b^2*a^2 + 4/7*x^7*f*c*a^3 + x^6*e*b^2*a^2 + 2/3*x^6*e
*c*a^3 + 6/5*x^5*d*b^2*a^2 + 4/5*x^5*d*c*a^3 + 4/5*x^5*f*b*a^3 +
x^4*e*b*a^3 + 4/3*x^3*d*b*a^3 + 1/3*x^3*f*a^4 + 1/2*x^2*e*a^4 + x
*d*a^4
```

Sympy [A] time = 0.359606, size = 503, normalized size = 1.21

$$a^4dx+\frac{a^4ex^2}{2}+a^3bex^4+\frac{bc^3ex^{16}}{4}+\frac{c^4ex^{18}}{18}+\frac{c^4fx^{19}}{19}+x^{17}\left(\frac{4bc^3f}{17}+\frac{c^4d}{17}\right)+x^{15}\left(\frac{4ac^3f}{15}+\frac{2b^2c^2f}{5}+\frac{4bc^3d}{15}\right)+x^{14}\left(\frac{2ac^3e}{7}+\frac{3b^2c^2e}{7}\right)+x^{13}\left(\frac{12abc^2f}{13}+\frac{4ac^3d}{13}+\frac{4b^3cf}{13}+\frac{6b^2c^2d}{13}\right)+x^{12}\left(abc^2e+\frac{b^3ce}{3}\right)+x^{11}\left(\frac{6a^2c^2f}{11}+\frac{12ab^2cf}{11}+\frac{12abc^2d}{11}+\frac{b^4f}{11}+\frac{4b^3cd}{11}\right)+x^{10}\left(\frac{3a^2c^2e}{5}+\frac{6ab^2ce}{5}+\frac{b^4e}{10}\right)+x^9\left(\frac{4a^2bcf}{3}+\frac{2a^2c^2d}{3}+\frac{4ab^3f}{9}+\frac{4ab^2cd}{3}+\frac{b^4d}{9}\right)+x^8\left(\frac{3a^2bce}{2}+\frac{ab^3e}{2}\right)+x^7\left(\frac{4a^3cf}{7}+\frac{6a^2b^2f}{7}+\frac{12a^2bcd}{7}+\frac{4ab^3d}{7}\right)+x^6\left(\frac{2a^3ce}{3}+a^2b^2e\right)+x^5\left(\frac{4a^3bf}{5}+\frac{4a^3cd}{5}+\frac{6a^2b^2d}{5}\right)+x^3\left(\frac{a^4f}{3}+\frac{4a^3bd}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**3*(a*d+a*e*x+(a*f+b*d)*x**2+b*e*x**3+(b*f+c*d)*x*
```

```
[Out] a**4*d*x + a**4*e*x**2/2 + a**3*b*e*x**4 + b*c**3*e*x**16/4 + c**
4*e*x**18/18 + c**4*f*x**19/19 + x**17*(4*b*c**3*f/17 + c**4*d/17
) + x**15*(4*a*c**3*f/15 + 2*b**2*c**2*f/5 + 4*b*c**3*d/15) + x**
14*(2*a*c**3*e/7 + 3*b**2*c**2*e/7) + x**13*(12*a*b*c**2*f/13 + 4
*a*c**3*d/13 + 4*b**3*c*f/13 + 6*b**2*c**2*d/13) + x**12*(a*b*c**
2*e + b**3*c*e/3) + x**11*(6*a**2*c**2*f/11 + 12*a*b**2*c*f/11 +
12*a*b*c**2*d/11 + b**4*f/11 + 4*b**3*c*d/11) + x**10*(3*a**2*c**
2*e/5 + 6*a*b**2*c*e/5 + b**4*e/10) + x**9*(4*a**2*b*c*f/3 + 2*a*
*2*c**2*d/3 + 4*a*b**3*f/9 + 4*a*b**2*c*d/3 + b**4*d/9) + x**8*(3
*a**2*b*c*e/2 + a*b**3*e/2) + x**7*(4*a**3*c*f/7 + 6*a**2*b**2*f/
7 + 12*a**2*b*c*d/7 + 4*a*b**3*d/7) + x**6*(2*a**3*c*e/3 + a**2*b
**2*e) + x**5*(4*a**3*b*f/5 + 4*a**3*c*d/5 + 6*a**2*b**2*d/5) + x
**3*(a**4*f/3 + 4*a**3*b*d/3)
```

<u>GIAC/XCAS</u> [A] time = 0.30358, size = 645, normalized size = 1.55

$$\frac{1}{19}\,c^4fx^{19}+\frac{1}{18}\,c^4x^{18}e+\frac{1}{17}\,c^4dx^{17}+\frac{4}{17}\,bc^3fx^{17}+\frac{1}{4}\,bc^3x^{16}e+\frac{4}{15}\,bc^3dx^{15}+\frac{2}{5}\,b^2c^2fx^{15}+\frac{4}{15}\,ac^3fx^{15}+\frac{3}{7}\,b^2c^2x^{14}e+\frac{2}{7}\,ac^3x^{14}e+\frac{6}{13}\,b^2c^2dx^{13}+\frac{4}{13}\,ac^3dx^{13}+\frac{4}{13}\,b^3cfx^{13}+\frac{12}{13}\,abc^2fx^{13}+\frac{1}{3}\,b^3cx^{12}e+abc^2x^{12}e+\frac{4}{11}\,b^3cdx^{11}+\frac{12}{11}\,abc^2dx^{11}+\frac{1}{11}\,b^4fx^{11}+\frac{12}{11}\,ab^2cfx^{11}+\frac{6}{11}\,a^2c^2fx^{11}+\frac{1}{10}\,b^4x^{10}e+\frac{6}{5}\,ab^2cx^{10}e+\frac{3}{5}\,a^2c^2x^{10}e+\frac{1}{9}\,b^4dx^9+\frac{4}{3}\,ab^2cdx^9+\frac{2}{3}\,a^2c^2dx^9+\frac{4}{9}\,ab^3fx^9+\frac{4}{3}\,a^2bcfx^9+\frac{1}{2}\,ab^3x^8e+\frac{3}{2}\,a^2bcx^8e+\frac{4}{7}\,ab^3dx^7+\frac{12}{7}\,a^2bcdx^7+\frac{6}{7}\,a^2b^2fx^7+\frac{4}{7}\,a^3cfx^7+a^2b^2x^6e+\frac{2}{3}\,a^3cx^6e+\frac{6}{5}\,a^2b^2dx^5+\frac{4}{5}\,a^3cdx^5+\frac{4}{5}\,a^3bfx^5+a^3bx^4e+\frac{4}{3}\,a^3bdx^3+\frac{1}{3}\,a^4fx^3+\frac{1}{2}\,a^4x^2e+a^4dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*f*x^6 + c*e*x^5 + b*e*x^3 + (c*d + b*f)*x^4 + a*e*x + (b*d + a*f)*x^2
```

```
[Out] 1/19*c^4*f*x^19 + 1/18*c^4*x^18*e + 1/17*c^4*d*x^17 + 4/17*b*c^3*
f*x^17 + 1/4*b*c^3*x^16*e + 4/15*b*c^3*d*x^15 + 2/5*b^2*c^2*f*x^1
5 + 4/15*a*c^3*f*x^15 + 3/7*b^2*c^2*x^14*e + 2/7*a*c^3*x^14*e + 6
/13*b^2*c^2*d*x^13 + 4/13*a*c^3*d*x^13 + 4/13*b^3*c*f*x^13 + 12/1
3*a*b*c^2*f*x^13 + 1/3*b^3*c*x^12*e + a*b*c^2*x^12*e + 4/11*b^3*c
*d*x^11 + 12/11*a*b*c^2*d*x^11 + 1/11*b^4*f*x^11 + 12/11*a*b^2*c*
f*x^11 + 6/11*a^2*c^2*f*x^11 + 1/10*b^4*x^10*e + 6/5*a*b^2*c*x^10
*e + 3/5*a^2*c^2*x^10*e + 1/9*b^4*d*x^9 + 4/3*a*b^2*c*d*x^9 + 2/3
*a^2*c^2*d*x^9 + 4/9*a*b^3*f*x^9 + 4/3*a^2*b*c*f*x^9 + 1/2*a*b^3*
x^8*e + 3/2*a^2*b*c*x^8*e + 4/7*a*b^3*d*x^7 + 12/7*a^2*b*c*d*x^7
+ 6/7*a^2*b^2*f*x^7 + 4/7*a^3*c*f*x^7 + a^2*b^2*x^6*e + 2/3*a^3*c
*x^6*e + 6/5*a^2*b^2*d*x^5 + 4/5*a^3*c*d*x^5 + 4/5*a^3*b*f*x^5 +
a^3*b*x^4*e + 4/3*a^3*b*d*x^3 + 1/3*a^4*f*x^3 + 1/2*a^4*x^2*e + a
^4*d*x
```

3.61 $\int \left(a+bx^2+cx^4\right)^2 \left(ad+aex+(bd+af)x^2+bex^3+(cd+bf)x^4+ce$

Optimal. Leaf size=259

$$a^3dx+\frac{1}{2}a^3ex^2+\frac{1}{7}x^7\left(3a^2cf+3ab^2f+6abcd+b^3d\right)+\frac{1}{3}a^2x^3(af+3bd)+\frac{3}{4}a^2bex^4+\frac{3}{11}cx^{11}\left(acf+b^2f+bcd\right)+\frac{3}{5}ax^5\left(abf+acd+b^2d\right)+\frac{3}{10}cex^{10}\left(ac+b^2\right)+\frac{1}{8}bex^8\left(6ac+b^2\right)+\frac{1}{2}aex^6\left(ac+b^2\right)+\frac{1}{9}x^9\left(6abcf+3ac^2d+b^3f+3b^2cd\right)+\frac{1}{13}c^2x^{13}(3bf+cd)+\frac{1}{4}bc^2ex^{12}+\frac{1}{14}c^3ex^{14}+\frac{1}{15}c^3fx^{15}$$

```
[Out] a^3*d*x + (a^3*e*x^2)/2 + (a^2*(3*b*d + a*f)*x^3)/3 + (3*a^2*b*e*
x^4)/4 + (3*a*(b^2*d + a*c*d + a*b*f)*x^5)/5 + (a*(b^2 + a*c)*e*x
^6)/2 + ((b^3*d + 6*a*b*c*d + 3*a*b^2*f + 3*a^2*c*f)*x^7)/7 + (b*
(b^2 + 6*a*c)*e*x^8)/8 + ((3*b^2*c*d + 3*a*c^2*d + b^3*f + 6*a*b*
c*f)*x^9)/9 + (3*c*(b^2 + a*c)*e*x^10)/10 + (3*c*(b*c*d + b^2*f +
 a*c*f)*x^11)/11 + (b*c^2*e*x^12)/4 + (c^2*(c*d + 3*b*f)*x^13)/13
 + (c^3*e*x^14)/14 + (c^3*f*x^15)/15
```

Rubi [A] time = 0.676424, antiderivative size = 259, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 63, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.016

$$a^3dx+\frac{1}{2}a^3ex^2+\frac{1}{7}x^7\left(3a^2cf+3ab^2f+6abcd+b^3d\right)+\frac{1}{3}a^2x^3(af+3bd)+\frac{3}{4}a^2bex^4+\frac{3}{11}cx^{11}\left(acf+b^2f+bcd\right)+\frac{3}{5}ax^5\left(abf+acd+b^2d\right)+\frac{3}{10}cex^{10}\left(ac+b^2\right)+\frac{1}{8}bex^8\left(6ac+b^2\right)+\frac{1}{2}aex^6\left(ac+b^2\right)+\frac{1}{9}x^9\left(6abcf+3ac^2d+b^3f+3b^2cd\right)+\frac{1}{13}c^2x^{13}(3bf+cd)+\frac{1}{4}bc^2ex^{12}+\frac{1}{14}c^3ex^{14}+\frac{1}{15}c^3fx^{15}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^2*(a*d + a*e*x + (b*d + a*f)*x^2 + b*e*x^3 + (c*d + b*f)*x^
```

```
[Out] a^3*d*x + (a^3*e*x^2)/2 + (a^2*(3*b*d + a*f)*x^3)/3 + (3*a^2*b*e*
x^4)/4 + (3*a*(b^2*d + a*c*d + a*b*f)*x^5)/5 + (a*(b^2 + a*c)*e*x
^6)/2 + ((b^3*d + 6*a*b*c*d + 3*a*b^2*f + 3*a^2*c*f)*x^7)/7 + (b*
(b^2 + 6*a*c)*e*x^8)/8 + ((3*b^2*c*d + 3*a*c^2*d + b^3*f + 6*a*b*
c*f)*x^9)/9 + (3*c*(b^2 + a*c)*e*x^10)/10 + (3*c*(b*c*d + b^2*f +
 a*c*f)*x^11)/11 + (b*c^2*e*x^12)/4 + (c^2*(c*d + 3*b*f)*x^13)/13
 + (c^3*e*x^14)/14 + (c^3*f*x^15)/15
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^3e\int x\,dx+a^3\int d\,dx+\frac{3a^2bex^4}{4}+\frac{a^2x^3(af+3bd)}{3}+\frac{aex^6(ac+b^2)}{2}+\frac{3ax^5(abf+acd+b^2d)}{5}+\frac{bc^2ex^{12}}{4}+\frac{bex^8(6ac+b^2)}{8}+\frac{c^3ex^{14}}{14}+\frac{c^3fx^{15}}{15}+\frac{c^2x^{13}(3bf+cd)}{13}+\frac{3cex^{10}(ac+b^2)}{10}+\frac{3cx^{11}(acf+b^2f+bcd)}{11}+x^9\left(\frac{2abcf}{3}+\frac{ac^2d}{3}+\frac{b^3f}{9}+\frac{b^2cd}{3}\right)+x^7\left(\frac{3a^2cf}{7}+\frac{3ab^2f}{7}+\frac{6abcd}{7}+\frac{b^3d}{7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((c*x**4+b*x**2+a)**2*(a*d+a*e*x+(a*f+b*d)*x**2+b*e*x**3+(b*f+c*
```

```
[Out] a**3*e*Integral(x, x) + a**3*Integral(d, x) + 3*a**2*b*e*x**4/4 +
 a**2*x**3*(a*f + 3*b*d)/3 + a*e*x**6*(a*c + b**2)/2 + 3*a*x**5*(
a*b*f + a*c*d + b**2*d)/5 + b*c**2*e*x**12/4 + b*e*x**8*(6*a*c +
b**2)/8 + c**3*e*x**14/14 + c**3*f*x**15/15 + c**2*x**13*(3*b*f +
 c*d)/13 + 3*c*e*x**10*(a*c + b**2)/10 + 3*c*x**11*(a*c*f + b**2*
f + b*c*d)/11 + x**9*(2*a*b*c*f/3 + a*c**2*d/3 + b**3*f/9 + b**2*
c*d/3) + x**7*(3*a**2*c*f/7 + 3*a*b**2*f/7 + 6*a*b*c*d/7 + b**3*d
/7)
```

Mathematica [A] time = 0.144787, size = 259, normalized size = 1.

$$a^3dx+\frac{1}{2}a^3ex^2+\frac{1}{7}x^7\left(3a^2cf+3ab^2f+6abcd+b^3d\right)+\frac{1}{3}a^2x^3(af+3bd)+\frac{3}{4}a^2bex^4+\frac{3}{11}cx^{11}\left(acf+b^2f+bcd\right)+\frac{3}{5}ax^5\left(abf+acd+b^2d\right)+\frac{3}{10}cex^{10}\left(ac+b^2\right)+\frac{1}{8}bex^8\left(6ac+b^2\right)+\frac{1}{2}aex^6\left(ac+b^2\right)+\frac{1}{9}x^9\left(6abcf+3ac^2d+b^3f+3b^2cd\right)+\frac{1}{13}c^2x^{13}(3bf+cd)+\frac{1}{4}bc^2ex^{12}+\frac{1}{14}c^3ex^{14}+\frac{1}{15}c^3fx^{15}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^2*(a*d + a*e*x + (b*d + a*f)*x^2 + b*e*x^3 + (c*d + b
```

```
[Out] a^3*d*x + (a^3*e*x^2)/2 + (a^2*(3*b*d + a*f)*x^3)/3 + (3*a^2*b*e*
x^4)/4 + (3*a*(b^2*d + a*c*d + a*b*f)*x^5)/5 + (a*(b^2 + a*c)*e*x
^6)/2 + ((b^3*d + 6*a*b*c*d + 3*a*b^2*f + 3*a^2*c*f)*x^7)/7 + (b*
(b^2 + 6*a*c)*e*x^8)/8 + ((3*b^2*c*d + 3*a*c^2*d + b^3*f + 6*a*b*
c*f)*x^9)/9 + (3*c*(b^2 + a*c)*e*x^10)/10 + (3*c*(b*c*d + b^2*f +
 a*c*f)*x^11)/11 + (b*c^2*e*x^12)/4 + (c^2*(c*d + 3*b*f)*x^13)/13
 + (c^3*e*x^14)/14 + (c^3*f*x^15)/15
```

Maple [A] time = 0.001, size = 354, normalized size = 1.4

$$\frac{c^3fx^{15}}{15}+\frac{c^3ex^{14}}{14}+\frac{\left(2bc^2f+c^2\left(bf+cd\right)\right)x^{13}}{13}+\frac{bc^2ex^{12}}{4}+\frac{\left(\left(2ac+b^2\right)cf+2bc\left(bf+cd\right)+c^2\left(fa+bd\right)\right)x^{11}}{11}+\frac{\left(\left(2ac+b^2\right)ce+2b^2ce+ac^2e\right)x^{10}}{10}+\frac{\left(2abcf+\left(2ac+b^2\right)\left(bf+cd\right)+2bc\left(fa+bd\right)+ac^2d\right)x^9}{9}+\frac{\left(4abce+\left(2ac+b^2\right)be\right)x^8}{8}+\frac{\left(a^2cf+2ab\left(bf+cd\right)+\left(2ac+b^2\right)\left(fa+bd\right)+2abcd\right)x^7}{7}+\frac{\left(a^2ce+2ab^2e+\left(2ac+b^2\right)ae\right)x^6}{6}+\frac{\left(a^2\left(bf+cd\right)+2ab\left(fa+bd\right)+\left(2ac+b^2\right)ad\right)x^5}{5}+\frac{3x^4a^2be}{4}+\frac{\left(a^2\left(fa+bd\right)+2a^2bd\right)x^3}{3}+\frac{x^2a^3e}{2}+a^3dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^2*(a*d+a*e*x+(a*f+b*d)*x^2+b*e*x^3+(b*f+c*d)*x^4+c*e*x^5+c
```

```
[Out] 1/15*c^3*f*x^15+1/14*c^3*e*x^14+1/13*(2*b*c^2*f+c^2*(b*f+c*d))*x^
13+1/4*b*c^2*e*x^12+1/11*((2*a*c+b^2)*c*f+2*b*c*(b*f+c*d)+c^2*(a*
f+b*d))*x^11+1/10*((2*a*c+b^2)*c*e+2*b^2*c*e+a*c^2*e)*x^10+1/9*(2
*a*b*c*f+(2*a*c+b^2)*(b*f+c*d)+2*b*c*(a*f+b*d)+a*c^2*d)*x^9+1/8*(
4*a*b*c*e+(2*a*c+b^2)*b*e)*x^8+1/7*(a^2*c*f+2*a*b*(b*f+c*d)+(2*a*
c+b^2)*(a*f+b*d)+2*a*b*c*d)*x^7+1/6*(a^2*c*e+2*a*b^2*e+(2*a*c+b^2
)*a*e)*x^6+1/5*(a^2*(b*f+c*d)+2*a*b*(a*f+b*d)+(2*a*c+b^2)*a*d)*x^
5+3/4*x^4*a^2*b*e+1/3*(a^2*(a*f+b*d)+2*a^2*b*d)*x^3+1/2*x^2*a^3*e
+a^3*d*x
```

Maxima [A] time = 0.704771, size = 339, normalized size = 1.31

$$\frac{1}{15}c^3fx^{15}+\frac{1}{14}c^3ex^{14}+\frac{1}{4}bc^2ex^{12}+\frac{1}{13}\left(c^3d+3bc^2f\right)x^{13}+\frac{3}{10}\left(b^2c+ac^2\right)ex^{10}+\frac{3}{11}\left(bc^2d+\left(b^2c+ac^2\right)f\right)x^{11}+\frac{1}{8}\left(b^3+6abc\right)ex^8+\frac{1}{9}\left(3\left(b^2c+ac^2\right)d+\left(b^3+6abc\right)f\right)x^9+\frac{3}{4}a^2bex^4+\frac{1}{2}\left(ab^2+a^2c\right)ex^6+\frac{1}{7}\left(\left(b^3+6abc\right)d+3\left(ab^2+a^2c\right)f\right)x^7+\frac{1}{2}a^3ex^2+\frac{3}{5}\left(a^2bf+\left(ab^2+a^2c\right)d\right)x^5+a^3dx+\frac{1}{3}\left(3a^2bd+a^3f\right)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*f*x^6 + c*e*x^5 + b*e*x^3 + (c*d + b*f)*x^4 + a*e*x + (b*d + a*f)*x^2
```

```
[Out] 1/15*c^3*f*x^15 + 1/14*c^3*e*x^14 + 1/4*b*c^2*e*x^12 + 1/13*(c^3*
d + 3*b*c^2*f)*x^13 + 3/10*(b^2*c + a*c^2)*e*x^10 + 3/11*(b*c^2*d
```

```
 + (b^2*c + a*c^2)*f)*x^11 + 1/8*(b^3 + 6*a*b*c)*e*x^8 + 1/9*(3*(
b^2*c + a*c^2)*d + (b^3 + 6*a*b*c)*f)*x^9 + 3/4*a^2*b*e*x^4 + 1/2
*(a*b^2 + a^2*c)*e*x^6 + 1/7*((b^3 + 6*a*b*c)*d + 3*(a*b^2 + a^2*
c)*f)*x^7 + 1/2*a^3*e*x^2 + 3/5*(a^2*b*f + (a*b^2 + a^2*c)*d)*x^5
 + a^3*d*x + 1/3*(3*a^2*b*d + a^3*f)*x^3
```

Fricas [A] time = 0.249155, size = 1, normalized size = 0.

$$\frac{1}{15}x^{15}fc^3+\frac{1}{14}x^{14}ec^3+\frac{1}{13}x^{13}dc^3+\frac{3}{13}x^{13}fc^2b+\frac{1}{4}x^{12}ec^2b+\frac{3}{11}x^{11}dc^2b+\frac{3}{11}x^{11}fcb^2+\frac{3}{11}x^{11}fc^2a+\frac{3}{10}x^{10}ecb^2+\frac{3}{10}x^{10}ec^2a+\frac{1}{3}x^9dcb^2+\frac{1}{9}x^9fb^3+\frac{1}{3}x^9dc^2a+\frac{2}{3}x^9fcba+\frac{1}{8}x^8eb^3+\frac{3}{4}x^8ecba+\frac{1}{7}x^7db^3+\frac{6}{7}x^7dcba+\frac{3}{7}x^7fb^2a+\frac{3}{7}x^7fca^2+\frac{1}{2}x^6eb^2a+\frac{1}{2}x^6eca^2+\frac{3}{5}x^5db^2a+\frac{3}{5}x^5dca^2+\frac{3}{5}x^5fba^2+\frac{3}{4}x^4eba^2+x^3dba^2+\frac{1}{3}x^3fa^3+\frac{1}{2}x^2ea^3+xda^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*f*x^6 + c*e*x^5 + b*e*x^3 + (c*d + b*f)*x^4 + a*e*x + (b*d + a*f)*x^2
```

```
[Out] 1/15*x^15*f*c^3 + 1/14*x^14*e*c^3 + 1/13*x^13*d*c^3 + 3/13*x^13*f
*c^2*b + 1/4*x^12*e*c^2*b + 3/11*x^11*d*c^2*b + 3/11*x^11*f*c*b^2
 + 3/11*x^11*f*c^2*a + 3/10*x^10*e*c*b^2 + 3/10*x^10*e*c^2*a + 1/
3*x^9*d*c*b^2 + 1/9*x^9*f*b^3 + 1/3*x^9*d*c^2*a + 2/3*x^9*f*c*b*a
 + 1/8*x^8*e*b^3 + 3/4*x^8*e*c*b*a + 1/7*x^7*d*b^3 + 6/7*x^7*d*c*
b*a + 3/7*x^7*f*b^2*a + 3/7*x^7*f*c*a^2 + 1/2*x^6*e*b^2*a + 1/2*x
^6*e*c*a^2 + 3/5*x^5*d*b^2*a + 3/5*x^5*d*c*a^2 + 3/5*x^5*f*b*a^2
+ 3/4*x^4*e*b*a^2 + x^3*d*b*a^2 + 1/3*x^3*f*a^3 + 1/2*x^2*e*a^3 +
 x*d*a^3
```

Sympy [A] time = 0.259141, size = 309, normalized size = 1.19

$$a^3dx+\frac{a^3ex^2}{2}+\frac{3a^2bex^4}{4}+\frac{bc^2ex^{12}}{4}+\frac{c^3ex^{14}}{14}+\frac{c^3fx^{15}}{15}+x^{13}\left(\frac{3bc^2f}{13}+\frac{c^3d}{13}\right)+x^{11}\left(\frac{3ac^2f}{11}+\frac{3b^2cf}{11}+\frac{3bc^2d}{11}\right)+x^{10}\left(\frac{3ac^2e}{10}+\frac{3b^2ce}{10}\right)+x^9\left(\frac{2abcf}{3}+\frac{ac^2d}{3}+\frac{b^3f}{9}+\frac{b^2cd}{3}\right)+x^8\left(\frac{3abce}{4}+\frac{b^3e}{8}\right)+x^7\left(\frac{3a^2cf}{7}+\frac{3ab^2f}{7}+\frac{6abcd}{7}+\frac{b^3d}{7}\right)+x^6\left(\frac{a^2ce}{2}+\frac{ab^2e}{2}\right)+x^5\left(\frac{3a^2bf}{5}+\frac{3a^2cd}{5}+\frac{3ab^2d}{5}\right)+x^3\left(\frac{a^3f}{3}+a^2bd\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**2*(a*d+a*e*x+(a*f+b*d)*x**2+b*e*x**3+(b*f+c*d)*x*
```

```
[Out] a**3*d*x + a**3*e*x**2/2 + 3*a**2*b*e*x**4/4 + b*c**2*e*x**12/4 +
 c**3*e*x**14/14 + c**3*f*x**15/15 + x**13*(3*b*c**2*f/13 + c**3*
d/13) + x**11*(3*a*c**2*f/11 + 3*b**2*c*f/11 + 3*b*c**2*d/11) + x
**10*(3*a*c**2*e/10 + 3*b**2*c*e/10) + x**9*(2*a*b*c*f/3 + a*c**2
*d/3 + b**3*f/9 + b**2*c*d/3) + x**8*(3*a*b*c*e/4 + b**3*e/8) + x
**7*(3*a**2*c*f/7 + 3*a*b**2*f/7 + 6*a*b*c*d/7 + b**3*d/7) + x**6
*(a**2*c*e/2 + a*b**2*e/2) + x**5*(3*a**2*b*f/5 + 3*a**2*c*d/5 +
3*a*b**2*d/5) + x**3*(a**3*f/3 + a**2*b*d)
```

GIAC/XCAS [A] time = 0.284078, size = 398, normalized size = 1.54

$$\frac{1}{15}c^3fx^{15} + \frac{1}{14}c^3x^{14}e + \frac{1}{13}c^3dx^{13} + \frac{3}{13}bc^2fx^{13} + \frac{1}{4}bc^2x^{12}e + \frac{3}{11}bc^2dx^{11} + \frac{3}{11}b^2cfx^{11} + \frac{3}{11}ac^2fx^{11} + \frac{3}{10}b^2cx^{10}e + \frac{3}{10}ac^2x^{10}e + \frac{1}{3}b^2cdx^9 + \frac{1}{3}ac^2dx^9 + \frac{1}{9}b^3fx^9 + \frac{2}{3}abcfx^9 + \frac{1}{8}b^3x^8e + \frac{3}{4}abcx^8e + \frac{1}{7}b^3dx^7 + \frac{6}{7}abcdx^7 + \frac{3}{7}ab^2fx^7 + \frac{3}{7}a^2cfx^7 + \frac{1}{2}ab^2x^6e + \frac{1}{2}a^2cx^6e + \frac{3}{5}ab^2dx^5 + \frac{3}{5}a^2cdx^5 + \frac{3}{5}a^2bfx^5 + \frac{3}{4}a^2bx^4e + a^2bdx^3 + \frac{1}{3}a^3fx^3 + \frac{1}{2}a^3x^2e + a^3dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*f*x^6 + c*e*x^5 + b*e*x^3 + (c*d + b*f)*x^4 + a*e*x + (b*d + a*f)*x^2
```

```
[Out] 1/15*c^3*f*x^15 + 1/14*c^3*x^14*e + 1/13*c^3*d*x^13 + 3/13*b*c^2*
f*x^13 + 1/4*b*c^2*x^12*e + 3/11*b*c^2*d*x^11 + 3/11*b^2*c*f*x^11
 + 3/11*a*c^2*f*x^11 + 3/10*b^2*c*x^10*e + 3/10*a*c^2*x^10*e + 1/
3*b^2*c*d*x^9 + 1/3*a*c^2*d*x^9 + 1/9*b^3*f*x^9 + 2/3*a*b*c*f*x^9
 + 1/8*b^3*x^8*e + 3/4*a*b*c*x^8*e + 1/7*b^3*d*x^7 + 6/7*a*b*c*d*
x^7 + 3/7*a*b^2*f*x^7 + 3/7*a^2*c*f*x^7 + 1/2*a*b^2*x^6*e + 1/2*a
^2*c*x^6*e + 3/5*a*b^2*d*x^5 + 3/5*a^2*c*d*x^5 + 3/5*a^2*b*f*x^5
+ 3/4*a^2*b*x^4*e + a^2*b*d*x^3 + 1/3*a^3*f*x^3 + 1/2*a^3*x^2*e +
 a^3*d*x
```

3.62 $\int \left(a+bx^2+cx^4\right) \left(ad+aex+(bd+af)x^2+bex^3+(cd+bf)x^4+cex\right.$

Optimal. Leaf size=154

$$a^2dx+\frac{1}{2}a^2ex^2+\frac{1}{7}x^7\left(2acf+b^2f+2bcd\right)+\frac{1}{5}x^5\left(2abf+2acd+b^2d\right)+\frac{1}{6}ex^6\left(2ac+b^2\right)+\frac{1}{3}ax^3(af+2bd)+\frac{1}{2}abex^4+\frac{1}{9}cx^9(2bf+cd)+\frac{1}{4}bcex^8+\frac{1}{10}c^2ex^{10}+\frac{1}{11}c^2fx^{11}$$

```
[Out] a^2*d*x + (a^2*e*x^2)/2 + (a*(2*b*d + a*f)*x^3)/3 + (a*b*e*x^4)/2
 + ((b^2*d + 2*a*c*d + 2*a*b*f)*x^5)/5 + ((b^2 + 2*a*c)*e*x^6)/6
+ ((2*b*c*d + b^2*f + 2*a*c*f)*x^7)/7 + (b*c*e*x^8)/4 + (c*(c*d +
 2*b*f)*x^9)/9 + (c^2*e*x^10)/10 + (c^2*f*x^11)/11
```

Rubi [A] time = 0.334447, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 61, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.016

$$a^2dx+\frac{1}{2}a^2ex^2+\frac{1}{7}x^7\left(2acf+b^2f+2bcd\right)+\frac{1}{5}x^5\left(2abf+2acd+b^2d\right)+\frac{1}{6}ex^6\left(2ac+b^2\right)+\frac{1}{3}ax^3(af+2bd)+\frac{1}{2}abex^4+\frac{1}{9}cx^9(2bf+cd)+\frac{1}{4}bcex^8+\frac{1}{10}c^2ex^{10}+\frac{1}{11}c^2fx^{11}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)*(a*d + a*e*x + (b*d + a*f)*x^2 + b*e*x^3 + (c*d + b*f)*x^4 +
```

```
[Out] a^2*d*x + (a^2*e*x^2)/2 + (a*(2*b*d + a*f)*x^3)/3 + (a*b*e*x^4)/2
 + ((b^2*d + 2*a*c*d + 2*a*b*f)*x^5)/5 + ((b^2 + 2*a*c)*e*x^6)/6
+ ((2*b*c*d + b^2*f + 2*a*c*f)*x^7)/7 + (b*c*e*x^8)/4 + (c*(c*d +
 2*b*f)*x^9)/9 + (c^2*e*x^10)/10 + (c^2*f*x^11)/11
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^2e\int x\,dx+a^2\int d\,dx+\frac{abex^4}{2}+\frac{ax^3\,(af+2bd)}{3}+\frac{bcex^8}{4}+\frac{c^2ex^{10}}{10}+\frac{c^2fx^{11}}{11}+\frac{cx^9\,(2bf+cd)}{9}+\frac{ex^6\,\left(2ac+b^2\right)}{6}+x^7\left(\frac{2acf}{7}+\frac{b^2f}{7}+\frac{2bcd}{7}\right)+x^5\left(\frac{2abf}{5}+\frac{2acd}{5}+\frac{b^2d}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((c*x**4+b*x**2+a)*(a*d+a*e*x+(a*f+b*d)*x**2+b*e*x**3+(b*f+c*d)*
```

```
[Out] a**2*e*Integral(x, x) + a**2*Integral(d, x) + a*b*e*x**4/2 + a*x*
*3*(a*f + 2*b*d)/3 + b*c*e*x**8/4 + c**2*e*x**10/10 + c**2*f*x**1
1/11 + c*x**9*(2*b*f + c*d)/9 + e*x**6*(2*a*c + b**2)/6 + x**7*(2
*a*c*f/7 + b**2*f/7 + 2*b*c*d/7) + x**5*(2*a*b*f/5 + 2*a*c*d/5 +
b**2*d/5)
```

Mathematica [A] time = 0.0947735, size = 154, normalized size = 1.

$$a^2dx+\frac{1}{2}a^2ex^2+\frac{1}{7}x^7\left(2acf+b^2f+2bcd\right)+\frac{1}{5}x^5\left(2abf+2acd+b^2d\right)+\frac{1}{6}ex^6\left(2ac+b^2\right)+\frac{1}{3}ax^3(af+2bd)+\frac{1}{2}abex^4+\frac{1}{9}cx^9(2bf+cd)+\frac{1}{4}bcex^8+\frac{1}{10}c^2ex^{10}+\frac{1}{11}c^2fx^{11}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)*(a*d + a*e*x + (b*d + a*f)*x^2 + b*e*x^3 + (c*d + b*f
```

```
[Out] a^2*d*x + (a^2*e*x^2)/2 + (a*(2*b*d + a*f)*x^3)/3 + (a*b*e*x^4)/2
 + ((b^2*d + 2*a*c*d + 2*a*b*f)*x^5)/5 + ((b^2 + 2*a*c)*e*x^6)/6
+ ((2*b*c*d + b^2*f + 2*a*c*f)*x^7)/7 + (b*c*e*x^8)/4 + (c*(c*d +
 2*b*f)*x^9)/9 + (c^2*e*x^10)/10 + (c^2*f*x^11)/11
```

Maple [A] time = 0.002, size = 161, normalized size = 1.1

$$\frac{c^2fx^{11}}{11}+\frac{c^2ex^{10}}{10}+\frac{(bcf+c(bf+cd))x^9}{9}+\frac{bcex^8}{4}+\frac{(acf+b(bf+cd)+c(fa+bd))x^7}{7}+\frac{\left(2ace+b^2e\right)x^6}{6}+\frac{(a(bf+cd)+b(fa+bd)+acd)x^5}{5}+\frac{abex^4}{2}+\frac{(a(fa+bd)+abd)x^3}{3}+\frac{a^2ex^2}{2}+a^2dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)*(a*d+a*e*x+(a*f+b*d)*x^2+b*e*x^3+(b*f+c*d)*x^4+c*e*x^5+c*f
```

```
[Out] 1/11*c^2*f*x^11+1/10*c^2*e*x^10+1/9*(b*c*f+c*(b*f+c*d))*x^9+1/4*b
*c*e*x^8+1/7*(a*c*f+b*(b*f+c*d)+c*(a*f+b*d))*x^7+1/6*(2*a*c*e+b^2
*e)*x^6+1/5*(a*(b*f+c*d)+b*(a*f+b*d)+a*c*d)*x^5+1/2*a*b*e*x^4+1/3
*(a*(a*f+b*d)+a*b*d)*x^3+1/2*a^2*e*x^2+a^2*d*x
```

Maxima [A] time = 0.704284, size = 186, normalized size = 1.21

$$\frac{1}{11}\,c^2fx^{11}+\frac{1}{10}\,c^2ex^{10}+\frac{1}{4}\,bcex^8+\frac{1}{9}\,\left(c^2d+2\,bcf\right)x^9+\frac{1}{6}\,\left(b^2+2\,ac\right)ex^6+\frac{1}{7}\,\left(2\,bcd+\left(b^2+2\,ac\right)f\right)x^7+\frac{1}{2}\,abex^4+\frac{1}{5}\,\left(2\,abf+\left(b^2+2\,ac\right)d\right)x^5+\frac{1}{2}\,a^2ex^2+a^2dx+\frac{1}{3}\,\left(2\,abd+a^2f\right)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*f*x^6 + c*e*x^5 + b*e*x^3 + (c*d + b*f)*x^4 + a*e*x + (b*d + a*f)*x^2
```

```
[Out] 1/11*c^2*f*x^11 + 1/10*c^2*e*x^10 + 1/4*b*c*e*x^8 + 1/9*(c^2*d +
2*b*c*f)*x^9 + 1/6*(b^2 + 2*a*c)*e*x^6 + 1/7*(2*b*c*d + (b^2 + 2*
a*c)*f)*x^7 + 1/2*a*b*e*x^4 + 1/5*(2*a*b*f + (b^2 + 2*a*c)*d)*x^5
 + 1/2*a^2*e*x^2 + a^2*d*x + 1/3*(2*a*b*d + a^2*f)*x^3
```

Fricas [A] time = 0.245856, size = 1, normalized size = 0.01

$$\frac{1}{11}x^{11}fc^2+\frac{1}{10}x^{10}ec^2+\frac{1}{9}x^9dc^2+\frac{2}{9}x^9fcb+\frac{1}{4}x^8ecb+\frac{2}{7}x^7dcb+\frac{1}{7}x^7fb^2+\frac{2}{7}x^7fca+\frac{1}{6}x^6eb^2+\frac{1}{3}x^6eca+\frac{1}{5}x^5db^2+\frac{2}{5}x^5dca+\frac{2}{5}x^5fba+\frac{1}{2}x^4eba+\frac{2}{3}x^3dba+\frac{1}{3}x^3fa^2+\frac{1}{2}x^2ea^2+xda^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*f*x^6 + c*e*x^5 + b*e*x^3 + (c*d + b*f)*x^4 + a*e*x + (b*d + a*f)*x^2
```

```
[Out] 1/11*x^11*f*c^2 + 1/10*x^10*e*c^2 + 1/9*x^9*d*c^2 + 2/9*x^9*f*c*b
 + 1/4*x^8*e*c*b + 2/7*x^7*d*c*b + 1/7*x^7*f*b^2 + 2/7*x^7*f*c*a
+ 1/6*x^6*e*b^2 + 1/3*x^6*e*c*a + 1/5*x^5*d*b^2 + 2/5*x^5*d*c*a +
 2/5*x^5*f*b*a + 1/2*x^4*e*b*a + 2/3*x^3*d*b*a + 1/3*x^3*f*a^2 +
1/2*x^2*e*a^2 + x*d*a^2
```

Sympy [A] time = 0.171624, size = 165, normalized size = 1.07

$$a^2dx+\frac{a^2ex^2}{2}+\frac{abex^4}{2}+\frac{bcex^8}{4}+\frac{c^2ex^{10}}{10}+\frac{c^2fx^{11}}{11}+x^9\left(\frac{2bcf}{9}+\frac{c^2d}{9}\right)+x^7\left(\frac{2acf}{7}+\frac{b^2f}{7}+\frac{2bcd}{7}\right)+x^6\left(\frac{ace}{3}+\frac{b^2e}{6}\right)+x^5\left(\frac{2abf}{5}+\frac{2acd}{5}+\frac{b^2d}{5}\right)+x^3\left(\frac{a^2f}{3}+\frac{2abd}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)*(a*d+a*e*x+(a*f+b*d)*x**2+b*e*x**3+(b*f+c*d)*x**4+
```

```
[Out] a**2*d*x + a**2*e*x**2/2 + a*b*e*x**4/2 + b*c*e*x**8/4 + c**2*e*x
**10/10 + c**2*f*x**11/11 + x**9*(2*b*c*f/9 + c**2*d/9) + x**7*(2
*a*c*f/7 + b**2*f/7 + 2*b*c*d/7) + x**6*(a*c*e/3 + b**2*e/6) + x*
*5*(2*a*b*f/5 + 2*a*c*d/5 + b**2*d/5) + x**3*(a**2*f/3 + 2*a*b*d/
3)
```

GIAC/XCAS [A] time = 0.286999, size = 212, normalized size = 1.38

$$\frac{1}{11}\,c^2fx^{11}+\frac{1}{10}\,c^2x^{10}e+\frac{1}{9}\,c^2dx^9+\frac{2}{9}\,bcfx^9+\frac{1}{4}\,bcx^8e+\frac{2}{7}\,bcdx^7+\frac{1}{7}\,b^2fx^7+\frac{2}{7}\,acfx^7+\frac{1}{6}\,b^2x^6e+\frac{1}{3}\,acx^6e+\frac{1}{5}\,b^2dx^5+\frac{2}{5}\,acdx^5+\frac{2}{5}\,abfx^5+\frac{1}{2}\,abx^4e+\frac{2}{3}\,abdx^3+\frac{1}{3}\,a^2fx^3+\frac{1}{2}\,a^2x^2e+a^2dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*f*x^6 + c*e*x^5 + b*e*x^3 + (c*d + b*f)*x^4 + a*e*x + (b*d + a*f)*x^2
```

```
[Out] 1/11*c^2*f*x^11 + 1/10*c^2*x^10*e + 1/9*c^2*d*x^9 + 2/9*b*c*f*x^9
 + 1/4*b*c*x^8*e + 2/7*b*c*d*x^7 + 1/7*b^2*f*x^7 + 2/7*a*c*f*x^7
+ 1/6*b^2*x^6*e + 1/3*a*c*x^6*e + 1/5*b^2*d*x^5 + 2/5*a*c*d*x^5 +
 2/5*a*b*f*x^5 + 1/2*a*b*x^4*e + 2/3*a*b*d*x^3 + 1/3*a^2*f*x^3 +
1/2*a^2*x^2*e + a^2*d*x
```

3.63 $\int \frac{ad+aex+(bd+af)x^2+bex^3+(cd+bf)x^4+cex^5+cfx^6}{a+bx^2+cx^4} \, dx$

Optimal. Leaf size=20

$$dx + \frac{ex^2}{2} + \frac{fx^3}{3}$$

```
[Out] d*x + (e*x^2)/2 + (f*x^3)/3
```

Rubi [A] time = 0.0291188, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 63, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.016

$$dx + \frac{ex^2}{2} + \frac{fx^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d + a*e*x + (b*d + a*f)*x^2 + b*e*x^3 + (c*d + b*f)*x^4 + c*e*x^5 + c*f*x^6)
```

```
[Out] d*x + (e*x^2)/2 + (f*x^3)/3
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$e \int x \, dx + \frac{fx^3}{3} + \int d \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((a*d+a*e*x+(a*f+b*d)*x**2+b*e*x**3+(b*f+c*d)*x**4+c*e*x**5+c*f*
```

```
[Out] e*Integral(x, x) + f*x**3/3 + Integral(d, x)
```

Mathematica [A] time = 0.00337774, size = 20, normalized size = 1.

$$dx + \frac{ex^2}{2} + \frac{fx^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d + a*e*x + (b*d + a*f)*x^2 + b*e*x^3 + (c*d + b*f)*x^4 + c*e*x^5 + c*
```

```
[Out] d*x + (e*x^2)/2 + (f*x^3)/3
```

Maple [A] time = 0.002, size = 17, normalized size = 0.9

$$dx + \frac{ex^2}{2} + \frac{fx^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d+a*e*x+(a*f+b*d)*x^2+b*e*x^3+(b*f+c*d)*x^4+c*e*x^5+c*f*x^6)/(c*x^4+b*x
```

```
[Out] d*x+1/2*e*x^2+1/3*f*x^3
```

Maxima [A] time = 0.701537, size = 22, normalized size = 1.1

$$\frac{1}{3} fx^3 + \frac{1}{2} ex^2 + dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*f*x^6 + c*e*x^5 + b*e*x^3 + (c*d + b*f)*x^4 + a*e*x + (b*d + a*f)*x^2
```

```
[Out] 1/3*f*x^3 + 1/2*e*x^2 + d*x
```

Fricas [A] time = 0.261822, size = 22, normalized size = 1.1

$$\frac{1}{3} fx^3 + \frac{1}{2} ex^2 + dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*f*x^6 + c*e*x^5 + b*e*x^3 + (c*d + b*f)*x^4 + a*e*x + (b*d + a*f)*x^2
```

```
[Out] 1/3*f*x^3 + 1/2*e*x^2 + d*x
```

Sympy [A] time = 0.144591, size = 15, normalized size = 0.75

$$dx + \frac{ex^2}{2} + \frac{fx^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d+a*e*x+(a*f+b*d)*x**2+b*e*x**3+(b*f+c*d)*x**4+c*e*x**5+c*f*x**6)
```

```
[Out] d*x + e*x**2/2 + f*x**3/3
```

GIAC/XCAS [A] time = 0.317661, size = 23, normalized size = 1.15

$$\frac{1}{3} fx^3 + \frac{1}{2} x^2 e + dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*f*x^6 + c*e*x^5 + b*e*x^3 + (c*d + b*f)*x^4 + a*e*x + (b*d + a*f)*x^2
```

```
[Out] 1/3*f*x^3 + 1/2*x^2*e + d*x
```

3.64 $\int \frac{ad+aex+(bd+af)x^2+bex^3+(cd+bf)x^4+cex^5+cfx^6}{(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=211

$$\frac{\left(\frac{2cd-bf}{\sqrt{b^2-4ac}}+f\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(f-\frac{2cd-bf}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}-\frac{e\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

```
[Out] ((f + (2*c*d - b*f)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)
/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b - Sqrt[b^2
 - 4*a*c]]) + ((f - (2*c*d - b*f)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt
[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqr
t[b + Sqrt[b^2 - 4*a*c]]) - (e*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4
*a*c]])/Sqrt[b^2 - 4*a*c]
```

Rubi [A] time = 0.652468, antiderivative size = 211, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 63, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.127

$$\frac{\left(\frac{2cd-bf}{\sqrt{b^2-4ac}}+f\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(f-\frac{2cd-bf}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}-\frac{e\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d + a*e*x + (b*d + a*f)*x^2 + b*e*x^3 + (c*d + b*f)*x^4 + c*e*x^5 + c*f*x^6)
```

```
[Out] ((f + (2*c*d - b*f)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)
/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b - Sqrt[b^2
 - 4*a*c]]) + ((f - (2*c*d - b*f)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt
[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqr
t[b + Sqrt[b^2 - 4*a*c]]) - (e*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4
*a*c]])/Sqrt[b^2 - 4*a*c]
```

Rubi in Sympy [A] time = 72.6825, size = 221, normalized size = 1.05

$$-\frac{e\operatorname{atanh}\left(\frac{b+2cx^2}{\sqrt{-4ac+b^2}}\right)}{\sqrt{-4ac+b^2}}+\frac{\sqrt{2}\left(bf-2cd+f\sqrt{-4ac+b^2}\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{-4ac+b^2}}}\right)}{2\sqrt{c}\sqrt{b+\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}}$$
$$-\frac{\sqrt{2}\left(bf-2cd-f\sqrt{-4ac+b^2}\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{-4ac+b^2}}}\right)}{2\sqrt{c}\sqrt{b-\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((a*d+a*e*x+(a*f+b*d)*x**2+b*e*x**3+(b*f+c*d)*x**4+c*e*x**5+c*f*
```

```
[Out] -e*atanh((b + 2*c*x**2)/sqrt(-4*a*c + b**2))/sqrt(-4*a*c + b**2)
+ sqrt(2)*(b*f - 2*c*d + f*sqrt(-4*a*c + b**2))*atan(sqrt(2)*sqrt
(c)*x/sqrt(b + sqrt(-4*a*c + b**2)))/(2*sqrt(c)*sqrt(b + sqrt(-4*
a*c + b**2))*sqrt(-4*a*c + b**2)) - sqrt(2)*(b*f - 2*c*d - f*sqrt
(-4*a*c + b**2))*atan(sqrt(2)*sqrt(c)*x/sqrt(b - sqrt(-4*a*c + b*
*2)))/(2*sqrt(c)*sqrt(b - sqrt(-4*a*c + b**2))*sqrt(-4*a*c + b**2
))
```

Mathematica [A] time = 0.466139, size = 234, normalized size = 1.11

$$\frac{\frac{\sqrt{2}\left(f\left(\sqrt{b^2-4ac}-b\right)+2cd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\left(f\left(\sqrt{b^2-4ac}+b\right)-2cd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}+e\log\left(\sqrt{b^2-4ac}-b-2cx^2\right)-e\log\left(\sqrt{b^2 \cdots\right.}{2\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d + a*e*x + (b*d + a*f)*x^2 + b*e*x^3 + (c*d + b*f)*x^4 + c*e*x^5 + c*
```

```
[Out] ((Sqrt[2]*(2*c*d + (-b + Sqrt[b^2 - 4*a*c])*f)*ArcTan[(Sqrt[2]*Sq
rt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*Sqrt[b - Sqrt[b^2
 - 4*a*c]]) + (Sqrt[2]*(-2*c*d + (b + Sqrt[b^2 - 4*a*c])*f)*ArcTa
n[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*Sqrt
[b + Sqrt[b^2 - 4*a*c]]) + e*Log[-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2
] - e*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(2*Sqrt[b^2 - 4*a*c])
```

Maple [B] time = 0.004, size = 616, normalized size = 2.9

$$\frac{e}{8ac-2b^2}\sqrt{-4ac+b^2}\ln\left(2cx^2+\sqrt{-4ac+b^2}+b\right)$$
$$+2\frac{c\sqrt{2}fa}{(4ac-b^2)\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\arctan\left(\frac{cx\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)$$
$$-\frac{\sqrt{2}fb^2}{8ac-2b^2}\arctan\left(cx\sqrt{2}\frac{1}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)\frac{1}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}$$
$$-\frac{b\sqrt{2}f}{8ac-2b^2}\sqrt{-4ac+b^2}\arctan\left(cx\sqrt{2}\frac{1}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)\frac{1}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}$$
$$+\frac{c\sqrt{2}d}{4ac-b^2}\sqrt{-4ac+b^2}\arctan\left(cx\sqrt{2}\frac{1}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)\frac{1}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}$$
$$-\frac{e}{8ac-2b^2}\sqrt{-4ac+b^2}\ln\left(-2cx^2+\sqrt{-4ac+b^2}-b\right)$$
$$-2\frac{c\sqrt{2}fa}{(4ac-b^2)\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}Artanh\left(\frac{cx\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)$$
$$+\frac{\sqrt{2}fb^2}{8ac-2b^2}Artanh\left(cx\sqrt{2}\frac{1}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)\frac{1}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}$$
$$-\frac{b\sqrt{2}f}{8ac-2b^2}\sqrt{-4ac+b^2}Artanh\left(cx\sqrt{2}\frac{1}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)\frac{1}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}$$
$$+\frac{c\sqrt{2}d}{4ac-b^2}\sqrt{-4ac+b^2}Artanh\left(cx\sqrt{2}\frac{1}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)\frac{1}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d+a*e*x+(a*f+b*d)*x^2+b*e*x^3+(b*f+c*d)*x^4+c*e*x^5+c*f*x^6)/(c*x^4+b*x
```

```
[Out] 1/2*(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)*e*ln(2*c*x^2+(-4*a*c+b^2)^(1/2
)+b)+2*c/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arc
tan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*f*a-1/2/(4*a*c-
b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/
((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*f*b^2-1/2*(-4*a*c+b^2)^(1/2)/(4
*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(
1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b*f+c*(-4*a*c+b^2)^(1/2)/(
4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^
(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*d-1/2*(-4*a*c+b^2)^(1/2)/
(4*a*c-b^2)*e*ln(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)-2*c/(4*a*c-b^2)*2
^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b
+(-4*a*c+b^2)^(1/2))*c)^(1/2))*f*a+1/2/(4*a*c-b^2)*2^(1/2)/((-b+(
-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)
^(1/2))*c)^(1/2))*f*b^2-1/2*(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)*2^(1/2
)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*
a*c+b^2)^(1/2))*c)^(1/2))*b*f+c*(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)*2^
(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+
(-4*a*c+b^2)^(1/2))*c)^(1/2))*d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{cfx^6 + cex^5 + bex^3 + (cd + bf)x^4 + aex + (bd + af)x^2 + ad}{(cx^4 + bx^2 + a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*f*x^6 + c*e*x^5 + b*e*x^3 + (c*d + b*f)*x^4 + a*e*x + (b*d + a*f)*x^2
```

```
[Out] integrate((c*f*x^6 + c*e*x^5 + b*e*x^3 + (c*d + b*f)*x^4 + a*e*x
+ (b*d + a*f)*x^2 + a*d)/(c*x^4 + b*x^2 + a)^2, x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*f*x^6 + c*e*x^5 + b*e*x^3 + (c*d + b*f)*x^4 + a*e*x + (b*d + a*f)*x^2
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d+a*e*x+(a*f+b*d)*x**2+b*e*x**3+(b*f+c*d)*x**4+c*e*x**5+c*f*x**6)
```

```
[Out] Timed out
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*f*x^6 + c*e*x^5 + b*e*x^3 + (c*d + b*f)*x^4 + a*e*x + (b*d + a*f)*x^2
```

```
[Out] Exception raised: TypeError
```

3.65 $\int \frac{ad+aex+(bd+af)x^2+bex^3+(cd+bf)x^4+cex^5+cfx^6}{(a+bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=368

$$\frac{x\left(cx^2(bd-2af)-abf-2acd+b^2d\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{c}\left(\frac{4abf-12acd+b^2d}{\sqrt{b^2-4ac}}-2af+bd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(-\frac{4abf-12acd+b^2d}{\sqrt{b^2-4ac}}-2af+bd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}a\left(b^2-4ac\right)\sqrt{\sqrt{b^2-4ac}+b}}+\frac{2ce\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}-\frac{e\left(b+2cx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

```
[Out] -(e*(b + 2*c*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (x*(b^
2*d - 2*a*c*d - a*b*f + c*(b*d - 2*a*f)*x^2))/(2*a*(b^2 - 4*a*c)*
(a + b*x^2 + c*x^4)) + (Sqrt[c]*(b*d - 2*a*f + (b^2*d - 12*a*c*d
+ 4*a*b*f)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b -
 Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^
2 - 4*a*c]]) + (Sqrt[c]*(b*d - 2*a*f - (b^2*d - 12*a*c*d + 4*a*b*
f)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^
2 - 4*a*c]]])/(2*Sqrt[2]*a*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*
c]]) + (2*c*e*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*
a*c)^(3/2)
```

Rubi [A] time = 1.89965, antiderivative size = 368, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 63, $\frac{\text{number of rules}}{\text{integrand size}} = 0.159$

$$\frac{x\left(cx^2(bd-2af)-abf-2acd+b^2d\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{c}\left(\frac{4abf-12acd+b^2d}{\sqrt{b^2-4ac}}-2af+bd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(-\frac{4abf-12acd+b^2d}{\sqrt{b^2-4ac}}-2af+bd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}a\left(b^2-4ac\right)\sqrt{\sqrt{b^2-4ac}+b}}+\frac{2ce\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}-\frac{e\left(b+2cx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d + a*e*x + (b*d + a*f)*x^2 + b*e*x^3 + (c*d + b*f)*x^4 + c*e*x^5 + c*f*x^6)
```

```
[Out] -(e*(b + 2*c*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (x*(b^
2*d - 2*a*c*d - a*b*f + c*(b*d - 2*a*f)*x^2))/(2*a*(b^2 - 4*a*c)*
(a + b*x^2 + c*x^4)) + (Sqrt[c]*(b*d - 2*a*f + (b^2*d - 12*a*c*d
+ 4*a*b*f)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b -
 Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^
2 - 4*a*c]]) + (Sqrt[c]*(b*d - 2*a*f - (b^2*d - 12*a*c*d + 4*a*b*
f)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^
2 - 4*a*c]]])/(2*Sqrt[2]*a*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*
c]]) + (2*c*e*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*
a*c)^(3/2)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((a*d+a*e*x+(a*f+b*d)*x**2+b*e*x**3+(b*f+c*d)*x**4+c*e*x**5+c*f*
```

```
[Out] Timed out
```

Mathematica [A] time = 2.55661, size = 398, normalized size = 1.08

$$\frac{1}{4}\left(\frac{2ab(e+fx)+4acx(d+x(e+fx))-2bdx\left(b+cx^2\right)}{a\left(4ac-b^2\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{2}\sqrt{c}\left(b\left(d\sqrt{b^2-4ac}+4af\right)-2a\left(f\sqrt{b^2-4ac}+6cd\right)+b^2d\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{a\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\sqrt{c}\left(bd\sqrt{b^2-4ac}-2af\sqrt{b^2-4ac}-4abf+12acd+b^2(-d)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{a\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}-\frac{4ce\log\left(\sqrt{b^2-4ac}-b-2cx^2\right)}{\left(b^2-4ac\right)^{3/2}}+\frac{4ce\log\left(\sqrt{b^2-4ac}+b+2cx^2\right)}{\left(b^2-4ac\right)^{3/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d + a*e*x + (b*d + a*f)*x^2 + b*e*x^3 + (c*d + b*f)*x^4 + c*e*x^5 + c*
```

```
[Out] ((2*a*b*(e + f*x) - 2*b*d*x*(b + c*x^2) + 4*a*c*x*(d + x*(e + f*x
)))/(a*(-b^2 + 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[2]*Sqrt[c]*(b^
2*d + b*(Sqrt[b^2 - 4*a*c]*d + 4*a*f) - 2*a*(6*c*d + Sqrt[b^2 - 4
*a*c]*f))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]]
)/(a*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*
Sqrt[c]*(-(b^2*d) + 12*a*c*d + b*Sqrt[b^2 - 4*a*c]*d - 4*a*b*f -
2*a*Sqrt[b^2 - 4*a*c]*f)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt
[b^2 - 4*a*c]]])/(a*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c
]]) - (4*c*e*Log[-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2])/(b^2 - 4*a*c)
^(3/2) + (4*c*e*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*
c)^(3/2))/4
```

Maple [B] time = 0.009, size = 2851, normalized size = 7.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d+a*e*x+(a*f+b*d)*x^2+b*e*x^3+(b*f+c*d)*x^4+c*e*x^5+c*f*x^6)/(c*x^4+b*x
```

```
[Out] 2*c^2/(4*a*c-b^2)^2*2^(1/2)/(4*a*c+3*b^2)/((-b+(-4*a*c+b^2)^(1/2)
)*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))
*b^3*d-3/4*c/(4*a*c-b^2)^2*2^(1/2)/(4*a*c+3*b^2)/a/((b+(-4*a*c+b^
2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^
(1/2))*(-4*a*c+b^2)^(1/2)*b^4*d+4*c^2/(4*a*c-b^2)^2*2^(1/2)/(4*a*
c+3*b^2)*a/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((
b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^2*f-4*c^3/(4*a*c-b^2)^2*2^(1/2)
/(4*a*c+3*b^2)*a/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1
/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b*d+3/4*c/(4*a*c-b^2)^2*2^(
1/2)/(4*a*c+3*b^2)/a/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*
2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^5*d+12*c^3/(4*a*c-b^2
)^2*2^(1/2)/(4*a*c+3*b^2)*a/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arc
tanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*(-4*a*c+b^2)^
(1/2)*d-3*c/(4*a*c-b^2)^2*2^(1/2)/(4*a*c+3*b^2)/((b+(-4*a*c+b^2)^
(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/
2))*(-4*a*c+b^2)^(1/2)*b^3*f+4*c^3/(4*a*c-b^2)^2*2^(1/2)/(4*a*c+3
*b^2)*a/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-
b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b*d-3/4*c/(4*a*c-b^2)^2*2^(1/2)/(
4*a*c+3*b^2)/a/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1
/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^5*d+8*c^2/(4*a*c-b^2)^2*
2^(1/2)/(4*a*c+3*b^2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x
*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*(-4*a*c+b^2)^(1/2)*b^2
*d-3*c/(4*a*c-b^2)^2*2^(1/2)/(4*a*c+3*b^2)/((-b+(-4*a*c+b^2)^(1/2
))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)
)*(-4*a*c+b^2)^(1/2)*b^3*f+8*c^2/(4*a*c-b^2)^2*2^(1/2)/(4*a*c+3*b
^2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-
4*a*c+b^2)^(1/2))*c)^(1/2))*(-4*a*c+b^2)^(1/2)*b^2*d-4*c^2/(4*a*c
-b^2)^2*2^(1/2)/(4*a*c+3*b^2)*a/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)
*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^2*f+12*
```

```
c^3/(4*a*c-b^2)^2*2^(1/2)/(4*a*c+3*b^2)*a/((b+(-4*a*c+b^2)^(1/2))
*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*(-
4*a*c+b^2)^(1/2)*d-1/4/(4*a*c-b^2)^2/(x^2+1/2/c*(-4*a*c+b^2)^(1/2
)+1/2*b/c)/a*d*x*b^2*(-4*a*c+b^2)^(1/2)+c/(4*a*c-b^2)^2/(x^2+1/2/
c*(-4*a*c+b^2)^(1/2)+1/2*b/c)*d*x*(-4*a*c+b^2)^(1/2)-c/(4*a*c-b^2
)^2/(x^2+1/2/c*(-4*a*c+b^2)^(1/2)+1/2*b/c)*d*x*b+1/4/(4*a*c-b^2)^
2/(x^2+1/2/c*(-4*a*c+b^2)^(1/2)+1/2*b/c)/a*d*x*b^3-c/(4*a*c-b^2)^
2/(x^2+1/2*b/c-1/2/c*(-4*a*c+b^2)^(1/2))*d*x*(-4*a*c+b^2)^(1/2)-c
/(4*a*c-b^2)^2/(x^2+1/2*b/c-1/2/c*(-4*a*c+b^2)^(1/2))*d*x*b+1/4/(
4*a*c-b^2)^2/(x^2+1/2*b/c-1/2/c*(-4*a*c+b^2)^(1/2))/a*d*x*b^3-4*c
^2/(4*a*c-b^2)^2*2^(1/2)/(4*a*c+3*b^2)*a/((-b+(-4*a*c+b^2)^(1/2))
*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*
(-4*a*c+b^2)^(1/2)*b*f-3/4*c/(4*a*c-b^2)^2*2^(1/2)/(4*a*c+3*b^2)/
a/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*
a*c+b^2)^(1/2))*c)^(1/2))*(-4*a*c+b^2)^(1/2)*b^4*d-4*c^2/(4*a*c-b
^2)^2*2^(1/2)/(4*a*c+3*b^2)*a/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*ar
ctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*(-4*a*c+b^2)^(
1/2)*b*f+1/4/(4*a*c-b^2)^2/(x^2+1/2*b/c-1/2/c*(-4*a*c+b^2)^(1/2))
/a*d*x*b^2*(-4*a*c+b^2)^(1/2)-8*c^3/(4*a*c-b^2)^2*2^(1/2)/(4*a*c+
3*b^2)*a^2/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/
((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*f+3/2*c/(4*a*c-b^2)^2*2^(1/2)/
(4*a*c+3*b^2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/
2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^4*f+2*c/(4*a*c-b^2)^2/(x^
2+1/2*b/c-1/2/c*(-4*a*c+b^2)^(1/2))*e*a-1/2/(4*a*c-b^2)^2/(x^2+1/
2/c*(-4*a*c+b^2)^(1/2)+1/2*b/c)*e*b^2-1/2/(4*a*c-b^2)^2/(x^2+1/2*
b/c-1/2/c*(-4*a*c+b^2)^(1/2))*e*b^2+2*c/(4*a*c-b^2)^2/(x^2+1/2/c*
(-4*a*c+b^2)^(1/2)+1/2*b/c)*e*a+8*c^3/(4*a*c-b^2)^2*2^(1/2)/(4*a*
c+3*b^2)*a^2/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/
((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*f-3/2*c/(4*a*c-b^2)^2*2^(1/2)/(
4*a*c+3*b^2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/
((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^4*f-2*c^2/(4*a*c-b^2)^2*2^(1/
2)/(4*a*c+3*b^2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1
/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^3*d+2*c/(4*a*c-b^2)^2/(x^
2+1/2*b/c-1/2/c*(-4*a*c+b^2)^(1/2))*a*x*f+2*c/(4*a*c-b^2)^2/(x^2+
1/2/c*(-4*a*c+b^2)^(1/2)+1/2*b/c)*a*x*f+4*c^2/(4*a*c-b^2)^2/(4*a*
c+3*b^2)*a*ln((4*a*c+3*b^2)*a*(2*c*x^2+(-4*a*c+b^2)^(1/2)+b))*(-4
*a*c+b^2)^(1/2)*e+3*c/(4*a*c-b^2)^2/(4*a*c+3*b^2)*ln((4*a*c+3*b^2
)*a*(2*c*x^2+(-4*a*c+b^2)^(1/2)+b))*(-4*a*c+b^2)^(1/2)*b^2*e-4*c^
2/(4*a*c-b^2)^2/(4*a*c+3*b^2)*a*ln((4*a*c+3*b^2)*a*(-2*c*x^2+(-4*
a*c+b^2)^(1/2)-b))*(-4*a*c+b^2)^(1/2)*e-3*c/(4*a*c-b^2)^2/(4*a*c+
3*b^2)*ln((4*a*c+3*b^2)*a*(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b))*(-4*a*
c+b^2)^(1/2)*b^2*e-1/2/(4*a*c-b^2)^2/(x^2+1/2*b/c-1/2/c*(-4*a*c+b
^2)^(1/2))*x*b^2*f-1/2/(4*a*c-b^2)^2/(x^2+1/2/c*(-4*a*c+b^2)^(1/2
)+1/2*b/c)*x*b^2*f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{2\,acex^2 - (bcd - 2\,acf)x^3 + abe + \left(abf - \left(b^2 - 2\,ac\right)d\right)x}{2\left((ab^2c - 4\,a^2c^2)x^4 + a^2b^2 - 4\,a^3c + (ab^3 - 4\,a^2bc)x^2\right)} - \frac{\int \frac{4\,acex - abf - (bcd - 2\,acf)x^2 - \left(b^2 - 6\,ac\right)d}{cx^4 + bx^2 + a}\,dx}{2\left(ab^2 - 4\,a^2c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*f*x^6 + c*e*x^5 + b*e*x^3 + (c*d + b*f)*x^4 + a*e*x + (b*d + a*f)*x^2
```

```
[Out] -1/2*(2*a*c*e*x^2 - (b*c*d - 2*a*c*f)*x^3 + a*b*e + (a*b*f - (b^2
 - 2*a*c)*d)*x)/((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c +
(a*b^3 - 4*a^2*b*c)*x^2) - 1/2*integrate((4*a*c*e*x - a*b*f - (b*
c*d - 2*a*c*f)*x^2 - (b^2 - 6*a*c)*d)/(c*x^4 + b*x^2 + a), x)/(a*
b^2 - 4*a^2*c)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*f*x^6 + c*e*x^5 + b*e*x^3 + (c*d + b*f)*x^4 + a*e*x + (b*d + a*f)*x^2
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d+a*e*x+(a*f+b*d)*x**2+b*e*x**3+(b*f+c*d)*x**4+c*e*x**5+c*f*x**6)
```

```
[Out] Timed out
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*f*x^6 + c*e*x^5 + b*e*x^3 + (c*d + b*f)*x^4 + a*e*x + (b*d + a*f)*x^2
```

```
[Out] Exception raised: TypeError
```

3.66 $\int \frac{ad+aex+(bd+af)x^2+bex^3+(cd+bf)x^4+cex^5+cfx^6}{(a+bx^2+cx^4)^4}\,dx$

Optimal. Leaf size=621

$$\frac{x\left(cx^2\left(20a^2cf+ab^2f-24abcd+3b^3d\right)+8a^2bcf+28a^2c^2d+ab^3f-25ab^2cd+3b^4d\right)}{8a^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}$$
$$+\frac{\sqrt{c}\left(-\frac{-52a^2bcf+168a^2c^2d+ab^3f-30ab^2cd+3b^4d}{\sqrt{b^2-4ac}}+20a^2cf+ab^2f-24abcd+3b^3d\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{8\sqrt{2}a^2\left(b^2-4ac\right)^2\sqrt{\sqrt{b^2-4ac}+b}}$$
$$+\frac{\sqrt{c}\left(4a^2c\left(5f\sqrt{b^2-4ac}+42cd\right)-ab^2\left(30cd-f\sqrt{b^2-4ac}\right)-4abc\left(6d\sqrt{b^2-4ac}+13af\right)+b^3\left(3d\sqrt{b^2-4ac}+af\right)+3}{8\sqrt{2}a^2\left(b^2-4ac\right)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}$$
$$-\frac{6c^2e\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/2}}+\frac{x\left(cx^2(bd-2af)-abf-2acd+b^2d\right)}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}$$
$$+\frac{3ce\left(b+2cx^2\right)}{2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}-\frac{e\left(b+2cx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}$$

```
[Out] -(e*(b + 2*c*x^2))/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (x*(
b^2*d - 2*a*c*d - a*b*f + c*(b*d - 2*a*f)*x^2))/(4*a*(b^2 - 4*a*c
)*(a + b*x^2 + c*x^4)^2) + (3*c*e*(b + 2*c*x^2))/(2*(b^2 - 4*a*c)
^2*(a + b*x^2 + c*x^4)) + (x*(3*b^4*d - 25*a*b^2*c*d + 28*a^2*c^2
*d + a*b^3*f + 8*a^2*b*c*f + c*(3*b^3*d - 24*a*b*c*d + a*b^2*f +
20*a^2*c*f)*x^2))/(8*a^2*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (
Sqrt[c]*(3*b^4*d + b^3*(3*Sqrt[b^2 - 4*a*c]*d + a*f) - 4*a*b*c*(6
*Sqrt[b^2 - 4*a*c]*d + 13*a*f) - a*b^2*(30*c*d - Sqrt[b^2 - 4*a*c
]*f) + 4*a^2*c*(42*c*d + 5*Sqrt[b^2 - 4*a*c]*f))*ArcTan[(Sqrt[2]*
Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a^2*(b^2 - 4*
a*c)^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[c]*(3*b^3*d - 24*
a*b*c*d + a*b^2*f + 20*a^2*c*f - (3*b^4*d - 30*a*b^2*c*d + 168*a^
2*c^2*d + a*b^3*f - 52*a^2*b*c*f)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt
[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a^2*(b^2
- 4*a*c)^2*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - (6*c^2*e*ArcTanh[(b + 2
*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(5/2)
```

Rubi [A] time = 10.3266, antiderivative size = 621, normalized size of antiderivative = 1., number of

steps used = 13, number of rules used = 10, integrand size = 63, $\frac{\text{number of rules}}{\text{integrand size}} = 0.159$

$$\frac{x\left(cx^2\left(20a^2cf+ab^2f-24abcd+3b^3d\right)+8a^2bcf+28a^2c^2d+ab^3f-25ab^2cd+3b^4d\right)}{8a^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \frac{\sqrt{c}\left(-\frac{-52a^2bcf+168a^2c^2d+ab^3f-30ab^2cd+3b^4d}{\sqrt{b^2-4ac}}+20a^2cf+ab^2f-24abcd+3b^3d\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{8\sqrt{2}a^2\left(b^2-4ac\right)^2\sqrt{\sqrt{b^2-4ac}+b}} + \frac{\sqrt{c}\left(4a^2c\left(5f\sqrt{b^2-4ac}+42cd\right)-ab^2\left(30cd-f\sqrt{b^2-4ac}\right)-4abc\left(6d\sqrt{b^2-4ac}+13af\right)+b^3\left(3d\sqrt{b^2-4ac}+af\right)+3}{8\sqrt{2}a^2\left(b^2-4ac\right)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{6c^2e\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/2}} + \frac{x\left(cx^2(bd-2af)-abf-2acd+b^2d\right)}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{3ce\left(b+2cx^2\right)}{2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} - \frac{e\left(b+2cx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d + a*e*x + (b*d + a*f)*x^2 + b*e*x^3 + (c*d + b*f)*x^4 + c*e*x^5 + c*f*x^6)

[Out] -(e*(b + 2*c*x^2))/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (x*(
b^2*d - 2*a*c*d - a*b*f + c*(b*d - 2*a*f)*x^2))/(4*a*(b^2 - 4*a*c
)*(a + b*x^2 + c*x^4)^2) + (3*c*e*(b + 2*c*x^2))/(2*(b^2 - 4*a*c)
^2*(a + b*x^2 + c*x^4)) + (x*(3*b^4*d - 25*a*b^2*c*d + 28*a^2*c^2
*d + a*b^3*f + 8*a^2*b*c*f + c*(3*b^3*d - 24*a*b*c*d + a*b^2*f +
20*a^2*c*f)*x^2))/(8*a^2*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (
Sqrt[c]*(3*b^4*d + b^3*(3*Sqrt[b^2 - 4*a*c]*d + a*f) - 4*a*b*c*(6
*Sqrt[b^2 - 4*a*c]*d + 13*a*f) - a*b^2*(30*c*d - Sqrt[b^2 - 4*a*c
]*f) + 4*a^2*c*(42*c*d + 5*Sqrt[b^2 - 4*a*c]*f))*ArcTan[(Sqrt[2]*
Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a^2*(b^2 - 4*
a*c)^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[c]*(3*b^3*d - 24*
a*b*c*d + a*b^2*f + 20*a^2*c*f - (3*b^4*d - 30*a*b^2*c*d + 168*a^
2*c^2*d + a*b^3*f - 52*a^2*b*c*f)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt
[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a^2*(b^2
- 4*a*c)^2*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - (6*c^2*e*ArcTanh[(b + 2
*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(5/2)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((a*d+a*e*x+(a*f+b*d)*x**2+b*e*x**3+(b*f+c*d)*x**4+c*e*x**5+c*f*
```

```
[Out] Timed out
```

Mathematica [A] time = 6.6056, size = 683, normalized size = 1.1

$$\frac{12a^2bce+8a^2bcfx+28a^2c^2dx+24a^2c^2ex^2+20a^2c^2fx^3+ab^3fx-25ab^2cdx+ab^2cfx^3-24abc^2dx^3+3b^4dx+3b^3cdx^3}{8a^2\left(4ac-b^2\right)^2\left(a+bx^2+cx^4\right)}$$
$$+\frac{\sqrt{c}\left(20a^2cf\sqrt{b^2-4ac}-52a^2bcf+168a^2c^2d+ab^3f-30ab^2cd-24abcd\sqrt{b^2-4ac}+ab^2f\sqrt{b^2-4ac}+3b^3d\sqrt{b^2-4ac}+\right.}{8\sqrt{2}a^2\left(b^2-4ac\right)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}$$
$$+\frac{\sqrt{c}\left(20a^2cf\sqrt{b^2-4ac}+52a^2bcf-168a^2c^2d-ab^3f+30ab^2cd-24abcd\sqrt{b^2-4ac}+ab^2f\sqrt{b^2-4ac}+3b^3d\sqrt{b^2-4ac}-\right.}{8\sqrt{2}a^2\left(b^2-4ac\right)^{5/2}\sqrt{\sqrt{b^2-4ac}+b}}$$
$$+\frac{3c^2e\log\left(\sqrt{b^2-4ac}-b-2cx^2\right)}{\left(b^2-4ac\right)^{5/2}}-\frac{3c^2e\log\left(\sqrt{b^2-4ac}+b+2cx^2\right)}{\left(b^2-4ac\right)^{5/2}}$$
$$+\frac{abe+abfx+2acdx+2acex^2+2acfx^3-b^2dx-bcdx^3}{4a\left(4ac-b^2\right)\left(a+bx^2+cx^4\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d + a*e*x + (b*d + a*f)*x^2 + b*e*x^3 + (c*d + b*f)*x^4 + c*e*x^5 + c*
```

```
[Out] (a*b*e - b^2*d*x + 2*a*c*d*x + a*b*f*x + 2*a*c*e*x^2 - b*c*d*x^3
+ 2*a*c*f*x^3)/(4*a*(-b^2 + 4*a*c)*(a + b*x^2 + c*x^4)^2) + (12*a
^2*b*c*e + 3*b^4*d*x - 25*a*b^2*c*d*x + 28*a^2*c^2*d*x + a*b^3*f*
x + 8*a^2*b*c*f*x + 24*a^2*c^2*e*x^2 + 3*b^3*c*d*x^3 - 24*a*b*c^2
*d*x^3 + a*b^2*c*f*x^3 + 20*a^2*c^2*f*x^3)/(8*a^2*(-b^2 + 4*a*c)^
2*(a + b*x^2 + c*x^4)) + (Sqrt[c]*(3*b^4*d - 30*a*b^2*c*d + 168*a
^2*c^2*d + 3*b^3*Sqrt[b^2 - 4*a*c]*d - 24*a*b*c*Sqrt[b^2 - 4*a*c]
*d + a*b^3*f - 52*a^2*b*c*f + a*b^2*Sqrt[b^2 - 4*a*c]*f + 20*a^2*
c*Sqrt[b^2 - 4*a*c]*f)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b
^2 - 4*a*c]]])/(8*Sqrt[2]*a^2*(b^2 - 4*a*c)^(5/2)*Sqrt[b - Sqrt[b
^2 - 4*a*c]]) + (Sqrt[c]*(-3*b^4*d + 30*a*b^2*c*d - 168*a^2*c^2*d
 + 3*b^3*Sqrt[b^2 - 4*a*c]*d - 24*a*b*c*Sqrt[b^2 - 4*a*c]*d - a*b
^3*f + 52*a^2*b*c*f + a*b^2*Sqrt[b^2 - 4*a*c]*f + 20*a^2*c*Sqrt[b
^2 - 4*a*c]*f)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a
*c]]])/(8*Sqrt[2]*a^2*(b^2 - 4*a*c)^(5/2)*Sqrt[b + Sqrt[b^2 - 4*a
*c]]) + (3*c^2*e*Log[-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2])/(b^2 - 4*
a*c)^(5/2) - (3*c^2*e*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2
- 4*a*c)^(5/2)
```

Maple [B] time = 0.015, size = 10809, normalized size = 17.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d+a*e*x+(a*f+b*d)*x^2+b*e*x^3+(b*f+c*d)*x^4+c*e*x^5+c*f*x^6)/(c*x^4+b*x
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{24\,a^2c^3ex^6 + 36\,a^2bc^2ex^4 + \left(3\left(b^3c^2 - 8\,abc^3\right)d + \left(ab^2c^2 + 20\,a^2c^3\right)f\right)x^7 + \left(\left(6\,b^4c - 49\,ab^2c^2 + 28\,a^2c^3\right)d + 2\left(ab^3c + 14\,a^2b\right.\right.}{8\left((a^2b^4c^2 - 8\,a^3b^2c^3 + 16\,a^4c^4)x^8 + a^4b^4 - 8\,a^5b^2c + 1\right.} + \frac{\int \frac{48\,a^2c^2ex+\left(3\left(b^3c-8\,abc^2\right)d+\left(ab^2c+20\,a^2c^2\right)f\right)x^2+3\left(b^4-9\,ab^2c+28\,a^2c^2\right)d+\left(ab^3-16\,a^2bc\right)f}{cx^4+bx^2+a}\,dx}{8\left(a^2b^4 - 8\,a^3b^2c + 16\,a^4c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*f*x^6 + c*e*x^5 + b*e*x^3 + (c*d + b*f)*x^4 + a*e*x + (b*d + a*f)*x^2
```

```
[Out] 1/8*(24*a^2*c^3*e*x^6 + 36*a^2*b*c^2*e*x^4 + (3*(b^3*c^2 - 8*a*b*
c^3)*d + (a*b^2*c^2 + 20*a^2*c^3)*f)*x^7 + ((6*b^4*c - 49*a*b^2*c
^2 + 28*a^2*c^3)*d + 2*(a*b^3*c + 14*a^2*b*c^2)*f)*x^5 + 8*(a^2*b
^2*c + 5*a^3*c^2)*e*x^2 + ((3*b^5 - 20*a*b^3*c - 4*a^2*b*c^2)*d +
 (a*b^4 + 5*a^2*b^2*c + 36*a^3*c^2)*f)*x^3 - 2*(a^2*b^3 - 10*a^3*
b*c)*e + ((5*a*b^4 - 37*a^2*b^2*c + 44*a^3*c^2)*d - (a^2*b^3 - 16
*a^3*b*c)*f)*x)/((a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*x^8 +
 a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2 + 2*(a^2*b^5*c - 8*a^3*b^3*c^
2 + 16*a^4*b*c^3)*x^6 + (a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3)*x^4
+ 2*(a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*x^2) + 1/8*integrate((
48*a^2*c^2*e*x + (3*(b^3*c - 8*a*b*c^2)*d + (a*b^2*c + 20*a^2*c^2
)*f)*x^2 + 3*(b^4 - 9*a*b^2*c + 28*a^2*c^2)*d + (a*b^3 - 16*a^2*b
*c)*f)/(c*x^4 + b*x^2 + a), x)/(a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^
2)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*f*x^6 + c*e*x^5 + b*e*x^3 + (c*d + b*f)*x^4 + a*e*x + (b*d + a*f)*x^2
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d+a*e*x+(a*f+b*d)*x**2+b*e*x**3+(b*f+c*d)*x**4+c*e*x**5+c*f*x**6)
```

```
[Out] Timed out
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*f*x^6 + c*e*x^5 + b*e*x^3 + (c*d + b*f)*x^4 + a*e*x + (b*d + a*f)*x^2
```

```
[Out] Exception raised: TypeError
```

3.67 $\int \frac{2-x-2x^2+x^3}{4-5x^2+x^4} \, dx$

Optimal. Leaf size=4

$$\log(x+2)$$

```
[Out] Log[2 + x]
```

Rubi [A] time = 0.0124115, antiderivative size = 4, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077

$$\log(x+2)$$

Antiderivative was successfully verified.

```
[In] Int[(2 - x - 2*x^2 + x^3)/(4 - 5*x^2 + x^4),x]
```

```
[Out] Log[2 + x]
```

Rubi in Sympy [A] time = 4.91714, size = 3, normalized size = 0.75

$$\log(x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((x**3-2*x**2-x+2)/(x**4-5*x**2+4),x)
```

```
[Out] log(x + 2)
```

Mathematica [A] time = 0.00133337, size = 4, normalized size = 1.

$$\log(x+2)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 - x - 2*x^2 + x^3)/(4 - 5*x^2 + x^4),x]
```

```
[Out] Log[2 + x]
```

Maple [A] time = 0.002, size = 5, normalized size = 1.3

$$\ln(2+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^3-2*x^2-x+2)/(x^4-5*x^2+4),x)

[Out] ln(2+x)
```

Maxima [A] time = 0.701242, size = 5, normalized size = 1.25

$$\log(x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 - 2*x^2 - x + 2)/(x^4 - 5*x^2 + 4),x, algorithm="maxima")

[Out] log(x + 2)
```

Fricas [A] time = 0.265592, size = 5, normalized size = 1.25

$$\log(x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 - 2*x^2 - x + 2)/(x^4 - 5*x^2 + 4),x, algorithm="fricas")

[Out] log(x + 2)
```

Sympy [A] time = 0.088752, size = 3, normalized size = 0.75

$$\log(x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**3-2*x**2-x+2)/(x**4-5*x**2+4),x)
```

```
[Out] log(x + 2)
```

GIAC/XCAS [A] time = 0.299861, size = 7, normalized size = 1.75

$$\ln(|x + 2|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3 - 2*x^2 - x + 2)/(x^4 - 5*x^2 + 4),x, algorithm="giac")
```

```
[Out] ln(abs(x + 2))
```

3.68 $\int \frac{(d+ex)(2-x-2x^2+x^3)}{4-5x^2+x^4}\,dx$

Optimal. Leaf size=14

$$(d-2e)\log(x+2)+ex$$

```
[Out] e*x + (d - 2*e)*Log[2 + x]
```

Rubi [A] time = 0.0397892, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065

$$(d-2e)\log(x+2)+ex$$

Antiderivative was successfully verified.

```
[In]  Int[((d + e*x)*(2 - x - 2*x^2 + x^3))/(4 - 5*x^2 + x^4),x]
```

```
[Out] e*x + (d - 2*e)*Log[2 + x]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$(d-2e)\log{(x+2)}+\int e\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x+d)*(x**3-2*x**2-x+2)/(x**4-5*x**2+4),x)
```

```
[Out] (d - 2*e)*log(x + 2) + Integral(e, x)
```

Mathematica [A] time = 0.00731641, size = 16, normalized size = 1.14

$$(d-2e)\log(x+2)+e(x+2)$$

Antiderivative was successfully verified.

```
[In]  Integrate[((d + e*x)*(2 - x - 2*x^2 + x^3))/(4 - 5*x^2 + x^4),x]
```

```
[Out] e*(2 + x) + (d - 2*e)*Log[2 + x]
```

Maple [A] time = 0.003, size = 18, normalized size = 1.3

$$ex + \ln(2+x)\,d - 2\,\ln(2+x)\,e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x+d)*(x^3-2*x^2-x+2)/(x^4-5*x^2+4),x)

[Out] e*x+ln(2+x)*d-2*ln(2+x)*e
```

Maxima [A] time = 0.708374, size = 19, normalized size = 1.36

$$ex + (d - 2\,e)\log(x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 - 2*x^2 - x + 2)*(e*x + d)/(x^4 - 5*x^2 + 4),x, algorithm="maxima")

[Out] e*x + (d - 2*e)*log(x + 2)
```

Fricas [A] time = 0.264118, size = 19, normalized size = 1.36

$$ex + (d - 2\,e)\log(x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 - 2*x^2 - x + 2)*(e*x + d)/(x^4 - 5*x^2 + 4),x, algorithm="fricas")

[Out] e*x + (d - 2*e)*log(x + 2)
```

Sympy [A] time = 1.026, size = 12, normalized size = 0.86

$$ex + (d - 2e)\log(x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x+d)*(x**3-2*x**2-x+2)/(x**4-5*x**2+4),x)
```

```
[Out] e*x + (d - 2*e)*log(x + 2)
```

GIAC/XCAS [A] time = 0.280264, size = 23, normalized size = 1.64

$$xe + (d - 2e)\ln(|x + 2|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 - 2*x^2 - x + 2)*(e*x + d)/(x^4 - 5*x^2 + 4),x, algorithm="giac")
```

```
[Out] x*e + (d - 2*e)*ln(abs(x + 2))
```

3.69 $\int \frac{(d+ex+fx^2)(2-x-2x^2+x^3)}{4-5x^2+x^4}\,dx$

Optimal. Leaf size=31

$$\log(x+2)(d-2e+4f)+x(e-4f)+\frac{1}{2}f(x+2)^2$$

```
[Out] (e - 4*f)*x + (f*(2 + x)^2)/2 + (d - 2*e + 4*f)*Log[2 + x]
```

Rubi [A] time = 0.071889, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056

$$\log(x+2)(d-2e+4f)+x(e-4f)+\frac{1}{2}f(x+2)^2$$

Antiderivative was successfully verified.

```
[In]  Int[((d + e*x + f*x^2)*(2 - x - 2*x^2 + x^3))/(4 - 5*x^2 + x^4),x]
```

```
[Out] (e - 4*f)*x + (f*(2 + x)^2)/2 + (d - 2*e + 4*f)*Log[2 + x]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-4fx+\frac{f(x+2)^2}{2}+(d-2e+4f)\log(x+2)+\int e\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**2+e*x+d)*(x**3-2*x**2-x+2)/(x**4-5*x**2+4),x)
```

```
[Out] -4*f*x + f*(x + 2)**2/2 + (d - 2*e + 4*f)*log(x + 2) + Integral(e
, x)
```

Mathematica [A] time = 0.0199852, size = 30, normalized size = 0.97

$$\log(x+2)(d-2e+4f)+\frac{1}{2}(x+2)(2e+f(x-6))$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x + f*x^2)*(2 - x - 2*x^2 + x^3))/(4 - 5*x^2 + x^4),x]
```

```
[Out] ((2*e + f*(-6 + x))*(2 + x))/2 + (d - 2*e + 4*f)*Log[2 + x]
```

Maple [A] time = 0.004, size = 35, normalized size = 1.1

$$\frac{f x^2}{2} + ex - 2\,f x + \ln(2+x)\,d - 2\,\ln(2+x)\,e + 4\,\ln(2+x)\,f$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x^2+e*x+d)*(x^3-2*x^2-x+2)/(x^4-5*x^2+4),x)
```

```
[Out] 1/2*f*x^2+e*x-2*f*x+ln(2+x)*d-2*ln(2+x)*e+4*ln(2+x)*f
```

Maxima [A] time = 0.700535, size = 36, normalized size = 1.16

$$\frac{1}{2}\,f x^2 + (e - 2\,f)x + (d - 2\,e + 4\,f)\log(x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2 + e*x + d)*(x^3 - 2*x^2 - x + 2)/(x^4 - 5*x^2 + 4),x, algorithm="m
```

```
[Out] 1/2*f*x^2 + (e - 2*f)*x + (d - 2*e + 4*f)*log(x + 2)
```

Fricas [A] time = 0.265825, size = 36, normalized size = 1.16

$$\frac{1}{2}\,f x^2 + (e - 2\,f)x + (d - 2\,e + 4\,f)\log(x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2 + e*x + d)*(x^3 - 2*x^2 - x + 2)/(x^4 - 5*x^2 + 4),x, algorithm="f
```

```
[Out] 1/2*f*x^2 + (e - 2*f)*x + (d - 2*e + 4*f)*log(x + 2)
```

Sympy [A] time = 1.10943, size = 26, normalized size = 0.84

$$\frac{fx^2}{2} + x\left(e - 2f\right) + \left(d - 2e + 4f\right)\log\left(x + 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x**2+e*x+d)*(x**3-2*x**2-x+2)/(x**4-5*x**2+4),x)
```

```
[Out] f*x**2/2 + x*(e - 2*f) + (d - 2*e + 4*f)*log(x + 2)
```

GIAC/XCAS [A] time = 0.283262, size = 41, normalized size = 1.32

$$\frac{1}{2}\, f x^2 - 2\, f x + x e + (d + 4\, f - 2\, e)\ln\left(|x + 2|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2 + e*x + d)*(x^3 - 2*x^2 - x + 2)/(x^4 - 5*x^2 + 4),x, algorithm="g
```

```
[Out] 1/2*f*x^2 - 2*f*x + x*e + (d + 4*f - 2*e)*ln(abs(x + 2))
```

3.70 $\int \frac{(2-x-2x^2+x^3)(d+ex+fx^2+gx^3)}{4-5x^2+x^4} \, dx$

Optimal. Leaf size=51

$$\log(x+2)(d-2e+4f-8g)+x(e-4f+12g)+\frac{1}{2}(x+2)^2(f-6g)+\frac{1}{3}g(x+2)^3$$

```
[Out] (e - 4*f + 12*g)*x + ((f - 6*g)*(2 + x)^2)/2 + (g*(2 + x)^3)/3 +
(d - 2*e + 4*f - 8*g)*Log[2 + x]
```

Rubi [A] time = 0.127868, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.049

$$\log(x+2)(d-2e+4f-8g)+x(e-4f+12g)+\frac{1}{2}(x+2)^2(f-6g)+\frac{1}{3}g(x+2)^3$$

Antiderivative was successfully verified.

```
[In] Int[((2 - x - 2*x^2 + x^3)*(d + e*x + f*x^2 + g*x^3))/(4 - 5*x^2 + x^4),x]
```

```
[Out] (e - 4*f + 12*g)*x + ((f - 6*g)*(2 + x)^2)/2 + (g*(2 + x)^3)/3 +
(d - 2*e + 4*f - 8*g)*Log[2 + x]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-4fx+12gx+\frac{g(x+2)^3}{3}+\left(\frac{f}{2}-3g\right)(x+2)^2+(d-2e+4f-8g)\log(x+2)+\int e\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((x**3-2*x**2-x+2)*(g*x**3+f*x**2+e*x+d)/(x**4-5*x**2+4),x)
```

```
[Out] -4*f*x + 12*g*x + g*(x + 2)**3/3 + (f/2 - 3*g)*(x + 2)**2 + (d -
2*e + 4*f - 8*g)*log(x + 2) + Integral(e, x)
```

Mathematica [A] time = 0.0432345, size = 45, normalized size = 0.88

$$\log(x+2)(d-2e+4f-8g)+\frac{1}{6}(x+2)\left(6e+3f(x-6)+2g\left(x^2-5x+22\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 - x - 2*x^2 + x^3)*(d + e*x + f*x^2 + g*x^3))/(4 - 5*x^2 + x^4),x]
```

```
[Out] ((2 + x)*(6*e + 3*f*(-6 + x) + 2*g*(22 - 5*x + x^2)))/6 + (d - 2*
e + 4*f - 8*g)*Log[2 + x]
```

Maple [A] time = 0.004, size = 58, normalized size = 1.1

$$\frac{gx^3}{3} + \frac{fx^2}{2} - gx^2 + ex - 2fx + 4gx + \ln(2+x)d - 2\ln(2+x)e + 4\ln(2+x)f - 8\ln(2+x)g$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-2*x^2-x+2)*(g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4),x)
```

```
[Out] 1/3*g*x^3+1/2*f*x^2-g*x^2+e*x-2*f*x+4*g*x+ln(2+x)*d-2*ln(2+x)*e+4
*ln(2+x)*f-8*ln(2+x)*g
```

Maxima [A] time = 0.693303, size = 58, normalized size = 1.14

$$\frac{1}{3}gx^3 + \frac{1}{2}(f - 2g)x^2 + (e - 2f + 4g)x + (d - 2e + 4f - 8g)\log(x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3 + f*x^2 + e*x + d)*(x^3 - 2*x^2 - x + 2)/(x^4 - 5*x^2 + 4),x, algor
```

```
[Out] 1/3*g*x^3 + 1/2*(f - 2*g)*x^2 + (e - 2*f + 4*g)*x + (d - 2*e + 4*
f - 8*g)*log(x + 2)
```

Fricas [A] time = 0.26788, size = 58, normalized size = 1.14

$$\frac{1}{3}gx^3 + \frac{1}{2}(f - 2g)x^2 + (e - 2f + 4g)x + (d - 2e + 4f - 8g)\log(x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3 + f*x^2 + e*x + d)*(x^3 - 2*x^2 - x + 2)/(x^4 - 5*x^2 + 4),x, algor
```

```
[Out] 1/3*g*x^3 + 1/2*(f - 2*g)*x^2 + (e - 2*f + 4*g)*x + (d - 2*e + 4*
f - 8*g)*log(x + 2)
```

Sympy [A] time = 1.21559, size = 41, normalized size = 0.8

$$\frac{gx^3}{3} + x^2\left(\frac{f}{2} - g\right) + x\,(e - 2f + 4g) + (d - 2e + 4f - 8g)\log{(x + 2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-2*x**2-x+2)*(g*x**3+f*x**2+e*x+d)/(x**4-5*x**2+4),x)
```

```
[Out] g*x**3/3 + x**2*(f/2 - g) + x*(e - 2*f + 4*g) + (d - 2*e + 4*f -
8*g)*log(x + 2)
```

GIAC/XCAS [A] time = 0.289, size = 66, normalized size = 1.29

$$\frac{1}{3}\,gx^3 + \frac{1}{2}\,fx^2 - gx^2 - 2\,fx + 4\,gx + xe + (d + 4\,f - 8\,g - 2\,e)\ln\left(|x + 2|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3 + f*x^2 + e*x + d)*(x^3 - 2*x^2 - x + 2)/(x^4 - 5*x^2 + 4),x, algor
```

```
[Out] 1/3*g*x^3 + 1/2*f*x^2 - g*x^2 - 2*f*x + 4*g*x + x*e + (d + 4*f -
8*g - 2*e)*ln(abs(x + 2))
```

3.71 $\int \frac{(2-x-2x^2+x^3)(d+ex+fx^2+gx^3+hx^4)}{4-5x^2+x^4}\,dx$

Optimal. Leaf size=68

$$\log(x+2)(d-2e+4f-8g+16h)+x(e-2f+4g-8h)+\frac{1}{2}x^2(f-2g+4h)+\frac{1}{3}x^3(g-2h)+\frac{hx^4}{4}$$

```
[Out] (e - 2*f + 4*g - 8*h)*x + ((f - 2*g + 4*h)*x^2)/2 + ((g - 2*h)*x^
3)/3 + (h*x^4)/4 + (d - 2*e + 4*f - 8*g + 16*h)*Log[2 + x]
```

Rubi [A] time = 0.200283, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043

$$\log(x+2)(d-2e+4f-8g+16h)+x(e-2f+4g-8h)+\frac{1}{2}x^2(f-2g+4h)+\frac{1}{3}x^3(g-2h)+\frac{hx^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[((2 - x - 2*x^2 + x^3)*(d + e*x + f*x^2 + g*x^3 + h*x^4))/(4 - 5*x^2 + x^4),x]
```

```
[Out] (e - 2*f + 4*g - 8*h)*x + ((f - 2*g + 4*h)*x^2)/2 + ((g - 2*h)*x^
3)/3 + (h*x^4)/4 + (d - 2*e + 4*f - 8*g + 16*h)*Log[2 + x]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-4fx+12gx-32hx+\frac{h(x+2)^4}{4}+\left(\frac{g}{3}-\frac{8h}{3}\right)(x+2)^3+(x+2)^2\left(\frac{f}{2}-3g+12h\right)+(d-2e+4f-8g+16h)\log(x+2)+\int e\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((x**3-2*x**2-x+2)*(h*x**4+g*x**3+f*x**2+e*x+d)/(x**4-
5*x**2+4),x)
```

```
[Out] -4*f*x + 12*g*x - 32*h*x + h*(x + 2)**4/4 + (g/3 - 8*h/3)*(x + 2)
**3 + (x + 2)**2*(f/2 - 3*g + 12*h) + (d - 2*e + 4*f - 8*g + 16*h
)*log(x + 2) + Integral(e, x)
```

Mathematica [A] time = 0.0407927, size = 68, normalized size = 1.

$$\log(x+2)(d-2e+4f-8g+16h)+x(e-2f+4g-8h)+\frac{1}{2}x^2(f-2g+4h)+\frac{1}{3}x^3(g-2h)+\frac{hx^4}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 - x - 2*x^2 + x^3)*(d + e*x + f*x^2 + g*x^3 + h*x^4))/(4 - 5*x^2 + x^4
```

```
[Out] (e - 2*f + 4*g - 8*h)*x + ((f - 2*g + 4*h)*x^2)/2 + ((g - 2*h)*x^
3)/3 + (h*x^4)/4 + (d - 2*e + 4*f - 8*g + 16*h)*Log[2 + x]
```

Maple [A] time = 0.005, size = 87, normalized size = 1.3

$$\frac{hx^4}{4}+\frac{gx^3}{3}-\frac{2\,hx^3}{3}+\frac{fx^2}{2}-gx^2+2\,hx^2+ex-2\,fx+4\,gx-8\,hx \\ +\ln(2+x)\,d-2\,\ln(2+x)\,e+4\,\ln(2+x)\,f-8\,\ln(2+x)\,g+16\,\ln(2+x)\,h$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^3-2*x^2-x+2)*(h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4),x)
```

```
[Out] 1/4*h*x^4+1/3*g*x^3-2/3*h*x^3+1/2*f*x^2-g*x^2+2*h*x^2+e*x-2*f*x+4
*g*x-8*h*x+ln(2+x)*d-2*ln(2+x)*e+4*ln(2+x)*f-8*ln(2+x)*g+16*ln(2+
x)*h
```

Maxima [A] time = 0.700153, size = 84, normalized size = 1.24

$$\frac{1}{4}\,hx^4+\frac{1}{3}\,(g-2\,h)x^3+\frac{1}{2}\,(f-2\,g+4\,h)x^2+(e-2\,f+4\,g-8\,h)x+(d-2\,e+4\,f-8\,g+16\,h)\log(x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^4 + g*x^3 + f*x^2 + e*x + d)*(x^3 - 2*x^2 - x + 2)/(x^4 - 5*x^2 + 4),
```

```
[Out] 1/4*h*x^4 + 1/3*(g - 2*h)*x^3 + 1/2*(f - 2*g + 4*h)*x^2 + (e - 2*
f + 4*g - 8*h)*x + (d - 2*e + 4*f - 8*g + 16*h)*log(x + 2)
```

Fricas [A] time = 0.246832, size = 84, normalized size = 1.24

$$\frac{1}{4}\,hx^4+\frac{1}{3}\,(g-2\,h)x^3+\frac{1}{2}\,(f-2\,g+4\,h)x^2+(e-2\,f+4\,g-8\,h)x+(d-2\,e+4\,f-8\,g+16\,h)\log(x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^4 + g*x^3 + f*x^2 + e*x + d)*(x^3 - 2*x^2 - x + 2)/(x^4 - 5*x^2 + 4),
```

```
[Out] 1/4*h*x^4 + 1/3*(g - 2*h)*x^3 + 1/2*(f - 2*g + 4*h)*x^2 + (e - 2*
f + 4*g - 8*h)*x + (d - 2*e + 4*f - 8*g + 16*h)*log(x + 2)
```

Sympy [A] time = 1.3224, size = 63, normalized size = 0.93

$$\frac{hx^4}{4} + x^3\left(\frac{g}{3} - \frac{2h}{3}\right) + x^2\left(\frac{f}{2} - g + 2h\right) + x\left(e - 2f + 4g - 8h\right) + \left(d - 2e + 4f - 8g + 16h\right)\log\left(x + 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-2*x**2-x+2)*(h*x**4+g*x**3+f*x**2+e*x+d)/(x**4-5*x**2+4),x)
```

```
[Out] h*x**4/4 + x**3*(g/3 - 2*h/3) + x**2*(f/2 - g + 2*h) + x*(e - 2*f
 + 4*g - 8*h) + (d - 2*e + 4*f - 8*g + 16*h)*log(x + 2)
```

GIAC/XCAS [A] time = 0.281569, size = 100, normalized size = 1.47

$$\frac{1}{4}hx^4 + \frac{1}{3}gx^3 - \frac{2}{3}hx^3 + \frac{1}{2}fx^2 - gx^2 + 2hx^2 - 2fx + 4gx - 8hx + xe + (d + 4f - 8g + 16h - 2e)\ln(|x + 2|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^4 + g*x^3 + f*x^2 + e*x + d)*(x^3 - 2*x^2 - x + 2)/(x^4 - 5*x^2 + 4),
```

```
[Out] 1/4*h*x^4 + 1/3*g*x^3 - 2/3*h*x^3 + 1/2*f*x^2 - g*x^2 + 2*h*x^2 -
 2*f*x + 4*g*x - 8*h*x + x*e + (d + 4*f - 8*g + 16*h - 2*e)*ln(ab
s(x + 2))
```

3.72 $\int \frac{(2-x-2x^2+x^3)(d+ex+fx^2+gx^3+hx^4+ix^5)}{4-5x^2+x^4}\,dx$

Optimal. Leaf size=92

$$\log(x+2)(d-2e+4f-8g+16h-32i)+x(e-2f+4g-8h+16i)+\frac{1}{2}x^2(f-2g+4h-8i)+\frac{1}{3}x^3(g-2h+4i)+\frac{1}{4}x^4(h-2i)+\frac{ix^5}{5}$$

```
[Out] (e - 2*f + 4*g - 8*h + 16*i)*x + ((f - 2*g + 4*h - 8*i)*x^2)/2 +
((g - 2*h + 4*i)*x^3)/3 + ((h - 2*i)*x^4)/4 + (i*x^5)/5 + (d - 2*
e + 4*f - 8*g + 16*h - 32*i)*Log[2 + x]
```

Rubi [A] time = 0.268006, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 51, $\frac{\text{number of rules}}{\text{integrand size}} = 0.039$

$$\log(x+2)(d-2e+4f-8g+16h-32i)+x(e-2f+4g-8h+16i)+\frac{1}{2}x^2(f-2g+4h-8i)+\frac{1}{3}x^3(g-2h+4i)+\frac{1}{4}x^4(h-2i)+\frac{ix^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[((2 - x - 2*x^2 + x^3)*(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5))/(4 - 5*x^2 + x^
```

```
[Out] (e - 2*f + 4*g - 8*h + 16*i)*x + ((f - 2*g + 4*h - 8*i)*x^2)/2 +
((g - 2*h + 4*i)*x^3)/3 + ((h - 2*i)*x^4)/4 + (i*x^5)/5 + (d - 2*
e + 4*f - 8*g + 16*h - 32*i)*Log[2 + x]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{ix^5}{5}+x^4\left(\frac{h}{4}-\frac{i}{2}\right)+x^3\left(\frac{g}{3}-\frac{2h}{3}+\frac{4i}{3}\right)+(f-2g+4h-8i)\int x\,dx+(d-2e+4f-8g+16h-32i)\log(x+2)+\frac{(e-2f+4g-8h+16i)\int e\,dx}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((x**3-2*x**2-x+2)*(i*x**5+h*x**4+g*x**3+f*x**2+e*x+d)/(x**4-
5*x**2+4),x)
```

```
[Out] i*x**5/5 + x**4*(h/4 - i/2) + x**3*(g/3 - 2*h/3 + 4*i/3) + (f - 2
*g + 4*h - 8*i)*Integral(x, x) + (d - 2*e + 4*f - 8*g + 16*h - 32
```

```
*i)*log(x + 2) + (e - 2*f + 4*g - 8*h + 16*i)*Integral(e, x)/e
```

Mathematica [A] time = 0.0638651, size = 92, normalized size = 1.

$$\log(x+2)(d-2e+4f-8g+16h-32i)+x(e-2f+4g-8h+16i)+\frac{1}{2}x^2(f-2g+4h-8i)+\frac{1}{3}x^3(g-2h+4i)+\frac{1}{4}x^4(h-2i)+\frac{ix^5}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 - x - 2*x^2 + x^3)*(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5))/(4 - 5*x
```

```
[Out] (e - 2*f + 4*g - 8*h + 16*i)*x + ((f - 2*g + 4*h - 8*i)*x^2)/2 +
((g - 2*h + 4*i)*x^3)/3 + ((h - 2*i)*x^4)/4 + (i*x^5)/5 + (d - 2*
e + 4*f - 8*g + 16*h - 32*i)*Log[2 + x]
```

Maple [A] time = 0.004, size = 122, normalized size = 1.3

$$\frac{ix^5}{5}+\frac{hx^4}{4}-\frac{ix^4}{2}+\frac{gx^3}{3}-\frac{2\,hx^3}{3}+\frac{4\,ix^3}{3}+\frac{fx^2}{2}-gx^2+2\,hx^2-4\,ix^2+ex-2\,fx+4\,gx-8\,hx+16\,ix+\ln(2+x)\,d-2\,\ln(2+x)\,e+4\,\ln(2+x)\,f-8\,\ln(2+x)\,g+16\,\ln(2+x)\,h-32\,\ln(2+x)\,i$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-2*x^2-x+2)*(i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4),x)
```

```
[Out] 1/5*i*x^5+1/4*h*x^4-1/2*i*x^4+1/3*g*x^3-2/3*h*x^3+4/3*i*x^3+1/2*f
*x^2-g*x^2+2*h*x^2-4*i*x^2+e*x-2*f*x+4*g*x-8*h*x+16*i*x+ln(2+x)*d
-2*ln(2+x)*e+4*ln(2+x)*f-8*ln(2+x)*g+16*ln(2+x)*h-32*ln(2+x)*i
```

Maxima [A] time = 0.70285, size = 113, normalized size = 1.23

$$\frac{1}{5}\,ix^5+\frac{1}{4}\,(h-2\,i)x^4+\frac{1}{3}\,(g-2\,h+4\,i)x^3+\frac{1}{2}\,(f-2\,g+4\,h-8\,i)x^2+(e-2\,f+4\,g-8\,h+16\,i)x+(d-2\,e+4\,f-8\,g+16\,h-32\,i)\log(x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^5 + h*x^4 + g*x^3 + f*x^2 + e*x + d)*(x^3 - 2*x^2 - x + 2)/(x^4 - 5*x
```

```
[Out] 1/5*i*x^5 + 1/4*(h - 2*i)*x^4 + 1/3*(g - 2*h + 4*i)*x^3 + 1/2*(f
- 2*g + 4*h - 8*i)*x^2 + (e - 2*f + 4*g - 8*h + 16*i)*x + (d - 2*
e + 4*f - 8*g + 16*h - 32*i)*log(x + 2)
```

Fricas [A] time = 0.282459, size = 113, normalized size = 1.23

$$\frac{1}{5}\,ix^5 + \frac{1}{4}\,(h - 2\,i)x^4 + \frac{1}{3}\,(g - 2\,h + 4\,i)x^3 + \frac{1}{2}\,(f - 2\,g + 4\,h - 8\,i)x^2 + (e - 2\,f + 4\,g - 8\,h + 16\,i)x + (d - 2\,e + 4\,f - 8\,g + 16\,h - 32\,i)\log(x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^5 + h*x^4 + g*x^3 + f*x^2 + e*x + d)*(x^3 - 2*x^2 - x + 2)/(x^4 - 5*x
```

```
[Out] 1/5*i*x^5 + 1/4*(h - 2*i)*x^4 + 1/3*(g - 2*h + 4*i)*x^3 + 1/2*(f
- 2*g + 4*h - 8*i)*x^2 + (e - 2*f + 4*g - 8*h + 16*i)*x + (d - 2*
e + 4*f - 8*g + 16*h - 32*i)*log(x + 2)
```

Sympy [A] time = 1.46319, size = 88, normalized size = 0.96

$$\frac{ix^5}{5} + x^4\left(\frac{h}{4} - \frac{i}{2}\right) + x^3\left(\frac{g}{3} - \frac{2h}{3} + \frac{4i}{3}\right) + x^2\left(\frac{f}{2} - g + 2h - 4i\right) + x\,(e - 2f + 4g - 8h + 16i) + (d - 2e + 4f - 8g + 16h - 32i)\log(x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-2*x**2-x+2)*(i*x**5+h*x**4+g*x**3+f*x**2+e*x+d)/(x**4-
5*x**2+4),x)
```

```
[Out] i*x**5/5 + x**4*(h/4 - i/2) + x**3*(g/3 - 2*h/3 + 4*i/3) + x**2*(
f/2 - g + 2*h - 4*i) + x*(e - 2*f + 4*g - 8*h + 16*i) + (d - 2*e
+ 4*f - 8*g + 16*h - 32*i)*log(x + 2)
```

GIAC/XCAS [A] time = 0.2854, size = 142, normalized size = 1.54

$$\frac{1}{5}\,ix^5 + \frac{1}{4}\,hx^4 - \frac{1}{2}\,ix^4 + \frac{1}{3}\,gx^3 - \frac{2}{3}\,hx^3 + \frac{4}{3}\,ix^3 + \frac{1}{2}\,fx^2 - gx^2 + 2\,hx^2 - 4\,ix^2 - 2\,fx + 4\,gx - 8\,hx + 16\,ix + xe + (d + 4\,f - 8\,g + 16\,h - 32\,i - 2\,e)\ln(|x + 2|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((i*x^5 + h*x^4 + g*x^3 + f*x^2 + e*x + d)*(x^3 - 2*x^2 - x + 2)/(x^4 - 5*x
```

```
[Out] 1/5*i*x^5 + 1/4*h*x^4 - 1/2*i*x^4 + 1/3*g*x^3 - 2/3*h*x^3 + 4/3*i
*x^3 + 1/2*f*x^2 - g*x^2 + 2*h*x^2 - 4*i*x^2 - 2*f*x + 4*g*x - 8*
h*x + 16*i*x + x*e + (d + 4*f - 8*g + 16*h - 32*i - 2*e)*ln(abs(x
 + 2))
```

3.73 $\int \frac{2-3x+x^2}{4-5x^2+x^4} \, dx$

Optimal. Leaf size=11

$$\log(x+1) - \log(x+2)$$

```
[Out] Log[1 + x] - Log[2 + x]
```

Rubi [A] time = 0.0157969, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$

$$\log(x+1) - \log(x+2)$$

Antiderivative was successfully verified.

```
[In]  Int[(2 - 3*x + x^2)/(4 - 5*x^2 + x^4),x]
```

```
[Out] Log[1 + x] - Log[2 + x]
```

Rubi in Sympy [A] time = 4.27421, size = 8, normalized size = 0.73

$$\log(x+1) - \log(x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**2-3*x+2)/(x**4-5*x**2+4),x)
```

```
[Out] log(x + 1) - log(x + 2)
```

Mathematica [A] time = 0.00490726, size = 11, normalized size = 1.

$$\log(x+1) - \log(x+2)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(2 - 3*x + x^2)/(4 - 5*x^2 + x^4),x]
```

```
[Out] Log[1 + x] - Log[2 + x]
```

Maple [A] time = 0.008, size = 12, normalized size = 1.1

$$\ln(1+x) - \ln(2+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^2-3*x+2)/(x^4-5*x^2+4),x)
```

```
[Out] ln(1+x)-ln(2+x)
```

Maxima [A] time = 0.694797, size = 15, normalized size = 1.36

$$-\log(x+2) + \log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 - 3*x + 2)/(x^4 - 5*x^2 + 4),x, algorithm="maxima")
```

```
[Out] -log(x + 2) + log(x + 1)
```

Fricas [A] time = 0.248909, size = 15, normalized size = 1.36

$$-\log(x+2) + \log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 - 3*x + 2)/(x^4 - 5*x^2 + 4),x, algorithm="fricas")
```

```
[Out] -log(x + 2) + log(x + 1)
```

Sympy [A] time = 0.181274, size = 8, normalized size = 0.73

$$\log(x+1) - \log(x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**2-3*x+2)/(x**4-5*x**2+4),x)
```

```
[Out] log(x + 1) - log(x + 2)
```

GIAC/XCAS [A] time = 0.283188, size = 18, normalized size = 1.64

$$-\ln(|x+2|)+\ln(|x+1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 - 3*x + 2)/(x^4 - 5*x^2 + 4),x, algorithm="giac")
```

```
[Out] -ln(abs(x + 2)) + ln(abs(x + 1))
```

3.74 $\int \frac{(d+ex)(2-3x+x^2)}{4-5x^2+x^4}\,dx$

Optimal. Leaf size=22

$$(d-e)\log(x+1)-(d-2e)\log(x+2)$$

```
[Out] (d - e)*Log[1 + x] - (d - 2*e)*Log[2 + x]
```

Rubi [A] time = 0.0504856, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115

$$(d-e)\log(x+1)-(d-2e)\log(x+2)$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)*(2 - 3*x + x^2))/(4 - 5*x^2 + x^4),x]

[Out] (d - e)*Log[1 + x] - (d - 2*e)*Log[2 + x]
```

Rubi in Sympy [A] time = 8.67422, size = 17, normalized size = 0.77

$$-(d-2e)\log(x+2)+(d-e)\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((e*x+d)*(x**2-3*x+2)/(x**4-5*x**2+4),x)

[Out] -(d - 2*e)*log(x + 2) + (d - e)*log(x + 1)
```

Mathematica [A] time = 0.0133759, size = 23, normalized size = 1.05

$$(d-e)\log(x+1)+(2e-d)\log(x+2)$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)*(2 - 3*x + x^2))/(4 - 5*x^2 + x^4),x]

[Out] (d - e)*Log[1 + x] + (-d + 2*e)*Log[2 + x]
```

Maple [A] time = 0.008, size = 29, normalized size = 1.3

$$-\ln(2+x)\,d+2\,\ln(2+x)\,e+\ln(1+x)\,d-\ln(1+x)\,e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x+d)*(x^2-3*x+2)/(x^4-5*x^2+4),x)

[Out] -ln(2+x)*d+2*ln(2+x)*e+ln(1+x)*d-ln(1+x)*e
```

Maxima [A] time = 0.703036, size = 30, normalized size = 1.36

$$-(d-2\,e)\log(x+2)+(d-e)\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x + d)*(x^2 - 3*x + 2)/(x^4 - 5*x^2 + 4),x, algorithm="maxima")

[Out] -(d - 2*e)*log(x + 2) + (d - e)*log(x + 1)
```

Fricas [A] time = 0.257359, size = 30, normalized size = 1.36

$$-(d-2\,e)\log(x+2)+(d-e)\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x + d)*(x^2 - 3*x + 2)/(x^4 - 5*x^2 + 4),x, algorithm="fricas")

[Out] -(d - 2*e)*log(x + 2) + (d - e)*log(x + 1)
```

Sympy [A] time = 0.72595, size = 29, normalized size = 1.32

$$(-d+2e)\log\left(x+\frac{4d-6e}{2d-3e}\right)+(d-e)\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x+d)*(x**2-3*x+2)/(x**4-5*x**2+4),x)
```

```
[Out] (-d + 2*e)*log(x + (4*d - 6*e)/(2*d - 3*e)) + (d - e)*log(x + 1)
```

GIAC/XCAS [A] time = 0.283366, size = 35, normalized size = 1.59

$$-(d - 2\,e)\ln\left(|x + 2|\right) + (d - e)\ln\left(|x + 1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x + d)*(x^2 - 3*x + 2)/(x^4 - 5*x^2 + 4),x, algorithm="giac")
```

```
[Out] -(d - 2*e)*ln(abs(x + 2)) + (d - e)*ln(abs(x + 1))
```

3.75 $\int \frac{(2-3x+x^2)(d+ex+fx^2)}{4-5x^2+x^4} \, dx$

Optimal. Leaf size=29

$$\log(x+1)(d-e+f)-\log(x+2)(d-2e+4f)+fx$$

```
[Out] f*x + (d - e + f)*Log[1 + x] - (d - 2*e + 4*f)*Log[2 + x]
```

Rubi [A] time = 0.0817662, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$

$$\log(x+1)(d-e+f)-\log(x+2)(d-2e+4f)+fx$$

Antiderivative was successfully verified.

```
[In]  Int[((2 - 3*x + x^2)*(d + e*x + f*x^2))/(4 - 5*x^2 + x^4),x]
```

```
[Out] f*x + (d - e + f)*Log[1 + x] - (d - 2*e + 4*f)*Log[2 + x]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-(d-2e+4f)\log(x+2)+(d-e+f)\log(x+1)+\int f\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**2-3*x+2)*(f*x**2+e*x+d)/(x**4-5*x**2+4),x)
```

```
[Out] -(d - 2*e + 4*f)*log(x + 2) + (d - e + f)*log(x + 1) + Integral(f
, x)
```

Mathematica [A] time = 0.0227694, size = 30, normalized size = 1.03

$$\log(x+1)(d-e+f)+\log(x+2)(-d+2e-4f)+fx$$

Antiderivative was successfully verified.

```
[In]  Integrate[((2 - 3*x + x^2)*(d + e*x + f*x^2))/(4 - 5*x^2 + x^4),x]
```

```
[Out] f*x + (d - e + f)*Log[1 + x] + (-d + 2*e - 4*f)*Log[2 + x]
```

Maple [A] time = 0.009, size = 45, normalized size = 1.6

$$fx - \ln(2+x)\,d + 2\,\ln(2+x)\,e - 4\,\ln(2+x)\,f + \ln(1+x)\,d - \ln(1+x)\,e + \ln(1+x)\,f$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^2-3*x+2)*(f*x^2+e*x+d)/(x^4-5*x^2+4),x)
```

```
[Out] f*x-ln(2+x)*d+2*ln(2+x)*e-4*ln(2+x)*f+ln(1+x)*d-ln(1+x)*e+ln(1+x)
*f
```

Maxima [A] time = 0.702563, size = 39, normalized size = 1.34

$$fx - (d - 2\,e + 4\,f)\log(x+2) + (d - e + f)\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2 + e*x + d)*(x^2 - 3*x + 2)/(x^4 - 5*x^2 + 4),x, algorithm="maxima
```

```
[Out] f*x - (d - 2*e + 4*f)*log(x + 2) + (d - e + f)*log(x + 1)
```

Fricas [A] time = 0.253867, size = 39, normalized size = 1.34

$$fx - (d - 2\,e + 4\,f)\log(x+2) + (d - e + f)\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2 + e*x + d)*(x^2 - 3*x + 2)/(x^4 - 5*x^2 + 4),x, algorithm="fricas
```

```
[Out] f*x - (d - 2*e + 4*f)*log(x + 2) + (d - e + f)*log(x + 1)
```

Sympy [A] time = 2.24305, size = 44, normalized size = 1.52

$$fx + (-d + 2e - 4f)\log\left(x + \frac{4d - 6e + 10f}{2d - 3e + 5f}\right) + (d - e + f)\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**2-3*x+2)*(f*x**2+e*x+d)/(x**4-5*x**2+4),x)
```

```
[Out] f*x + (-d + 2*e - 4*f)*log(x + (4*d - 6*e + 10*f)/(2*d - 3*e + 5*
f)) + (d - e + f)*log(x + 1)
```

GIAC/XCAS [A] time = 0.282943, size = 45, normalized size = 1.55

$$fx - (d + 4f - 2e)\ln(|x + 2|) + (d + f - e)\ln(|x + 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^2 + e*x + d)*(x^2 - 3*x + 2)/(x^4 - 5*x^2 + 4),x, algorithm="giac")
```

```
[Out] f*x - (d + 4*f - 2*e)*ln(abs(x + 2)) + (d + f - e)*ln(abs(x + 1))
```

3.76 $\int \frac{(2-3x+x^2)(d+ex+fx^2+gx^3)}{4-5x^2+x^4}\,dx$

Optimal. Leaf size=47

$$\log(x+1)(d-e+f-g)-\log(x+2)(d-2e+4f-8g)+x(f-3g)+\frac{gx^2}{2}$$

```
[Out] (f - 3*g)*x + (g*x^2)/2 + (d - e + f - g)*Log[1 + x] - (d - 2*e +
 4*f - 8*g)*Log[2 + x]
```

Rubi [A] time = 0.122068, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111

$$\log(x+1)(d-e+f-g)-\log(x+2)(d-2e+4f-8g)+x(f-3g)+\frac{gx^2}{2}$$

Antiderivative was successfully verified.

```
[In]  Int[((2 - 3*x + x^2)*(d + e*x + f*x^2 + g*x^3))/(4 - 5*x^2 + x^4),x]
```

```
[Out] (f - 3*g)*x + (g*x^2)/2 + (d - e + f - g)*Log[1 + x] - (d - 2*e +
 4*f - 8*g)*Log[2 + x]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-3gx+g\int x\,dx-(d-2e+4f-8g)\log(x+2)+(d-e+f-g)\log(x+1)+\int f\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((x**2-3*x+2)*(g*x**3+f*x**2+e*x+d)/(x**4-5*x**2+4),x)
```

```
[Out] -3*g*x + g*Integral(x, x) - (d - 2*e + 4*f - 8*g)*log(x + 2) + (d
 - e + f - g)*log(x + 1) + Integral(f, x)
```

Mathematica [A] time = 0.0413789, size = 44, normalized size = 0.94

$$\log(x+1)(d-e+f-g)-\log(x+2)(d-2e+4f-8g)+fx+\frac{1}{2}g(x-6)x$$

Antiderivative was successfully verified.

```
[In]  Integrate[((2 - 3*x + x^2)*(d + e*x + f*x^2 + g*x^3))/(4 - 5*x^2 + x^4),x]
```

```
[Out] f*x + (g*(-6 + x)*x)/2 + (d - e + f - g)*Log[1 + x] - (d - 2*e +
4*f - 8*g)*Log[2 + x]
```

Maple [A] time = 0.009, size = 69, normalized size = 1.5

$$\frac{gx^2}{2} + fx - 3gx - \ln(2+x)d + 2\ln(2+x)e - 4\ln(2+x)f + 8\ln(2+x)g + \ln(1+x)d - \ln(1+x)e + \ln(1+x)f - \ln(1+x)g$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^2-3*x+2)*(g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4),x)
```

```
[Out] 1/2*g*x^2+f*x-3*g*x-ln(2+x)*d+2*ln(2+x)*e-4*ln(2+x)*f+8*ln(2+x)*g
+ln(1+x)*d-ln(1+x)*e+ln(1+x)*f-ln(1+x)*g
```

Maxima [A] time = 0.704417, size = 61, normalized size = 1.3

$$\frac{1}{2}gx^2 + (f-3g)x - (d-2e+4f-8g)\log(x+2) + (d-e+f-g)\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((g*x^3 + f*x^2 + e*x + d)*(x^2 - 3*x + 2)/(x^4 - 5*x^2 + 4),x, algorithm=
```

```
[Out] 1/2*g*x^2 + (f - 3*g)*x - (d - 2*e + 4*f - 8*g)*log(x + 2) + (d -
 e + f - g)*log(x + 1)
```

Fricas [A] time = 0.255763, size = 61, normalized size = 1.3

$$\frac{1}{2}gx^2 + (f-3g)x - (d-2e+4f-8g)\log(x+2) + (d-e+f-g)\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((g*x^3 + f*x^2 + e*x + d)*(x^2 - 3*x + 2)/(x^4 - 5*x^2 + 4),x, algorithm=
```

```
[Out] 1/2*g*x^2 + (f - 3*g)*x - (d - 2*e + 4*f - 8*g)*log(x + 2) + (d -
 e + f - g)*log(x + 1)
```

Sympy [A] time = 3.35304, size = 66, normalized size = 1.4

$$\frac{gx^2}{2} + x(f - 3g) + (-d + 2e - 4f + 8g)\log\left(x + \frac{4d - 6e + 10f - 18g}{2d - 3e + 5f - 9g}\right) + (d - e + f - g)\log(x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**2-3*x+2)*(g*x**3+f*x**2+e*x+d)/(x**4-5*x**2+4),x)
```

```
[Out] g*x**2/2 + x*(f - 3*g) + (-d + 2*e - 4*f + 8*g)*log(x + (4*d - 6*
e + 10*f - 18*g)/(2*d - 3*e + 5*f - 9*g)) + (d - e + f - g)*log(x
 + 1)
```

GIAC/XCAS [A] time = 0.286063, size = 66, normalized size = 1.4

$$\frac{1}{2}gx^2 + fx - 3gx - (d + 4f - 8g - 2e)\ln(|x + 2|) + (d + f - g - e)\ln(|x + 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3 + f*x^2 + e*x + d)*(x^2 - 3*x + 2)/(x^4 - 5*x^2 + 4),x, algorithm=
```

```
[Out] 1/2*g*x^2 + f*x - 3*g*x - (d + 4*f - 8*g - 2*e)*ln(abs(x + 2)) +
(d + f - g - e)*ln(abs(x + 1))
```

3.77 $\int \frac{(2-3x+x^2)(d+ex+fx^2+gx^3+hx^4)}{4-5x^2+x^4}\,dx$

Optimal. Leaf size=66

$$\log(x+1)(d-e+f-g+h)-\log(x+2)(d-2e+4f-8g+16h)+x(f-3g+7h)+\frac{1}{2}x^2(g-3h)+\frac{hx^3}{3}$$

```
[Out] (f - 3*g + 7*h)*x + ((g - 3*h)*x^2)/2 + (h*x^3)/3 + (d - e + f -
g + h)*Log[1 + x] - (d - 2*e + 4*f - 8*g + 16*h)*Log[2 + x]
```

Rubi [A] time = 0.172661, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.098

$$\log(x+1)(d-e+f-g+h)-\log(x+2)(d-2e+4f-8g+16h)+x(f-3g+7h)+\frac{1}{2}x^2(g-3h)+\frac{hx^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[((2 - 3*x + x^2)*(d + e*x + f*x^2 + g*x^3 + h*x^4))/(4 - 5*x^2 + x^4),x]
```

```
[Out] (f - 3*g + 7*h)*x + ((g - 3*h)*x^2)/2 + (h*x^3)/3 + (d - e + f -
g + h)*Log[1 + x] - (d - 2*e + 4*f - 8*g + 16*h)*Log[2 + x]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-3gx+\frac{hx^3}{3}+7hx+(g-3h)\int x\,dx-(d-2e+4f-8g+16h)\log(x+2)+(d-e+f-g+h)\log(x+1)+\int f\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((x**2-3*x+2)*(h*x**4+g*x**3+f*x**2+e*x+d)/(x**4-5*x**2+4),x)
```

```
[Out] -3*g*x + h*x**3/3 + 7*h*x + (g - 3*h)*Integral(x, x) - (d - 2*e +
 4*f - 8*g + 16*h)*log(x + 2) + (d - e + f - g + h)*log(x + 1) +
Integral(f, x)
```

Mathematica [A] time = 0.0552303, size = 67, normalized size = 1.02

$$\log(x+1)(d-e+f-g+h)+\log(x+2)(-d+2e-4f+8g-16h)+x(f-3g+7h)+\frac{1}{2}x^2(g-3h)+\frac{hx^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 - 3*x + x^2)*(d + e*x + f*x^2 + g*x^3 + h*x^4))/(4 - 5*x^2 + x^4),x]
```

```
[Out] (f - 3*g + 7*h)*x + ((g - 3*h)*x^2)/2 + (h*x^3)/3 + (d - e + f -
g + h)*Log[1 + x] + (-d + 2*e - 4*f + 8*g - 16*h)*Log[2 + x]
```

Maple [A] time = 0.01, size = 98, normalized size = 1.5

$$\frac{hx^3}{3}+\frac{gx^2}{2}-\frac{3hx^2}{2}+fx-3gx+7hx-\ln(2+x)d+2\ln(2+x)e-4\ln(2+x)f+8\ln(2+x)g-16\ln(2+x)h+\ln(1+x)d-\ln(1+x)e+\ln(1+x)f-\ln(1+x)g+\ln(1+x)h$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-3*x+2)*(h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4),x)
```

```
[Out] 1/3*h*x^3+1/2*g*x^2-3/2*h*x^2+f*x-3*g*x+7*h*x-ln(2+x)*d+2*ln(2+x)
*e-4*ln(2+x)*f+8*ln(2+x)*g-16*ln(2+x)*h+ln(1+x)*d-ln(1+x)*e+ln(1+
x)*f-ln(1+x)*g+ln(1+x)*h
```

Maxima [A] time = 0.705171, size = 84, normalized size = 1.27

$$\frac{1}{3}hx^3+\frac{1}{2}(g-3h)x^2+(f-3g+7h)x-(d-2e+4f-8g+16h)\log(x+2)+(d-e+f-g+h)\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^4 + g*x^3 + f*x^2 + e*x + d)*(x^2 - 3*x + 2)/(x^4 - 5*x^2 + 4),x, alg
```

```
[Out] 1/3*h*x^3 + 1/2*(g - 3*h)*x^2 + (f - 3*g + 7*h)*x - (d - 2*e + 4*
f - 8*g + 16*h)*log(x + 2) + (d - e + f - g + h)*log(x + 1)
```

Fricas [A] time = 0.259409, size = 84, normalized size = 1.27

$$\frac{1}{3}hx^3+\frac{1}{2}(g-3h)x^2+(f-3g+7h)x-(d-2e+4f-8g+16h)\log(x+2)+(d-e+f-g+h)\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^4 + g*x^3 + f*x^2 + e*x + d)*(x^2 - 3*x + 2)/(x^4 - 5*x^2 + 4),x, alg
```

```
[Out] 1/3*h*x^3 + 1/2*(g - 3*h)*x^2 + (f - 3*g + 7*h)*x - (d - 2*e + 4*
f - 8*g + 16*h)*log(x + 2) + (d - e + f - g + h)*log(x + 1)
```

Sympy [A] time = 5.44209, size = 94, normalized size = 1.42

$$\frac{hx^3}{3} + x^2\left(\frac{g}{2} - \frac{3h}{2}\right) + x\,(f - 3g + 7h) + (-d + 2e - 4f + 8g - 16h)\log\left(x + \frac{4d - 6e + 10f - 18g + 34h}{2d - 3e + 5f - 9g + 17h}\right) + (d - e + f - g + h)\log\,(x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-3*x+2)*(h*x**4+g*x**3+f*x**2+e*x+d)/(x**4-5*x**2+4),x)
```

```
[Out] h*x**3/3 + x**2*(g/2 - 3*h/2) + x*(f - 3*g + 7*h) + (-d + 2*e - 4
*f + 8*g - 16*h)*log(x + (4*d - 6*e + 10*f - 18*g + 34*h)/(2*d -
3*e + 5*f - 9*g + 17*h)) + (d - e + f - g + h)*log(x + 1)
```

GIAC/XCAS [A] time = 0.285038, size = 93, normalized size = 1.41

$$\frac{1}{3}\,hx^3 + \frac{1}{2}\,gx^2 - \frac{3}{2}\,hx^2 + f\,x - 3\,gx + 7\,hx - (d + 4\,f - 8\,g + 16\,h - 2\,e)\ln\left(|x + 2|\right) + (d + f - g + h - e)\ln\left(|x + 1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^4 + g*x^3 + f*x^2 + e*x + d)*(x^2 - 3*x + 2)/(x^4 - 5*x^2 + 4),x, alg
```

```
[Out] 1/3*h*x^3 + 1/2*g*x^2 - 3/2*h*x^2 + f*x - 3*g*x + 7*h*x - (d + 4*
f - 8*g + 16*h - 2*e)*ln(abs(x + 2)) + (d + f - g + h - e)*ln(abs
(x + 1))
```

3.78 $\int \frac{(2-3x+x^2)(d+ex+fx^2+gx^3+hx^4+ix^5)}{4-5x^2+x^4}\,dx$

Optimal. Leaf size=90

$$\log(x+1)(d-e+f-g+h-i)-\log(x+2)(d-2e+4f-8g+16h-32i)+x(f-3g+7h-15i)+\frac{1}{2}x^2(g-3h+7i)+\frac{1}{3}x^3(h-3i)+\frac{ix^4}{4}$$

```
[Out] (f - 3*g + 7*h - 15*i)*x + ((g - 3*h + 7*i)*x^2)/2 + ((h - 3*i)*x
^3)/3 + (i*x^4)/4 + (d - e + f - g + h - i)*Log[1 + x] - (d - 2*e
 + 4*f - 8*g + 16*h - 32*i)*Log[2 + x]
```

Rubi [A] time = 0.213773, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$

$$\log(x+1)(d-e+f-g+h-i)-\log(x+2)(d-2e+4f-8g+16h-32i)+x(f-3g+7h-15i)+\frac{1}{2}x^2(g-3h+7i)+\frac{1}{3}x^3(h-3i)+\frac{ix^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[((2 - 3*x + x^2)*(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5))/(4 - 5*x^2 + x^4),x]
```

```
[Out] (f - 3*g + 7*h - 15*i)*x + ((g - 3*h + 7*i)*x^2)/2 + ((h - 3*i)*x
^3)/3 + (i*x^4)/4 + (d - e + f - g + h - i)*Log[1 + x] - (d - 2*e
 + 4*f - 8*g + 16*h - 32*i)*Log[2 + x]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-3gx+7hx+\frac{ix^4}{4}-15ix+x^3\left(\frac{h}{3}-i\right)+(g-3h+7i)\int x\,dx-(d-2e+4f-8g+16h-32i)\log(x+2)+(d-e+f-g+h-i)\log(x+1)+\int f\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((x**2-3*x+2)*(i*x**5+h*x**4+g*x**3+f*x**2+e*x+d)/(x**4-
5*x**2+4),x)
```

```
[Out] -3*g*x + 7*h*x + i*x**4/4 - 15*i*x + x**3*(h/3 - i) + (g - 3*h +
7*i)*Integral(x, x) - (d - 2*e + 4*f - 8*g + 16*h - 32*i)*log(x +
```

```
 2) + (d - e + f - g + h - i)*log(x + 1) + Integral(f, x)
```

Mathematica [A] time = 0.0757102, size = 91, normalized size = 1.01

$$\log(x+1)(d-e+f-g+h-i)+\log(x+2)(-d+2e-4f+8g-16h+32i)+x(f-3g+7h-15i)+\frac{1}{2}x^2(g-3h+7i)+\frac{1}{3}x^3(h-3i)+\frac{ix^4}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 - 3*x + x^2)*(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5))/(4 - 5*x^2 + x
```

```
[Out] (f - 3*g + 7*h - 15*i)*x + ((g - 3*h + 7*i)*x^2)/2 + ((h - 3*i)*x
^3)/3 + (i*x^4)/4 + (d - e + f - g + h - i)*Log[1 + x] + (-d + 2*
e - 4*f + 8*g - 16*h + 32*i)*Log[2 + x]
```

Maple [A] time = 0.01, size = 134, normalized size = 1.5

$$\frac{ix^4}{4}+\frac{hx^3}{3}-ix^3+\frac{gx^2}{2}-\frac{3\,hx^2}{2}+\frac{7\,ix^2}{2}+fx-3\,gx+7\,hx-15\,ix-\ln(2+x)\,d+2\,\ln(2+x)\,e-4\,\ln(2+x)\,f+8\,\ln(2+x)\,g-16\,\ln(2+x)\,h+32\,\ln(2+x)\,i+\ln(1+x)\,d-\ln(1+x)\,e+\ln(1+x)\,f-\ln(1+x)\,g+\ln(1+x)\,h-\ln(1+x)\,i$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-3*x+2)*(i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4),x)
```

```
[Out] 1/4*i*x^4+1/3*h*x^3-i*x^3+1/2*g*x^2-3/2*h*x^2+7/2*i*x^2+f*x-3*g*x
+7*h*x-15*i*x-ln(2+x)*d+2*ln(2+x)*e-4*ln(2+x)*f+8*ln(2+x)*g-16*ln
(2+x)*h+32*ln(2+x)*i+ln(1+x)*d-ln(1+x)*e+ln(1+x)*f-ln(1+x)*g+ln(1
+x)*h-ln(1+x)*i
```

Maxima [A] time = 0.709519, size = 113, normalized size = 1.26

$$\frac{1}{4}\,ix^4+\frac{1}{3}\,(h-3\,i)x^3+\frac{1}{2}\,(g-3\,h+7\,i)x^2+(f-3\,g+7\,h-15\,i)x-(d-2\,e+4\,f-8\,g+16\,h-32\,i)\log(x+2)+(d-e+f-g+h-i)\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^5 + h*x^4 + g*x^3 + f*x^2 + e*x + d)*(x^2 - 3*x + 2)/(x^4 - 5*x^2 + 4
```

```
[Out] 1/4*i*x^4 + 1/3*(h - 3*i)*x^3 + 1/2*(g - 3*h + 7*i)*x^2 + (f - 3*
g + 7*h - 15*i)*x - (d - 2*e + 4*f - 8*g + 16*h - 32*i)*log(x + 2
) + (d - e + f - g + h - i)*log(x + 1)
```

Fricas [A] time = 0.273541, size = 113, normalized size = 1.26

$$\frac{1}{4}ix^4 + \frac{1}{3}(h - 3i)x^3 + \frac{1}{2}(g - 3h + 7i)x^2 + (f - 3g + 7h - 15i)x - (d - 2e + 4f - 8g + 16h - 32i)\log(x + 2) + (d - e + f - g + h - i)\log(x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.


```
[In] integrate((i*x^5 + h*x^4 + g*x^3 + f*x^2 + e*x + d)*(x^2 - 3*x + 2)/(x^4 - 5*x^2 + 4
```

```
[Out] 1/4*i*x^4 + 1/3*(h - 3*i)*x^3 + 1/2*(g - 3*h + 7*i)*x^2 + (f - 3*
g + 7*h - 15*i)*x - (d - 2*e + 4*f - 8*g + 16*h - 32*i)*log(x + 2
) + (d - e + f - g + h - i)*log(x + 1)
```

Sympy [A] time = 8.60097, size = 122, normalized size = 1.36

$$\frac{ix^4}{4} + x^3\left(\frac{h}{3} - i\right) + x^2\left(\frac{g}{2} - \frac{3h}{2} + \frac{7i}{2}\right) + x(f - 3g + 7h - 15i) + (-d + 2e - 4f + 8g - 16h + 32i)\log\left(x + \frac{4d - 6e + 10f - 18g + 34h - 66i}{2d - 3e + 5f - 9g + 17h - 33i}\right) + (d - e + f - g + h - i)\log(x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-3*x+2)*(i*x**5+h*x**4+g*x**3+f*x**2+e*x+d)/(x**4-
5*x**2+4),x)
```

```
[Out] i*x**4/4 + x**3*(h/3 - i) + x**2*(g/2 - 3*h/2 + 7*i/2) + x*(f - 3
*g + 7*h - 15*i) + (-d + 2*e - 4*f + 8*g - 16*h + 32*i)*log(x + (
4*d - 6*e + 10*f - 18*g + 34*h - 66*i)/(2*d - 3*e + 5*f - 9*g + 1
7*h - 33*i)) + (d - e + f - g + h - i)*log(x + 1)
```

GIAC/XCAS [A] time = 0.286156, size = 131, normalized size = 1.46

$$\frac{1}{4}\,ix^{4} + \frac{1}{3}\,hx^{3} - ix^{3} + \frac{1}{2}\,gx^{2} - \frac{3}{2}\,hx^{2} + \frac{7}{2}\,ix^{2} + fx - 3\,gx + 7\,hx - 15\,ix - (d + 4\,f - 8\,g + 16\,h - 32\,i - 2\,e)\ln\left(|x + 2|\right) + (d + f - g + h - i - e)\ln\left(|x + 1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^5 + h*x^4 + g*x^3 + f*x^2 + e*x + d)*(x^2 - 3*x + 2)/(x^4 - 5*x^2 + 4
```

```
[Out] 1/4*i*x^4 + 1/3*h*x^3 - i*x^3 + 1/2*g*x^2 - 3/2*h*x^2 + 7/2*i*x^2
 + f*x - 3*g*x + 7*h*x - 15*i*x - (d + 4*f - 8*g + 16*h - 32*i -
2*e)*ln(abs(x + 2)) + (d + f - g + h - i - e)*ln(abs(x + 1))
```

3.79 $\int \frac{2+x}{4-5x^2+x^4} \, dx$

Optimal. Leaf size=29

$$-\frac{1}{2}\log(1-x)+\frac{1}{3}\log(2-x)+\frac{1}{6}\log(x+1)$$

```
[Out] -Log[1 - x]/2 + Log[2 - x]/3 + Log[1 + x]/6
```

Rubi [A] time = 0.0385765, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$-\frac{1}{2}\log(1-x)+\frac{1}{3}\log(2-x)+\frac{1}{6}\log(x+1)$$

Antiderivative was successfully verified.

```
[In]  Int[(2 + x)/(4 - 5*x^2 + x^4),x]

[Out] -Log[1 - x]/2 + Log[2 - x]/3 + Log[1 + x]/6
```

Rubi in Sympy [A] time = 26.8304, size = 22, normalized size = 0.76

$$-3\log\left(-x+\frac{8}{3}\right)+2\log\left(-x+\frac{26}{3}\right)+\log\left(x+\frac{28}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((2+x)/(x**4-5*x**2+4),x)

[Out] -3*log(-x + 8/3) + 2*log(-x + 26/3) + log(x + 28/3)
```

Mathematica [A] time = 0.00967629, size = 29, normalized size = 1.

$$-\frac{1}{2}\log(1-x)+\frac{1}{3}\log(2-x)+\frac{1}{6}\log(x+1)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(2 + x)/(4 - 5*x^2 + x^4),x]
```

```
[Out] -Log[1 - x]/2 + Log[2 - x]/3 + Log[1 + x]/6
```

Maple [A] time = 0.01, size = 20, normalized size = 0.7

$$-\frac{\ln(-1+x)}{2}+\frac{\ln(1+x)}{6}+\frac{\ln(x-2)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((2+x)/(x^4-5*x^2+4),x)
```

```
[Out] -1/2*ln(-1+x)+1/6*ln(1+x)+1/3*ln(x-2)
```

Maxima [A] time = 0.700268, size = 26, normalized size = 0.9

$$\frac{1}{6}\log(x+1)-\frac{1}{2}\log(x-1)+\frac{1}{3}\log(x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + 2)/(x^4 - 5*x^2 + 4),x, algorithm="maxima")
```

```
[Out] 1/6*log(x + 1) - 1/2*log(x - 1) + 1/3*log(x - 2)
```

Fricas [A] time = 0.26206, size = 26, normalized size = 0.9

$$\frac{1}{6}\log(x+1)-\frac{1}{2}\log(x-1)+\frac{1}{3}\log(x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + 2)/(x^4 - 5*x^2 + 4),x, algorithm="fricas")
```

```
[Out] 1/6*log(x + 1) - 1/2*log(x - 1) + 1/3*log(x - 2)
```

Sympy [A] time = 0.27699, size = 19, normalized size = 0.66

$$\frac{\log(x-2)}{3}-\frac{\log(x-1)}{2}+\frac{\log(x+1)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2+x)/(x**4-5*x**2+4),x)

[Out] log(x - 2)/3 - log(x - 1)/2 + log(x + 1)/6
```

GIAC/XCAS [A] time = 0.28784, size = 30, normalized size = 1.03

$$\frac{1}{6}\ln\left(|x+1|\right)-\frac{1}{2}\ln\left(|x-1|\right)+\frac{1}{3}\ln\left(|x-2|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + 2)/(x^4 - 5*x^2 + 4),x, algorithm="giac")

[Out] 1/6*ln(abs(x + 1)) - 1/2*ln(abs(x - 1)) + 1/3*ln(abs(x - 2))
```

3.80 $\int \frac{(2+x)(d+ex)}{4-5x^2+x^4}\,dx$

Optimal. Leaf size=42

$$-\frac{1}{2}(d+e)\log(1-x)+\frac{1}{3}(d+2e)\log(2-x)+\frac{1}{6}(d-e)\log(x+1)$$

```
[Out] -((d + e)*Log[1 - x])/2 + ((d + 2*e)*Log[2 - x])/3 + ((d - e)*Log
[1 + x])/6
```

Rubi [A] time = 0.0984844, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$

$$-\frac{1}{2}(d+e)\log(1-x)+\frac{1}{3}(d+2e)\log(2-x)+\frac{1}{6}(d-e)\log(x+1)$$

Antiderivative was successfully verified.

```
[In]  Int[((2 + x)*(d + e*x))/(4 - 5*x^2 + x^4),x]
```

```
[Out] -((d + e)*Log[1 - x])/2 + ((d + 2*e)*Log[2 - x])/3 + ((d - e)*Log
[1 + x])/6
```

Rubi in Sympy [A] time = 23.5605, size = 36, normalized size = 0.86

$$\left(\frac{d}{6}-\frac{e}{6}\right)\log(x+1)+\left(\frac{d}{3}+\frac{2e}{3}\right)\log(-x+2)-\left(\frac{d}{2}+\frac{e}{2}\right)\log(-x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((2+x)*(e*x+d)/(x**4-5*x**2+4),x)
```

```
[Out] (d/6 - e/6)*log(x + 1) + (d/3 + 2*e/3)*log(-x + 2) - (d/2 + e/2)*
log(-x + 1)
```

Mathematica [A] time = 0.0282561, size = 39, normalized size = 0.93

$$\frac{1}{6}(-3(d+e)\log(1-x)+2(d+2e)\log(2-x)+(d-e)\log(x+1))$$

Antiderivative was successfully verified.

```
[In]  Integrate[((2 + x)*(d + e*x))/(4 - 5*x^2 + x^4),x]
```

```
[Out] (-3*(d + e)*Log[1 - x] + 2*(d + 2*e)*Log[2 - x] + (d - e)*Log[1 +
 x])/6
```

Maple [A] time = 0.009, size = 44, normalized size = 1.1

$$-\frac{\ln\left(-1+x\right)d}{2}-\frac{\ln\left(-1+x\right)e}{2}+\frac{\ln\left(1+x\right)d}{6}-\frac{\ln\left(1+x\right)e}{6}+\frac{\ln\left(x-2\right)d}{3}+\frac{2\ \ln\left(x-2\right)e}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((2+x)*(e*x+d)/(x^4-5*x^2+4),x)
```

```
[Out] -1/2*ln(-1+x)*d-1/2*ln(-1+x)*e+1/6*ln(1+x)*d-1/6*ln(1+x)*e+1/3*ln
(x-2)*d+2/3*ln(x-2)*e
```

Maxima [A] time = 0.699116, size = 43, normalized size = 1.02

$$\frac{1}{6}\left(d-e\right)\log\left(x+1\right)-\frac{1}{2}\left(d+e\right)\log\left(x-1\right)+\frac{1}{3}\left(d+2\,e\right)\log\left(x-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x + d)*(x + 2)/(x^4 - 5*x^2 + 4),x, algorithm="maxima")
```

```
[Out] 1/6*(d - e)*log(x + 1) - 1/2*(d + e)*log(x - 1) + 1/3*(d + 2*e)*l
og(x - 2)
```

Fricas [A] time = 0.297686, size = 43, normalized size = 1.02

$$\frac{1}{6}\left(d-e\right)\log\left(x+1\right)-\frac{1}{2}\left(d+e\right)\log\left(x-1\right)+\frac{1}{3}\left(d+2\,e\right)\log\left(x-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x + d)*(x + 2)/(x^4 - 5*x^2 + 4),x, algorithm="fricas")
```

```
[Out] 1/6*(d - e)*log(x + 1) - 1/2*(d + e)*log(x - 1) + 1/3*(d + 2*e)*l
og(x - 2)
```

Sympy [A] time = 3.68231, size = 304, normalized size = 7.24

$$\frac{(d-e)\log\left(x+\frac{26d^3+66d^2e-9d^2(d-e)+78de^2-12de(d-e)-7d(d-e)^2+46e^3+3e^2(d-e)-8e(d-e)^2}{10d^3+69d^2e+102de^2+35e^3}\right)}{6} - \frac{(d+e)\log\left(x+\frac{26d^3+66d^2e+27d^2(d+e)+78de^2+36de(d+e)-63d(d+e)^2+46e^3-9e^2(d+e)-72e(d+e)^2}{10d^3+69d^2e+102de^2+35e^3}\right)}{2} + \frac{(d+2e)\log\left(x+\frac{26d^3+66d^2e-18d^2(d+2e)+78de^2-24de(d+2e)-28d(d+2e)^2+46e^3+6e^2(d+2e)-32e(d+2e)^2}{10d^3+69d^2e+102de^2+35e^3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2+x)*(e*x+d)/(x**4-5*x**2+4),x)
```

```
[Out] (d - e)*log(x + (26*d**3 + 66*d**2*e - 9*d**2*(d - e) + 78*d*e**2
 - 12*d*e*(d - e) - 7*d*(d - e)**2 + 46*e**3 + 3*e**2*(d - e) - 8
*e*(d - e)**2)/(10*d**3 + 69*d**2*e + 102*d*e**2 + 35*e**3))/6 -
(d + e)*log(x + (26*d**3 + 66*d**2*e + 27*d**2*(d + e) + 78*d*e**
2 + 36*d*e*(d + e) - 63*d*(d + e)**2 + 46*e**3 - 9*e**2*(d + e) -
 72*e*(d + e)**2)/(10*d**3 + 69*d**2*e + 102*d*e**2 + 35*e**3))/2
 + (d + 2*e)*log(x + (26*d**3 + 66*d**2*e - 18*d**2*(d + 2*e) + 7
8*d*e**2 - 24*d*e*(d + 2*e) - 28*d*(d + 2*e)**2 + 46*e**3 + 6*e**
2*(d + 2*e) - 32*e*(d + 2*e)**2)/(10*d**3 + 69*d**2*e + 102*d*e**
2 + 35*e**3))/3
```

GIAC/XCAS [A] time = 0.284256, size = 51, normalized size = 1.21

$$\frac{1}{6}(d-e)\ln(|x+1|) - \frac{1}{2}(d+e)\ln(|x-1|) + \frac{1}{3}(d+2e)\ln(|x-2|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x + d)*(x + 2)/(x^4 - 5*x^2 + 4),x, algorithm="giac")
```

```
[Out] 1/6*(d - e)*ln(abs(x + 1)) - 1/2*(d + e)*ln(abs(x - 1)) + 1/3*(d
+ 2*e)*ln(abs(x - 2))
```

3.81 $\int \frac{(2+x)\left(d+ex+fx^2\right)}{4-5x^2+x^4}\,dx$

Optimal. Leaf size=47

$$-\frac{1}{2}\log(1-x)(d+e+f)+\frac{1}{3}\log(2-x)(d+2e+4f)+\frac{1}{6}\log(x+1)(d-e+f)$$

```
[Out] -((d + e + f)*Log[1 - x])/2 + ((d + 2*e + 4*f)*Log[2 - x])/3 + ((
d - e + f)*Log[1 + x])/6
```

Rubi [A] time = 0.124664, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077

$$-\frac{1}{2}\log(1-x)(d+e+f)+\frac{1}{3}\log(2-x)(d+2e+4f)+\frac{1}{6}\log(x+1)(d-e+f)$$

Antiderivative was successfully verified.

```
[In]  Int[((2 + x)*(d + e*x + f*x^2))/(4 - 5*x^2 + x^4),x]
```

```
[Out] -((d + e + f)*Log[1 - x])/2 + ((d + 2*e + 4*f)*Log[2 - x])/3 + ((
d - e + f)*Log[1 + x])/6
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((2+x)*(f*x**2+e*x+d)/(x**4-5*x**2+4),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.0401857, size = 44, normalized size = 0.94

$$\frac{1}{6}(-3\log(1-x)(d+e+f)+2\log(2-x)(d+2e+4f)+\log(x+1)(d-e+f))$$

Antiderivative was successfully verified.

```
[In]  Integrate[((2 + x)*(d + e*x + f*x^2))/(4 - 5*x^2 + x^4),x]
```

```
[Out] (-3*(d + e + f)*Log[1 - x] + 2*(d + 2*e + 4*f)*Log[2 - x] + (d -
e + f)*Log[1 + x])/6
```

Maple [A] time = 0.01, size = 65, normalized size = 1.4

$$-\frac{\ln\left(-1+x\right)d}{2}-\frac{\ln\left(-1+x\right)e}{2}-\frac{\ln\left(-1+x\right)f}{2}+\frac{\ln\left(1+x\right)d}{6}-\frac{\ln\left(1+x\right)e}{6}+\frac{\ln\left(1+x\right)f}{6}+\frac{\ln\left(x-2\right)d}{3}+\frac{2\ln\left(x-2\right)e}{3}+\frac{4\ln\left(x-2\right)f}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((2+x)*(f*x^2+e*x+d)/(x^4-5*x^2+4),x)
```

```
[Out] -1/2*ln(-1+x)*d-1/2*ln(-1+x)*e-1/2*ln(-1+x)*f+1/6*ln(1+x)*d-1/6*l
n(1+x)*e+1/6*ln(1+x)*f+1/3*ln(x-2)*d+2/3*ln(x-2)*e+4/3*ln(x-2)*f
```

Maxima [A] time = 0.700647, size = 50, normalized size = 1.06

$$\frac{1}{6}\left(d-e+f\right)\log\left(x+1\right)-\frac{1}{2}\left(d+e+f\right)\log\left(x-1\right)+\frac{1}{3}\left(d+2e+4f\right)\log\left(x-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2 + e*x + d)*(x + 2)/(x^4 - 5*x^2 + 4),x, algorithm="maxima")
```

```
[Out] 1/6*(d - e + f)*log(x + 1) - 1/2*(d + e + f)*log(x - 1) + 1/3*(d
+ 2*e + 4*f)*log(x - 2)
```

Fricas [A] time = 0.261257, size = 50, normalized size = 1.06

$$\frac{1}{6}\left(d-e+f\right)\log\left(x+1\right)-\frac{1}{2}\left(d+e+f\right)\log\left(x-1\right)+\frac{1}{3}\left(d+2e+4f\right)\log\left(x-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2 + e*x + d)*(x + 2)/(x^4 - 5*x^2 + 4),x, algorithm="fricas")
```

```
[Out] 1/6*(d - e + f)*log(x + 1) - 1/2*(d + e + f)*log(x - 1) + 1/3*(d
+ 2*e + 4*f)*log(x - 2)
```

Sympy [A] time = 15.6017, size = 716, normalized size = 15.23

$$\frac{(d-e+f)\log\left(x+\frac{26d^3+66d^2e+132d^2f-9d^2(d-e+f)+78de^2+276def-12de(d-e+f)+222df^2+6df(d-e+f)-7d(d-e+f)^2+46e^3+204e^2f+3e^2(d-e+f)+28}{10d^3+69d^2e+102d^2f+102de^2+318def+246df^2+35e^3+174e^2f+285ef^2}\right.}{6} - \frac{(d+e+f)\log\left(x+\frac{26d^3+66d^2e+132d^2f+27d^2(d+e+f)+78de^2+276def+36de(d+e+f)+222df^2-18df(d+e+f)-63d(d+e+f)^2+46e^3+204e^2f-9e^2(d+e+f}{10d^3+69d^2e+102d^2f+102de^2+318def+246df^2+35e^3+174e^2f+28}\right.}{2} + \frac{(d+2e+4f)\log\left(x+\frac{26d^3+66d^2e+132d^2f-18d^2(d+2e+4f)+78de^2+276def-24de(d+2e+4f)+222df^2+12df(d+2e+4f)-28d(d+2e+4f)^2+46e^3+204e^2f+}{10d^3+69d^2e+102d^2f+102de^2+318def+246df^2+35e^3+}\right.}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2+x)*(f*x**2+e*x+d)/(x**4-5*x**2+4),x)
```

```
[Out] (d - e + f)*log(x + (26*d**3 + 66*d**2*e + 132*d**2*f - 9*d**2*(d
 - e + f) + 78*d*e**2 + 276*d*e*f - 12*d*e*(d - e + f) + 222*d*f*
*2 + 6*d*f*(d - e + f) - 7*d*(d - e + f)**2 + 46*e**3 + 204*e**2*
f + 3*e**2*(d - e + f) + 282*e*f**2 + 36*e*f*(d - e + f) - 8*e*(d
 - e + f)**2 + 116*f**3 + 51*f**2*(d - e + f) - 13*f*(d - e + f)*
*2)/(10*d**3 + 69*d**2*e + 102*d**2*f + 102*d*e**2 + 318*d*e*f +
246*d*f**2 + 35*e**3 + 174*e**2*f + 285*e*f**2 + 154*f**3))/6 - (
d + e + f)*log(x + (26*d**3 + 66*d**2*e + 132*d**2*f + 27*d**2*(d
 + e + f) + 78*d*e**2 + 276*d*e*f + 36*d*e*(d + e + f) + 222*d*f*
*2 - 18*d*f*(d + e + f) - 63*d*(d + e + f)**2 + 46*e**3 + 204*e**
2*f - 9*e**2*(d + e + f) + 282*e*f**2 - 108*e*f*(d + e + f) - 72*
e*(d + e + f)**2 + 116*f**3 - 153*f**2*(d + e + f) - 117*f*(d + e
 + f)**2)/(10*d**3 + 69*d**2*e + 102*d**2*f + 102*d*e**2 + 318*d*
e*f + 246*d*f**2 + 35*e**3 + 174*e**2*f + 285*e*f**2 + 154*f**3))
/2 + (d + 2*e + 4*f)*log(x + (26*d**3 + 66*d**2*e + 132*d**2*f -
18*d**2*(d + 2*e + 4*f) + 78*d*e**2 + 276*d*e*f - 24*d*e*(d + 2*e
 + 4*f) + 222*d*f**2 + 12*d*f*(d + 2*e + 4*f) - 28*d*(d + 2*e + 4
*f)**2 + 46*e**3 + 204*e**2*f + 6*e**2*(d + 2*e + 4*f) + 282*e*f*
*2 + 72*e*f*(d + 2*e + 4*f) - 32*e*(d + 2*e + 4*f)**2 + 116*f**3
+ 102*f**2*(d + 2*e + 4*f) - 52*f*(d + 2*e + 4*f)**2)/(10*d**3 +
69*d**2*e + 102*d**2*f + 102*d*e**2 + 318*d*e*f + 246*d*f**2 + 35
*e**3 + 174*e**2*f + 285*e*f**2 + 154*f**3))/3
```

GIAC/XCAS [A] time = 0.285921, size = 58, normalized size = 1.23

$$\frac{1}{6}(d+f-e)\ln(|x+1|) - \frac{1}{2}(d+f+e)\ln(|x-1|) + \frac{1}{3}(d+4f+2e)\ln(|x-2|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2 + e*x + d)*(x + 2)/(x^4 - 5*x^2 + 4),x, algorithm="giac")
```

```
[Out] 1/6*(d + f - e)*ln(abs(x + 1)) - 1/2*(d + f + e)*ln(abs(x - 1)) +
 1/3*(d + 4*f + 2*e)*ln(abs(x - 2))
```

3.82 $\int \frac{(2+x)(d+ex+fx^2+gx^3)}{4-5x^2+x^4}\,dx$

Optimal. Leaf size=57

$$-\frac{1}{2}\log(1-x)(d+e+f+g)+\frac{1}{3}\log(2-x)(d+2e+4f+8g)+\frac{1}{6}\log(x+1)(d-e+f-g)+gx$$

```
[Out] g*x - ((d + e + f + g)*Log[1 - x])/2 + ((d + 2*e + 4*f + 8*g)*Log
[2 - x])/3 + ((d - e + f - g)*Log[1 + x])/6
```

Rubi [A] time = 0.147466, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$

$$-\frac{1}{2}\log(1-x)(d+e+f+g)+\frac{1}{3}\log(2-x)(d+2e+4f+8g)+\frac{1}{6}\log(x+1)(d-e+f-g)+gx$$

Antiderivative was successfully verified.

```
[In]  Int[((2 + x)*(d + e*x + f*x^2 + g*x^3))/(4 - 5*x^2 + x^4),x]
```

```
[Out] g*x - ((d + e + f + g)*Log[1 - x])/2 + ((d + 2*e + 4*f + 8*g)*Log
[2 - x])/3 + ((d - e + f - g)*Log[1 + x])/6
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((2+x)*(g*x**3+f*x**2+e*x+d)/(x**4-5*x**2+4),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.0519937, size = 55, normalized size = 0.96

$$\frac{1}{6}(-3\log(1-x)(d+e+f+g)+2\log(2-x)(d+2e+4f+8g)+\log(x+1)(d-e+f-g)+6gx)$$

Antiderivative was successfully verified.

```
[In]  Integrate[((2 + x)*(d + e*x + f*x^2 + g*x^3))/(4 - 5*x^2 + x^4),x]
```

```
[Out] (6*g*x - 3*(d + e + f + g)*Log[1 - x] + 2*(d + 2*e + 4*f + 8*g)*L
og[2 - x] + (d - e + f - g)*Log[1 + x])/6
```

Maple [A] time = 0.011, size = 89, normalized size = 1.6

$$gx - \frac{\ln(-1+x)\,d}{2} - \frac{\ln(-1+x)\,e}{2} - \frac{\ln(-1+x)\,f}{2} - \frac{\ln(-1+x)\,g}{2} + \frac{\ln(1+x)\,d}{6} - \frac{\ln(1+x)\,e}{6} + \frac{\ln(1+x)\,f}{6} - \frac{\ln(1+x)\,g}{6} + \frac{\ln(x-2)\,d}{3} + \frac{2\ln(x-2)\,e}{3} + \frac{4\ln(x-2)\,f}{3} + \frac{8\ln(x-2)\,g}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((2+x)*(g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4),x)
```

```
[Out] g*x-1/2*ln(-1+x)*d-1/2*ln(-1+x)*e-1/2*ln(-1+x)*f-1/2*ln(-1+x)*g+1
/6*ln(1+x)*d-1/6*ln(1+x)*e+1/6*ln(1+x)*f-1/6*ln(1+x)*g+1/3*ln(x-2
)*d+2/3*ln(x-2)*e+4/3*ln(x-2)*f+8/3*ln(x-2)*g
```

Maxima [A] time = 0.700485, size = 63, normalized size = 1.11

$$gx + \frac{1}{6}(d - e + f - g)\log(x+1) - \frac{1}{2}(d + e + f + g)\log(x-1) + \frac{1}{3}(d + 2e + 4f + 8g)\log(x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((g*x^3 + f*x^2 + e*x + d)*(x + 2)/(x^4 - 5*x^2 + 4),x, algorithm="maxima
```

```
[Out] g*x + 1/6*(d - e + f - g)*log(x + 1) - 1/2*(d + e + f + g)*log(x
- 1) + 1/3*(d + 2*e + 4*f + 8*g)*log(x - 2)
```

Fricas [A] time = 0.287316, size = 63, normalized size = 1.11

$$gx + \frac{1}{6}(d - e + f - g)\log(x+1) - \frac{1}{2}(d + e + f + g)\log(x-1) + \frac{1}{3}(d + 2e + 4f + 8g)\log(x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((g*x^3 + f*x^2 + e*x + d)*(x + 2)/(x^4 - 5*x^2 + 4),x, algorithm="fricas
```

```
[Out] g*x + 1/6*(d - e + f - g)*log(x + 1) - 1/2*(d + e + f + g)*log(x
- 1) + 1/3*(d + 2*e + 4*f + 8*g)*log(x - 2)
```

Sympy [A] time = 71.2712, size = 1389, normalized size = 24.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2+x)*(g*x**3+f*x**2+e*x+d)/(x**4-5*x**2+4),x)
```

```
[Out] g*x + (d - e + f - g)*log(x + (26*d**3 + 66*d**2*e + 132*d**2*f +
 174*d**2*g - 9*d**2*(d - e + f - g) + 78*d*e**2 + 276*d*e*f + 44
4*d*e*g - 12*d*e*(d - e + f - g) + 222*d*f**2 + 636*d*f*g + 6*d*f
*(d - e + f - g) + 510*d*g**2 + 36*d*g*(d - e + f - g) - 7*d*(d -
 e + f - g)**2 + 46*e**3 + 204*e**2*f + 390*e**2*g + 3*e**2*(d -
e + f - g) + 282*e*f**2 + 984*e*f*g + 36*e*f*(d - e + f - g) + 93
0*e*g**2 + 102*e*g*(d - e + f - g) - 8*e*(d - e + f - g)**2 + 116
*f**3 + 534*f**2*g + 51*f**2*(d - e + f - g) + 924*f*g**2 + 228*f
*g*(d - e + f - g) - 13*f*(d - e + f - g)**2 + 586*g**3 + 243*g**
2*(d - e + f - g) - 20*g*(d - e + f - g)**2)/(10*d**3 + 69*d**2*e
 + 102*d**2*f + 213*d**2*g + 102*d*e**2 + 318*d*e*f + 564*d*e*g +
 246*d*f**2 + 894*d*f*g + 750*d*g**2 + 35*e**3 + 174*e**2*f + 249
*e**2*g + 285*e*f**2 + 852*e*f*g + 537*e*g**2 + 154*f**3 + 717*f*
*2*g + 966*f*g**2 + 323*g**3))/6 - (d + e + f + g)*log(x + (26*d*
*3 + 66*d**2*e + 132*d**2*f + 174*d**2*g + 27*d**2*(d + e + f + g
) + 78*d*e**2 + 276*d*e*f + 444*d*e*g + 36*d*e*(d + e + f + g) +
222*d*f**2 + 636*d*f*g - 18*d*f*(d + e + f + g) + 510*d*g**2 - 10
8*d*g*(d + e + f + g) - 63*d*(d + e + f + g)**2 + 46*e**3 + 204*e
**2*f + 390*e**2*g - 9*e**2*(d + e + f + g) + 282*e*f**2 + 984*e*
f*g - 108*e*f*(d + e + f + g) + 930*e*g**2 - 306*e*g*(d + e + f +
 g) - 72*e*(d + e + f + g)**2 + 116*f**3 + 534*f**2*g - 153*f**2*
(d + e + f + g) + 924*f*g**2 - 684*f*g*(d + e + f + g) - 117*f*(d
 + e + f + g)**2 + 586*g**3 - 729*g**2*(d + e + f + g) - 180*g*(d
 + e + f + g)**2)/(10*d**3 + 69*d**2*e + 102*d**2*f + 213*d**2*g
+ 102*d*e**2 + 318*d*e*f + 564*d*e*g + 246*d*f**2 + 894*d*f*g + 7
50*d*g**2 + 35*e**3 + 174*e**2*f + 249*e**2*g + 285*e*f**2 + 852*
e*f*g + 537*e*g**2 + 154*f**3 + 717*f**2*g + 966*f*g**2 + 323*g**
3))/2 + (d + 2*e + 4*f + 8*g)*log(x + (26*d**3 + 66*d**2*e + 132*
d**2*f + 174*d**2*g - 18*d**2*(d + 2*e + 4*f + 8*g) + 78*d*e**2 +
 276*d*e*f + 444*d*e*g - 24*d*e*(d + 2*e + 4*f + 8*g) + 222*d*f**
2 + 636*d*f*g + 12*d*f*(d + 2*e + 4*f + 8*g) + 510*d*g**2 + 72*d*
g*(d + 2*e + 4*f + 8*g) - 28*d*(d + 2*e + 4*f + 8*g)**2 + 46*e**3
 + 204*e**2*f + 390*e**2*g + 6*e**2*(d + 2*e + 4*f + 8*g) + 282*e
*f**2 + 984*e*f*g + 72*e*f*(d + 2*e + 4*f + 8*g) + 930*e*g**2 + 2
04*e*g*(d + 2*e + 4*f + 8*g) - 32*e*(d + 2*e + 4*f + 8*g)**2 + 11
6*f**3 + 534*f**2*g + 102*f**2*(d + 2*e + 4*f + 8*g) + 924*f*g**2
 + 456*f*g*(d + 2*e + 4*f + 8*g) - 52*f*(d + 2*e + 4*f + 8*g)**2
+ 586*g**3 + 486*g**2*(d + 2*e + 4*f + 8*g) - 80*g*(d + 2*e + 4*f
```

```
 + 8*g)**2)/(10*d**3 + 69*d**2*e + 102*d**2*f + 213*d**2*g + 102*
d*e**2 + 318*d*e*f + 564*d*e*g + 246*d*f**2 + 894*d*f*g + 750*d*g
**2 + 35*e**3 + 174*e**2*f + 249*e**2*g + 285*e*f**2 + 852*e*f*g
+ 537*e*g**2 + 154*f**3 + 717*f**2*g + 966*f*g**2 + 323*g**3))/3
```

GIAC/XCAS [A] time = 0.295258, size = 72, normalized size = 1.26

$$gx + \frac{1}{6}(d + f - g - e)\ln(|x + 1|) - \frac{1}{2}(d + f + g + e)\ln(|x - 1|) + \frac{1}{3}(d + 4f + 8g + 2e)\ln(|x - 2|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3 + f*x^2 + e*x + d)*(x + 2)/(x^4 - 5*x^2 + 4),x, algorithm="giac")
```

```
[Out] g*x + 1/6*(d + f - g - e)*ln(abs(x + 1)) - 1/2*(d + f + g + e)*ln
(abs(x - 1)) + 1/3*(d + 4*f + 8*g + 2*e)*ln(abs(x - 2))
```

3.83 $\int \frac{(2+x)\left(d+ex+fx^2+gx^3+hx^4\right)}{4-5x^2+x^4}\,dx$

Optimal. Leaf size=74

$$-\frac{1}{2}\log(1-x)(d+e+f+g+h)+\frac{1}{3}\log(2-x)(d+2e+4f+8g+16h)+\frac{1}{6}\log(x+1)(d-e+f-g+h)+x(g+2h)+\frac{hx^2}{2}$$

```
[Out] (g + 2*h)*x + (h*x^2)/2 - ((d + e + f + g + h)*Log[1 - x])/2 + ((
d + 2*e + 4*f + 8*g + 16*h)*Log[2 - x])/3 + ((d - e + f - g + h)*
Log[1 + x])/6
```

Rubi [A] time = 0.237427, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056

$$-\frac{1}{2}\log(1-x)(d+e+f+g+h)+\frac{1}{3}\log(2-x)(d+2e+4f+8g+16h)+\frac{1}{6}\log(x+1)(d-e+f-g+h)+x(g+2h)+\frac{hx^2}{2}$$

Antiderivative was successfully verified.

```
[In]  Int[((2 + x)*(d + e*x + f*x^2 + g*x^3 + h*x^4))/(4 - 5*x^2 + x^4),x]
```

```
[Out] (g + 2*h)*x + (h*x^2)/2 - ((d + e + f + g + h)*Log[1 - x])/2 + ((
d + 2*e + 4*f + 8*g + 16*h)*Log[2 - x])/3 + ((d - e + f - g + h)*
Log[1 + x])/6
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((2+x)*(h*x**4+g*x**3+f*x**2+e*x+d)/(x**4-5*x**2+4),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.0752171, size = 71, normalized size = 0.96

$$\frac{1}{6}\left(-3\log(1-x)(d+e+f+g+h)+2\log(2-x)(d+2(e+2f+4g+8h))+\log(x+1)(d-e+f-g+h)+6x(g+2h)+3hx^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + x)*(d + e*x + f*x^2 + g*x^3 + h*x^4))/(4 - 5*x^2 + x^4),x]
```

```
[Out] (6*(g + 2*h)*x + 3*h*x^2 - 3*(d + e + f + g + h)*Log[1 - x] + 2*(
d + 2*(e + 2*f + 4*g + 8*h))*Log[2 - x] + (d - e + f - g + h)*Log
[1 + x])/6
```

Maple [A] time = 0.011, size = 120, normalized size = 1.6

$$\frac{hx^2}{2}+gx+2hx-\frac{\ln(-1+x)d}{2}-\frac{\ln(-1+x)e}{2}-\frac{\ln(-1+x)f}{2}-\frac{\ln(-1+x)g}{2}-\frac{\ln(-1+x)h}{2}+\frac{\ln(1+x)d}{6}-\frac{\ln(1+x)e}{6}+\frac{\ln(1+x)f}{6}-\frac{\ln(1+x)g}{6}+\frac{\ln(1+x)h}{6}+\frac{\ln(x-2)d}{3}+\frac{2\ln(x-2)e}{3}+\frac{4\ln(x-2)f}{3}+\frac{8\ln(x-2)g}{3}+\frac{16\ln(x-2)h}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+x)*(h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4),x)
```

```
[Out] 1/2*h*x^2+g*x+2*h*x-1/2*ln(-1+x)*d-1/2*ln(-1+x)*e-1/2*ln(-1+x)*f-
1/2*ln(-1+x)*g-1/2*ln(-1+x)*h+1/6*ln(1+x)*d-1/6*ln(1+x)*e+1/6*ln(
1+x)*f-1/6*ln(1+x)*g+1/6*ln(1+x)*h+1/3*ln(x-2)*d+2/3*ln(x-2)*e+4/
3*ln(x-2)*f+8/3*ln(x-2)*g+16/3*ln(x-2)*h
```

Maxima [A] time = 0.702753, size = 84, normalized size = 1.14

$$\frac{1}{2}hx^2+(g+2h)x+\frac{1}{6}(d-e+f-g+h)\log(x+1)-\frac{1}{2}(d+e+f+g+h)\log(x-1)+\frac{1}{3}(d+2e+4f+8g+16h)\log(x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^4 + g*x^3 + f*x^2 + e*x + d)*(x + 2)/(x^4 - 5*x^2 + 4),x, algorithm=
```

```
[Out] 1/2*h*x^2 + (g + 2*h)*x + 1/6*(d - e + f - g + h)*log(x + 1) - 1/
2*(d + e + f + g + h)*log(x - 1) + 1/3*(d + 2*e + 4*f + 8*g + 16*
h)*log(x - 2)
```

Fricas [A] time = 0.286027, size = 84, normalized size = 1.14

$$\frac{1}{2}hx^2 + (g + 2h)x + \frac{1}{6}(d - e + f - g + h)\log(x + 1) - \frac{1}{2}(d + e + f + g + h)\log(x - 1) + \frac{1}{3}(d + 2e + 4f + 8g + 16h)\log(x - 2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^4 + g*x^3 + f*x^2 + e*x + d)*(x + 2)/(x^4 - 5*x^2 + 4),x, algorithm=
```

```
[Out] 1/2*h*x^2 + (g + 2*h)*x + 1/6*(d - e + f - g + h)*log(x + 1) - 1/
2*(d + e + f + g + h)*log(x - 1) + 1/3*(d + 2*e + 4*f + 8*g + 16*
h)*log(x - 2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)*(h*x**4+g*x**3+f*x**2+e*x+d)/(x**4-5*x**2+4),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.283007, size = 92, normalized size = 1.24

$$\frac{1}{2}hx^2 + gx + 2hx + \frac{1}{6}(d + f - g + h - e)\ln(|x + 1|) - \frac{1}{2}(d + f + g + h + e)\ln(|x - 1|) + \frac{1}{3}(d + 4f + 8g + 16h + 2e)\ln(|x - 2|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^4 + g*x^3 + f*x^2 + e*x + d)*(x + 2)/(x^4 - 5*x^2 + 4),x, algorithm=
```

```
[Out] 1/2*h*x^2 + g*x + 2*h*x + 1/6*(d + f - g + h - e)*ln(abs(x + 1))
- 1/2*(d + f + g + h + e)*ln(abs(x - 1)) + 1/3*(d + 4*f + 8*g + 1
6*h + 2*e)*ln(abs(x - 2))
```

3.84 $\int \frac{(2+x)\left(d+ex+fx^2+gx^3+hx^4+ix^5\right)}{4-5x^2+x^4}\,dx$

Optimal. Leaf size=96

$$-\frac{1}{2}\log(1-x)(d+e+f+g+h+i)+\frac{1}{3}\log(2-x)(d+2e+4f+8g+16h+32i)+\frac{1}{6}\log(x+1)(d-e+f-g+h-i)+x(g+2h+5i)+\frac{1}{2}x^2(h+2i)+\frac{ix^3}{3}$$

```
[Out] (g + 2*h + 5*i)*x + ((h + 2*i)*x^2)/2 + (i*x^3)/3 - ((d + e + f +
 g + h + i)*Log[1 - x])/2 + ((d + 2*e + 4*f + 8*g + 16*h + 32*i)*
Log[2 - x])/3 + ((d - e + f - g + h - i)*Log[1 + x])/6
```

Rubi [A] time = 0.274685, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.049

$$-\frac{1}{2}\log(1-x)(d+e+f+g+h+i)+\frac{1}{3}\log(2-x)(d+2e+4f+8g+16h+32i)+\frac{1}{6}\log(x+1)(d-e+f-g+h-i)+x(g+2h+5i)+\frac{1}{2}x^2(h+2i)+\frac{ix^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[((2 + x)*(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5))/(4 - 5*x^2 + x^4),x]
```

```
[Out] (g + 2*h + 5*i)*x + ((h + 2*i)*x^2)/2 + (i*x^3)/3 - ((d + e + f +
 g + h + i)*Log[1 - x])/2 + ((d + 2*e + 4*f + 8*g + 16*h + 32*i)*
Log[2 - x])/3 + ((d - e + f - g + h - i)*Log[1 + x])/6
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((2+x)*(i*x**5+h*x**4+g*x**3+f*x**2+e*x+d)/(x**4-5*x**2+4),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.103942, size = 91, normalized size = 0.95

$$\frac{1}{6}\left(-3\log(1-x)(d+e+f+g+h+i)+2\log(2-x)(d+2e+4(f+2g+4h+8i))+\log(x+1)(d-e+f-g+h-i)+6x(g+2h+5i)+3x^2(h+2i)+2ix^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + x)*(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5))/(4 - 5*x^2 + x^4),x]
```

```
[Out] (6*(g + 2*h + 5*i)*x + 3*(h + 2*i)*x^2 + 2*i*x^3 - 3*(d + e + f +
 g + h + i)*Log[1 - x] + 2*(d + 2*e + 4*(f + 2*g + 4*h + 8*i))*Lo
g[2 - x] + (d - e + f - g + h - i)*Log[1 + x])/6
```

Maple [A] time = 0.011, size = 156, normalized size = 1.6

$$\frac{ix^3}{3}+\frac{hx^2}{2}+ix^2+gx+2hx+5ix-\frac{\ln(-1+x)d}{2}-\frac{\ln(-1+x)e}{2}-\frac{\ln(-1+x)f}{2}-\frac{\ln(-1+x)g}{2}-\frac{\ln(-1+x)h}{2}-\frac{\ln(-1+x)i}{2}+\frac{\ln(1+x)d}{6}-\frac{\ln(1+x)e}{6}+\frac{\ln(1+x)f}{6}-\frac{\ln(1+x)g}{6}+\frac{\ln(1+x)h}{6}-\frac{\ln(1+x)i}{6}+\frac{\ln(x-2)d}{3}+\frac{2\ln(x-2)e}{3}+\frac{4\ln(x-2)f}{3}+\frac{8\ln(x-2)g}{3}+\frac{16\ln(x-2)h}{3}+\frac{32\ln(x-2)i}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+x)*(i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4),x)
```

```
[Out] 1/3*i*x^3+1/2*h*x^2+i*x^2+g*x+2*h*x+5*i*x-1/2*ln(-1+x)*d-1/2*ln(-
1+x)*e-1/2*ln(-1+x)*f-1/2*ln(-1+x)*g-1/2*ln(-1+x)*h-1/2*ln(-1+x)*
i+1/6*ln(1+x)*d-1/6*ln(1+x)*e+1/6*ln(1+x)*f-1/6*ln(1+x)*g+1/6*ln(
1+x)*h-1/6*ln(1+x)*i+1/3*ln(x-2)*d+2/3*ln(x-2)*e+4/3*ln(x-2)*f+8/
3*ln(x-2)*g+16/3*ln(x-2)*h+32/3*ln(x-2)*i
```

Maxima [A] time = 0.704455, size = 111, normalized size = 1.16

$$\frac{1}{3}ix^3+\frac{1}{2}(h+2i)x^2+(g+2h+5i)x+\frac{1}{6}(d-e+f-g+h-i)\log(x+1)-\frac{1}{2}(d+e+f+g+h+i)\log(x-1)+\frac{1}{3}(d+2e+4f+8g+16h+32i)\log(x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^5 + h*x^4 + g*x^3 + f*x^2 + e*x + d)*(x + 2)/(x^4 - 5*x^2 + 4),x, alg
```

```
[Out] 1/3*i*x^3 + 1/2*(h + 2*i)*x^2 + (g + 2*h + 5*i)*x + 1/6*(d - e +
f - g + h - i)*log(x + 1) - 1/2*(d + e + f + g + h + i)*log(x - 1
) + 1/3*(d + 2*e + 4*f + 8*g + 16*h + 32*i)*log(x - 2)
```

Fricas [A] time = 0.32984, size = 111, normalized size = 1.16

$$\frac{1}{3}\,ix^3+\frac{1}{2}\,(h+2\,i)x^2+(g+2\,h+5\,i)x+\frac{1}{6}\,(d-e+f-g+h-i)\log{(x+1)}\\-\frac{1}{2}\,(d+e+f+g+h+i)\log{(x-1)}+\frac{1}{3}\,(d+2\,e+4\,f+8\,g+16\,h+32\,i)\log{(x-2)}$$

Verification of antiderivative is not currently implemented for this CAS.


```
[In] integrate((i*x^5 + h*x^4 + g*x^3 + f*x^2 + e*x + d)*(x + 2)/(x^4 - 5*x^2 + 4),x, alg
```

```
[Out] 1/3*i*x^3 + 1/2*(h + 2*i)*x^2 + (g + 2*h + 5*i)*x + 1/6*(d - e +
f - g + h - i)*log(x + 1) - 1/2*(d + e + f + g + h + i)*log(x - 1
) + 1/3*(d + 2*e + 4*f + 8*g + 16*h + 32*i)*log(x - 2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)*(i*x**5+h*x**4+g*x**3+f*x**2+e*x+d)/(x**4-5*x**2+4),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.283505, size = 122, normalized size = 1.27

$$\frac{1}{3}\,ix^3+\frac{1}{2}\,hx^2+ix^2+gx+2\,hx+5\,ix+\frac{1}{6}\,(d+f-g+h-i-e)\ln{(|x+1|)}\\-\frac{1}{2}\,(d+f+g+h+i+e)\ln{(|x-1|)}+\frac{1}{3}\,(d+4\,f+8\,g+16\,h+32\,i+2\,e)\ln{(|x-2|)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^5 + h*x^4 + g*x^3 + f*x^2 + e*x + d)*(x + 2)/(x^4 - 5*x^2 + 4),x, alg

[Out] 1/3*i*x^3 + 1/2*h*x^2 + i*x^2 + g*x + 2*h*x + 5*i*x + 1/6*(d + f
- g + h - i - e)*ln(abs(x + 1)) - 1/2*(d + f + g + h + i + e)*ln(
abs(x - 1)) + 1/3*(d + 4*f + 8*g + 16*h + 32*i + 2*e)*ln(abs(x -
2))
```

3.85 $\int \frac{2-x-2x^2+x^3}{(4-5x^2+x^4)^2}\,dx$

Optimal. Leaf size=46

$$\frac{1}{12(x+2)}-\frac{1}{18}\log(1-x)+\frac{1}{48}\log(2-x)+\frac{1}{6}\log(x+1)-\frac{19}{144}\log(x+2)$$

```
[Out] 1/(12*(2 + x)) - Log[1 - x]/18 + Log[2 - x]/48 + Log[1 + x]/6 - (
19*Log[2 + x])/144
```

Rubi [A] time = 0.0889306, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077

$$\frac{1}{12(x+2)}-\frac{1}{18}\log(1-x)+\frac{1}{48}\log(2-x)+\frac{1}{6}\log(x+1)-\frac{19}{144}\log(x+2)$$

Antiderivative was successfully verified.

```
[In]  Int[(2 - x - 2*x^2 + x^3)/(4 - 5*x^2 + x^4)^2,x]
```

```
[Out] 1/(12*(2 + x)) - Log[1 - x]/18 + Log[2 - x]/48 + Log[1 + x]/6 - (
19*Log[2 + x])/144
```

Rubi in Sympy [A] time = 23.0545, size = 34, normalized size = 0.74

$$-\frac{\log(-x+1)}{18}+\frac{\log(-x+2)}{48}+\frac{\log(x+1)}{6}-\frac{19\log(x+2)}{144}+\frac{1}{12(x+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**3-2*x**2-x+2)/(x**4-5*x**2+4)**2,x)
```

```
[Out] -log(-x + 1)/18 + log(-x + 2)/48 + log(x + 1)/6 - 19*log(x + 2)/1
44 + 1/(12*(x + 2))
```

Mathematica [A] time = 0.0366656, size = 42, normalized size = 0.91

$$\frac{1}{144}\left(\frac{12}{x+2}+24\log(-x-1)-8\log(1-x)+3\log(2-x)-19\log(x+2)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(2 - x - 2*x^2 + x^3)/(4 - 5*x^2 + x^4)^2,x]
```

```
[Out] (12/(2 + x) + 24*Log[-1 - x] - 8*Log[1 - x] + 3*Log[2 - x] - 19*L
og[2 + x])/144
```

Maple [A] time = 0.015, size = 33, normalized size = 0.7

$$\frac{1}{24+12\,x}-\frac{19\,\ln\left(2+x\right)}{144}-\frac{\ln\left(-1+x\right)}{18}+\frac{\ln\left(1+x\right)}{6}+\frac{\ln\left(x-2\right)}{48}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^3-2*x^2-x+2)/(x^4-5*x^2+4)^2,x)
```

```
[Out] 1/12/(2+x)-19/144*ln(2+x)-1/18*ln(-1+x)+1/6*ln(1+x)+1/48*ln(x-2)
```

Maxima [A] time = 0.700617, size = 43, normalized size = 0.93

$$\frac{1}{12\,(x+2)}-\frac{19}{144}\,\log\left(x+2\right)+\frac{1}{6}\,\log\left(x+1\right)-\frac{1}{18}\,\log\left(x-1\right)+\frac{1}{48}\,\log\left(x-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 - 2*x^2 - x + 2)/(x^4 - 5*x^2 + 4)^2,x, algorithm="maxima")
```

```
[Out] 1/12/(x + 2) - 19/144*log(x + 2) + 1/6*log(x + 1) - 1/18*log(x -
1) + 1/48*log(x - 2)
```

Fricas [A] time = 0.25596, size = 61, normalized size = 1.33

$$-\frac{19\,(x+2)\log\left(x+2\right)-24\,(x+2)\log\left(x+1\right)+8\,(x+2)\log\left(x-1\right)-3\,(x+2)\log\left(x-2\right)-12}{144\,(x+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 - 2*x^2 - x + 2)/(x^4 - 5*x^2 + 4)^2,x, algorithm="fricas")
```

```
[Out] -1/144*(19*(x + 2)*log(x + 2) - 24*(x + 2)*log(x + 1) + 8*(x + 2)
*log(x - 1) - 3*(x + 2)*log(x - 2) - 12)/(x + 2)
```

Sympy [A] time = 0.688523, size = 34, normalized size = 0.74

$$\frac{\log(x-2)}{48} - \frac{\log(x-1)}{18} + \frac{\log(x+1)}{6} - \frac{19\log(x+2)}{144} + \frac{1}{12x+24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**3-2*x**2-x+2)/(x**4-5*x**2+4)**2,x)
```

```
[Out] log(x - 2)/48 - log(x - 1)/18 + log(x + 1)/6 - 19*log(x + 2)/144
+ 1/(12*x + 24)
```

GIAC/XCAS [A] time = 0.286789, size = 49, normalized size = 1.07

$$\frac{1}{12(x+2)} - \frac{19}{144}\ln(|x+2|) + \frac{1}{6}\ln(|x+1|) - \frac{1}{18}\ln(|x-1|) + \frac{1}{48}\ln(|x-2|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 - 2*x^2 - x + 2)/(x^4 - 5*x^2 + 4)^2,x, algorithm="giac")
```

```
[Out] 1/12/(x + 2) - 19/144*ln(abs(x + 2)) + 1/6*ln(abs(x + 1)) - 1/18*
ln(abs(x - 1)) + 1/48*ln(abs(x - 2))
```

3.86 $\int \frac{(d+ex)(2-x-2x^2+x^3)}{(4-5x^2+x^4)^2}\,dx$

Optimal. Leaf size=71

$$\frac{d-2e}{12(x+2)}-\frac{1}{18}(d+e)\log(1-x)+\frac{1}{48}(d+2e)\log(2-x)+\frac{1}{6}(d-e)\log(x+1)-\frac{1}{144}(19d-26e)\log(x+2)$$

```
[Out] (d - 2*e)/(12*(2 + x)) - ((d + e)*Log[1 - x])/18 + ((d + 2*e)*Log
[2 - x])/48 + ((d - e)*Log[1 + x])/6 - ((19*d - 26*e)*Log[2 + x])
/144
```

Rubi [A] time = 0.328723, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$

$$\frac{d-2e}{12(x+2)}-\frac{1}{18}(d+e)\log(1-x)+\frac{1}{48}(d+2e)\log(2-x)+\frac{1}{6}(d-e)\log(x+1)-\frac{1}{144}(19d-26e)\log(x+2)$$

Antiderivative was successfully verified.

```
[In]  Int[((d + e*x)*(2 - x - 2*x^2 + x^3))/(4 - 5*x^2 + x^4)^2,x]
```

```
[Out] (d - 2*e)/(12*(2 + x)) - ((d + e)*Log[1 - x])/18 + ((d + 2*e)*Log
[2 - x])/48 + ((d - e)*Log[1 + x])/6 - ((19*d - 26*e)*Log[2 + x])
/144
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x+d)*(x**3-2*x**2-x+2)/(x**4-5*x**2+4)**2,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.0705662, size = 66, normalized size = 0.93

$$\frac{1}{144}\left(\frac{12(d-2e)}{x+2}+24(d-e)\log(-x-1)-8(d+e)\log(1-x)+3(d+2e)\log(2-x)+(26e-19d)\log(x+2)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[((d + e*x)*(2 - x - 2*x^2 + x^3))/(4 - 5*x^2 + x^4)^2,x]
```

```
[Out] ((12*(d - 2*e))/(2 + x) + 24*(d - e)*Log[-1 - x] - 8*(d + e)*Log[
1 - x] + 3*(d + 2*e)*Log[2 - x] + (-19*d + 26*e)*Log[2 + x])/144
```

Maple [A] time = 0.016, size = 74, normalized size = 1.

$$-\frac{19\ln(2+x)d}{144}+\frac{13\ln(2+x)e}{72}+\frac{d}{24+12x}-\frac{e}{12+6x}-\frac{\ln(-1+x)d}{18}-\frac{\ln(-1+x)e}{18}+\frac{\ln(1+x)d}{6}-\frac{\ln(1+x)e}{6}+\frac{\ln(x-2)d}{48}+\frac{\ln(x-2)e}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x+d)*(x^3-2*x^2-x+2)/(x^4-5*x^2+4)^2,x)
```

```
[Out] -19/144*ln(2+x)*d+13/72*ln(2+x)*e+1/12/(2+x)*d-1/6/(2+x)*e-1/18*l
n(-1+x)*d-1/18*ln(-1+x)*e+1/6*ln(1+x)*d-1/6*ln(1+x)*e+1/48*ln(x-2
)*d+1/24*ln(x-2)*e
```

Maxima [A] time = 0.70171, size = 77, normalized size = 1.08

$$-\frac{1}{144}(19d-26e)\log(x+2)+\frac{1}{6}(d-e)\log(x+1)-\frac{1}{18}(d+e)\log(x-1)+\frac{1}{48}(d+2e)\log(x-2)+\frac{d-2e}{12(x+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3 - 2*x^2 - x + 2)*(e*x + d)/(x^4 - 5*x^2 + 4)^2,x, algorithm="maxima
```

```
[Out] -1/144*(19*d - 26*e)*log(x + 2) + 1/6*(d - e)*log(x + 1) - 1/18*(
d + e)*log(x - 1) + 1/48*(d + 2*e)*log(x - 2) + 1/12*(d - 2*e)/(x
 + 2)
```

Fricas [A] time = 0.270795, size = 126, normalized size = 1.77

$$-\frac{((19d-26e)x+38d-52e)\log(x+2)-24((d-e)x+2d-2e)\log(x+1)+8((d+e)x+2d+2e)\log(x-1)-3((d+}{144(x+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 - 2*x^2 - x + 2)*(e*x + d)/(x^4 - 5*x^2 + 4)^2,x, algorithm="fricas
```

```
[Out] -1/144*(((19*d - 26*e)*x + 38*d - 52*e)*log(x + 2) - 24*((d - e)*
x + 2*d - 2*e)*log(x + 1) + 8*((d + e)*x + 2*d + 2*e)*log(x - 1)
- 3*((d + 2*e)*x + 2*d + 4*e)*log(x - 2) - 12*d + 24*e)/(x + 2)
```

Sympy [A] time = 16.6657, size = 1188, normalized size = 16.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x+d)*(x**3-2*x**2-x+2)/(x**4-5*x**2+4)**2,x)
```

```
[Out] (d - 2*e)/(12*x + 24) + (d - e)*log(x + (-1534775*d**6 + 8032360*
d**5*e - 984027*d**5*(d - e) - 12991180*d**4*e**2 + 11797266*d**4
*e*(d - e) + 3567168*d**4*(d - e)**2 + 1075200*d**3*e**3 - 327215
28*d**3*e**2*(d - e) - 8725248*d**3*e*(d - e)**2 - 247104*d**3*(d
 - e)**3 + 16959280*d**2*e**4 + 38977296*d**2*e**3*(d - e) - 2820
096*d**2*e**2*(d - e)**2 - 10357632*d**2*e*(d - e)**3 - 15836800*
d*e**5 - 21294960*d*e**4*(d - e) + 15436800*d*e**3*(d - e)**2 + 1
6277760*d*e**2*(d - e)**3 + 4283840*e**6 + 3876000*e**5*(d - e) -
 6865920*e**4*(d - e)**2 - 4078080*e**3*(d - e)**3)/(801262*d**6
- 4662251*d**5*e + 7296938*d**4*e**2 + 1388616*d**3*e**3 - 124474
40*d**2*e**4 + 9990800*d*e**5 - 2380000*e**6))/6 - (d + e)*log(x
+ (-1534775*d**6 + 8032360*d**5*e + 328009*d**5*(d + e) - 1299118
0*d**4*e**2 - 3932422*d**4*e*(d + e) + 396352*d**4*(d + e)**2 + 1
075200*d**3*e**3 + 10907176*d**3*e**2*(d + e) - 969472*d**3*e*(d
+ e)**2 + 9152*d**3*(d + e)**3 + 16959280*d**2*e**4 - 12992432*d*
*2*e**3*(d + e) - 313344*d**2*e**2*(d + e)**2 + 383616*d**2*e*(d
+ e)**3 - 15836800*d*e**5 + 7098320*d*e**4*(d + e) + 1715200*d*e*
*3*(d + e)**2 - 602880*d*e**2*(d + e)**3 + 4283840*e**6 - 1292000
*e**5*(d + e) - 762880*e**4*(d + e)**2 + 151040*e**3*(d + e)**3)/
(801262*d**6 - 4662251*d**5*e + 7296938*d**4*e**2 + 1388616*d**3*
e**3 - 12447440*d**2*e**4 + 9990800*d*e**5 - 2380000*e**6))/18 +
(d + 2*e)*log(x + (-1534775*d**6 + 8032360*d**5*e - 984027*d**5*(
d + 2*e)/8 - 12991180*d**4*e**2 + 5898633*d**4*e*(d + 2*e)/4 + 55
737*d**4*(d + 2*e)**2 + 1075200*d**3*e**3 - 4090191*d**3*e**2*(d
+ 2*e) - 136332*d**3*e*(d + 2*e)**2 - 3861*d**3*(d + 2*e)**3/8 +
16959280*d**2*e**4 + 4872162*d**2*e**3*(d + 2*e) - 44064*d**2*e**
2*(d + 2*e)**2 - 80919*d**2*e*(d + 2*e)**3/4 - 15836800*d*e**5 -
2661870*d*e**4*(d + 2*e) + 241200*d*e**3*(d + 2*e)**2 + 63585*d*e
**2*(d + 2*e)**3/2 + 4283840*e**6 + 484500*e**5*(d + 2*e) - 10728
0*e**4*(d + 2*e)**2 - 7965*e**3*(d + 2*e)**3)/(801262*d**6 - 4662
251*d**5*e + 7296938*d**4*e**2 + 1388616*d**3*e**3 - 12447440*d**
2*e**4 + 9990800*d*e**5 - 2380000*e**6))/48 - (19*d - 26*e)*log(x
```

```
 + (-1534775*d**6 + 8032360*d**5*e + 328009*d**5*(19*d - 26*e)/8
- 12991180*d**4*e**2 - 1966211*d**4*e*(19*d - 26*e)/4 + 6193*d**4
*(19*d - 26*e)**2 + 1075200*d**3*e**3 + 1363397*d**3*e**2*(19*d -
 26*e) - 15148*d**3*e*(19*d - 26*e)**2 + 143*d**3*(19*d - 26*e)**
3/8 + 16959280*d**2*e**4 - 1624054*d**2*e**3*(19*d - 26*e) - 4896
*d**2*e**2*(19*d - 26*e)**2 + 2997*d**2*e*(19*d - 26*e)**3/4 - 15
836800*d*e**5 + 887290*d*e**4*(19*d - 26*e) + 26800*d*e**3*(19*d
- 26*e)**2 - 2355*d*e**2*(19*d - 26*e)**3/2 + 4283840*e**6 - 1615
00*e**5*(19*d - 26*e) - 11920*e**4*(19*d - 26*e)**2 + 295*e**3*(1
9*d - 26*e)**3)/(801262*d**6 - 4662251*d**5*e + 7296938*d**4*e**2
 + 1388616*d**3*e**3 - 12447440*d**2*e**4 + 9990800*d*e**5 - 2380
000*e**6))/144
```

GIAC/XCAS [A] time = 0.283728, size = 89, normalized size = 1.25

$$-\frac{1}{144}(19\,d-26\,e)\ln(|x+2|)+\frac{1}{6}(d-e)\ln(|x+1|)-\frac{1}{18}(d+e)\ln(|x-1|)+\frac{1}{48}(d+2\,e)\ln(|x-2|)+\frac{d-2\,e}{12\,(x+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 - 2*x^2 - x + 2)*(e*x + d)/(x^4 - 5*x^2 + 4)^2,x, algorithm="giac")
```

```
[Out] -1/144*(19*d - 26*e)*ln(abs(x + 2)) + 1/6*(d - e)*ln(abs(x + 1))
- 1/18*(d + e)*ln(abs(x - 1)) + 1/48*(d + 2*e)*ln(abs(x - 2)) + 1
/12*(d - 2*e)/(x + 2)
```

3.87 $\int \frac{(d+ex+fx^2)(2-x-2x^2+x^3)}{(4-5x^2+x^4)^2} dx$

Optimal. Leaf size=82

$$\frac{d-2e+4f}{12(x+2)} - \frac{1}{18}\log(1-x)(d+e+f) + \frac{1}{48}\log(2-x)(d+2e+4f) + \frac{1}{6}\log(x+1)(d-e+f) - \frac{1}{144}\log(x+2)(19d-26e+28f)$$

```
[Out] (d - 2*e + 4*f)/(12*(2 + x)) - ((d + e + f)*Log[1 - x])/18 + ((d
+ 2*e + 4*f)*Log[2 - x])/48 + ((d - e + f)*Log[1 + x])/6 - ((19*d
 - 26*e + 28*f)*Log[2 + x])/144
```

Rubi [A] time = 0.363242, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$

$$\frac{d-2e+4f}{12(x+2)} - \frac{1}{18}\log(1-x)(d+e+f) + \frac{1}{48}\log(2-x)(d+2e+4f) + \frac{1}{6}\log(x+1)(d-e+f) - \frac{1}{144}\log(x+2)(19d-26e+28f)$$

Antiderivative was successfully verified.

```
[In]  Int[((d + e*x + f*x^2)*(2 - x - 2*x^2 + x^3))/(4 - 5*x^2 + x^4)^2,x]
```

```
[Out] (d - 2*e + 4*f)/(12*(2 + x)) - ((d + e + f)*Log[1 - x])/18 + ((d
+ 2*e + 4*f)*Log[2 - x])/48 + ((d - e + f)*Log[1 + x])/6 - ((19*d
 - 26*e + 28*f)*Log[2 + x])/144
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((f*x**2+e*x+d)*(x**3-2*x**2-x+2)/(x**4-5*x**2+4)**2,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.102384, size = 77, normalized size = 0.94

$$\frac{1}{144}\left(\frac{12(d-2e+4f)}{x+2}+24\log(-x-1)(d-e+f)-8\log(1-x)(d+e+f)+3\log(2-x)(d+2e+4f)+\log(x+2)(-19d+26e-28f)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x + f*x^2)*(2 - x - 2*x^2 + x^3))/(4 - 5*x^2 + x^4)^2,x]
```

```
[Out] ((12*(d - 2*e + 4*f))/(2 + x) + 24*(d - e + f)*Log[-1 - x] - 8*(d
 + e + f)*Log[1 - x] + 3*(d + 2*e + 4*f)*Log[2 - x] + (-19*d + 26
*e - 28*f)*Log[2 + x])/144
```

Maple [A] time = 0.017, size = 110, normalized size = 1.3

$$\frac{13\ln(2+x)e}{72}-\frac{7\ln(2+x)f}{36}-\frac{19\ln(2+x)d}{144}+\frac{d}{24+12x}-\frac{e}{12+6x}+\frac{f}{6+3x}-\frac{\ln(-1+x)d}{18}-\frac{\ln(-1+x)e}{18}-\frac{\ln(-1+x)f}{18}+\frac{\ln(1+x)d}{6}-\frac{\ln(1+x)e}{6}+\frac{\ln(1+x)f}{6}+\frac{\ln(x-2)d}{48}+\frac{\ln(x-2)e}{24}+\frac{\ln(x-2)f}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)*(x^3-2*x^2-x+2)/(x^4-5*x^2+4)^2,x)
```

```
[Out] 13/72*ln(2+x)*e-7/36*ln(2+x)*f-19/144*ln(2+x)*d+1/12/(2+x)*d-1/6/
(2+x)*e+1/3/(2+x)*f-1/18*ln(-1+x)*d-1/18*ln(-1+x)*e-1/18*ln(-1+x)
*f+1/6*ln(1+x)*d-1/6*ln(1+x)*e+1/6*ln(1+x)*f+1/48*ln(x-2)*d+1/24*
ln(x-2)*e+1/12*ln(x-2)*f
```

Maxima [A] time = 0.700742, size = 92, normalized size = 1.12

$$-\frac{1}{144}(19d-26e+28f)\log(x+2)+\frac{1}{6}(d-e+f)\log(x+1)-\frac{1}{18}(d+e+f)\log(x-1)+\frac{1}{48}(d+2e+4f)\log(x-2)+\frac{d-2e+4f}{12(x+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2 + e*x + d)*(x^3 - 2*x^2 - x + 2)/(x^4 - 5*x^2 + 4)^2,x, algorithm=
```

```
[Out] -1/144*(19*d - 26*e + 28*f)*log(x + 2) + 1/6*(d - e + f)*log(x +
1) - 1/18*(d + e + f)*log(x - 1) + 1/48*(d + 2*e + 4*f)*log(x - 2
) + 1/12*(d - 2*e + 4*f)/(x + 2)
```

Fricas [A] time = 0.357387, size = 157, normalized size = 1.91

$$-\frac{((19\,d - 26\,e + 28\,f)x + 38\,d - 52\,e + 56\,f)\log(x+2) - 24\,((d - e + f)x + 2\,d - 2\,e + 2\,f)\log(x+1) + 8\,((d + e + f)x + 2\,\ldots}{144\,(x+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2 + e*x + d)*(x^3 - 2*x^2 - x + 2)/(x^4 - 5*x^2 + 4)^2,x, algorithm=
```

```
[Out] -1/144*(((19*d - 26*e + 28*f)*x + 38*d - 52*e + 56*f)*log(x + 2)
- 24*((d - e + f)*x + 2*d - 2*e + 2*f)*log(x + 1) + 8*((d + e + f
)*x + 2*d + 2*e + 2*f)*log(x - 1) - 3*((d + 2*e + 4*f)*x + 2*d +
4*e + 8*f)*log(x - 2) - 12*d + 24*e - 48*f)/(x + 2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)*(x**3-2*x**2-x+2)/(x**4-5*x**2+4)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.28489, size = 104, normalized size = 1.27

$$-\frac{1}{144}\,(19\,d + 28\,f - 26\,e)\ln(|x+2|) + \frac{1}{6}\,(d + f - e)\ln(|x+1|) - \frac{1}{18}\,(d + f + e)\ln(|x-1|) + \frac{1}{48}\,(d + 4\,f + 2\,e)\ln(|x-2|) + \frac{d + 4\,f - 2\,e}{12\,(x+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2 + e*x + d)*(x^3 - 2*x^2 - x + 2)/(x^4 - 5*x^2 + 4)^2,x, algorithm=
```

```
[Out] -1/144*(19*d + 28*f - 26*e)*ln(abs(x + 2)) + 1/6*(d + f - e)*ln(a
bs(x + 1)) - 1/18*(d + f + e)*ln(abs(x - 1)) + 1/48*(d + 4*f + 2*
e)*ln(abs(x - 2)) + 1/12*(d + 4*f - 2*e)/(x + 2)
```

3.88 $\int \frac{(2-x-2x^2+x^3)(d+ex+fx^2+gx^3)}{(4-5x^2+x^4)^2}\,dx$

Optimal. Leaf size=95

$$\frac{d-2e+4f-8g}{12(x+2)}-\frac{1}{18}\log(1-x)(d+e+f+g)+\frac{1}{48}\log(2-x)(d+2e+4f+8g)+\frac{1}{6}\log(x+1)(d-e+f-g)-\frac{1}{144}\log(x+2)(19d-26e+28f-8g)$$

```
[Out] (d - 2*e + 4*f - 8*g)/(12*(2 + x)) - ((d + e + f + g)*Log[1 - x])
/18 + ((d + 2*e + 4*f + 8*g)*Log[2 - x])/48 + ((d - e + f - g)*Lo
g[1 + x])/6 - ((19*d - 26*e + 28*f - 8*g)*Log[2 + x])/144
```

Rubi [A] time = 0.4201, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.049

$$\frac{d-2e+4f-8g}{12(x+2)}-\frac{1}{18}\log(1-x)(d+e+f+g)+\frac{1}{48}\log(2-x)(d+2e+4f+8g)+\frac{1}{6}\log(x+1)(d-e+f-g)-\frac{1}{144}\log(x+2)(19d-26e+28f-8g)$$

Antiderivative was successfully verified.

```
[In] Int[((2 - x - 2*x^2 + x^3)*(d + e*x + f*x^2 + g*x^3))/(4 - 5*x^2 + x^4)^2,x]
```

```
[Out] (d - 2*e + 4*f - 8*g)/(12*(2 + x)) - ((d + e + f + g)*Log[1 - x])
/18 + ((d + 2*e + 4*f + 8*g)*Log[2 - x])/48 + ((d - e + f - g)*Lo
g[1 + x])/6 - ((19*d - 26*e + 28*f - 8*g)*Log[2 + x])/144
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((x**3-2*x**2-x+2)*(g*x**3+f*x**2+e*x+d)/(x**4-5*x**2+4)**2,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.089585, size = 90, normalized size = 0.95

$$\frac{1}{144}\left(\frac{12(d-2e+4f-8g)}{x+2}+24\log(-x-1)(d-e+f-g)-8\log(1-x)(d+e+f+g)+3\log(2-x)(d+2e+4f+8g)+\log(x+2)(-19d+26e-28f+8g)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 - x - 2*x^2 + x^3)*(d + e*x + f*x^2 + g*x^3))/(4 - 5*x^2 + x^4)^2,x]
```

```
[Out] ((12*(d - 2*e + 4*f - 8*g))/(2 + x) + 24*(d - e + f - g)*Log[-1 -
 x] - 8*(d + e + f + g)*Log[1 - x] + 3*(d + 2*e + 4*f + 8*g)*Log[
2 - x] + (-19*d + 26*e - 28*f + 8*g)*Log[2 + x])/144
```

Maple [A] time = 0.016, size = 146, normalized size = 1.5

$$\frac{13\ln(2+x)e}{72}-\frac{7\ln(2+x)f}{36}+\frac{\ln(2+x)g}{18}-\frac{19\ln(2+x)d}{144}+\frac{d}{24+12x}-\frac{e}{12+6x}+\frac{f}{6+3x}-\frac{2g}{6+3x}-\frac{\ln(-1+x)d}{18}-\frac{\ln(-1+x)e}{18}-\frac{\ln(-1+x)f}{18}-\frac{\ln(-1+x)g}{18}+\frac{\ln(1+x)d}{6}-\frac{\ln(1+x)e}{6}+\frac{\ln(1+x)f}{6}-\frac{\ln(1+x)g}{6}+\frac{\ln(x-2)d}{48}+\frac{\ln(x-2)e}{24}+\frac{\ln(x-2)f}{12}+\frac{\ln(x-2)g}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-2*x^2-x+2)*(g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x)
```

```
[Out] 13/72*ln(2+x)*e-7/36*ln(2+x)*f+1/18*ln(2+x)*g-19/144*ln(2+x)*d+1/
12/(2+x)*d-1/6/(2+x)*e+1/3/(2+x)*f-2/3/(2+x)*g-1/18*ln(-1+x)*d-1/
18*ln(-1+x)*e-1/18*ln(-1+x)*f-1/18*ln(-1+x)*g+1/6*ln(1+x)*d-1/6*l
n(1+x)*e+1/6*ln(1+x)*f-1/6*ln(1+x)*g+1/48*ln(x-2)*d+1/24*ln(x-2)*
e+1/12*ln(x-2)*f+1/6*ln(x-2)*g
```

Maxima [A] time = 0.702063, size = 109, normalized size = 1.15

$$-\frac{1}{144}(19d-26e+28f-8g)\log(x+2)+\frac{1}{6}(d-e+f-g)\log(x+1)-\frac{1}{18}(d+e+f+g)\log(x-1)+\frac{1}{48}(d+2e+4f+8g)\log(x-2)+\frac{d-2e+4f-8g}{12(x+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3 + f*x^2 + e*x + d)*(x^3 - 2*x^2 - x + 2)/(x^4 - 5*x^2 + 4)^2,x, alg
```

```
[Out] -1/144*(19*d - 26*e + 28*f - 8*g)*log(x + 2) + 1/6*(d - e + f - g
)*log(x + 1) - 1/18*(d + e + f + g)*log(x - 1) + 1/48*(d + 2*e +
4*f + 8*g)*log(x - 2) + 1/12*(d - 2*e + 4*f - 8*g)/(x + 2)
```

Fricas [A] time = 0.874593, size = 190, normalized size = 2.

$$-\frac{((19\,d-26\,e+28\,f-8\,g)x+38\,d-52\,e+56\,f-16\,g)\log\left(x+2\right)-24\left((d-e+f-g)x+2\,d-2\,e+2\,f-2\,g\right)\log\left(x+1\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3 + f*x^2 + e*x + d)*(x^3 - 2*x^2 - x + 2)/(x^4 - 5*x^2 + 4)^2,x, alg
```

```
[Out] -1/144*(((19*d - 26*e + 28*f - 8*g)*x + 38*d - 52*e + 56*f - 16*g
)*log(x + 2) - 24*((d - e + f - g)*x + 2*d - 2*e + 2*f - 2*g)*log
(x + 1) + 8*((d + e + f + g)*x + 2*d + 2*e + 2*f + 2*g)*log(x - 1
) - 3*((d + 2*e + 4*f + 8*g)*x + 2*d + 4*e + 8*f + 16*g)*log(x -
2) - 12*d + 24*e - 48*f + 96*g)/(x + 2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-2*x**2-x+2)*(g*x**3+f*x**2+e*x+d)/(x**4-5*x**2+4)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.292295, size = 122, normalized size = 1.28

$$-\frac{1}{144}\left(19\,d+28\,f-8\,g-26\,e\right)\ln\left(|x+2|\right)+\frac{1}{6}\left(d+f-g-e\right)\ln\left(|x+1|\right)-\frac{1}{18}\left(d+f+g+e\right)\ln\left(|x-1|\right)+\frac{1}{48}\left(d+4\,f+8\,g+2\,e\right)\ln\left(|x-2|\right)+\frac{d+4\,f-8\,g-2\,e}{12\,(x+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3 + f*x^2 + e*x + d)*(x^3 - 2*x^2 - x + 2)/(x^4 - 5*x^2 + 4)^2,x, alg

[Out] -1/144*(19*d + 28*f - 8*g - 26*e)*ln(abs(x + 2)) + 1/6*(d + f - g
 - e)*ln(abs(x + 1)) - 1/18*(d + f + g + e)*ln(abs(x - 1)) + 1/48
*(d + 4*f + 8*g + 2*e)*ln(abs(x - 2)) + 1/12*(d + 4*f - 8*g - 2*e
)/(x + 2)
```

3.89 $\int \frac{(2-x-2x^2+x^3)(d+ex+fx^2+gx^3+hx^4)}{(4-5x^2+x^4)^2}\,dx$

Optimal. Leaf size=106

$$\frac{d-2e+4f-8g+16h}{12(x+2)}-\frac{1}{18}\log(1-x)(d+e+f+g+h)+\frac{1}{48}\log(2-x)(d+2e+4f+8g+16h)+\frac{1}{6}\log(x+1)(d-e+f-g+h)-\frac{1}{144}\log(x+2)(19d-26e+28f-8g-80h)$$

```
[Out] (d - 2*e + 4*f - 8*g + 16*h)/(12*(2 + x)) - ((d + e + f + g + h)*
Log[1 - x])/18 + ((d + 2*e + 4*f + 8*g + 16*h)*Log[2 - x])/48 + (
(d - e + f - g + h)*Log[1 + x])/6 - ((19*d - 26*e + 28*f - 8*g -
80*h)*Log[2 + x])/144
```

Rubi [A] time = 0.485613, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043

$$\frac{d-2e+4f-8g+16h}{12(x+2)}-\frac{1}{18}\log(1-x)(d+e+f+g+h)+\frac{1}{48}\log(2-x)(d+2e+4f+8g+16h)+\frac{1}{6}\log(x+1)(d-e+f-g+h)-\frac{1}{144}\log(x+2)(19d-26e+28f-8g-80h)$$

Antiderivative was successfully verified.

```
[In] Int[((2 - x - 2*x^2 + x^3)*(d + e*x + f*x^2 + g*x^3 + h*x^4))/(4 - 5*x^2 + x^4)^2,x]
```

```
[Out] (d - 2*e + 4*f - 8*g + 16*h)/(12*(2 + x)) - ((d + e + f + g + h)*
Log[1 - x])/18 + ((d + 2*e + 4*f + 8*g + 16*h)*Log[2 - x])/48 + (
(d - e + f - g + h)*Log[1 + x])/6 - ((19*d - 26*e + 28*f - 8*g -
80*h)*Log[2 + x])/144
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((x**3-2*x**2-x+2)*(h*x**4+g*x**3+f*x**2+e*x+d)/(x**4-
5*x**2+4)**2,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.114195, size = 102, normalized size = 0.96

$$\frac{1}{144}\left(\frac{12(d-2e+4f-8g+16h)}{x+2}+24\log(-x-1)(d-e+f-g+h)-8\log(1-x)(d+e+f+g+h)\right.$$
$$\left.+3\log(2-x)(d+2(e+2f+4g+8h))+\log(x+2)(-19d+26e-28f+8g+80h)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 - x - 2*x^2 + x^3)*(d + e*x + f*x^2 + g*x^3 + h*x^4))/(4 - 5*x^2 + x^4
```

```
[Out] ((12*(d - 2*e + 4*f - 8*g + 16*h))/(2 + x) + 24*(d - e + f - g +
h)*Log[-1 - x] - 8*(d + e + f + g + h)*Log[1 - x] + 3*(d + 2*(e +
 2*f + 4*g + 8*h))*Log[2 - x] + (-19*d + 26*e - 28*f + 8*g + 80*h
)*Log[2 + x])/144
```

Maple [A] time = 0.017, size = 182, normalized size = 1.7

$$\frac{4h}{6+3x}-\frac{2g}{6+3x}+\frac{d}{24+12x}-\frac{e}{12+6x}+\frac{f}{6+3x}+\frac{\ln(1+x)d}{6}-\frac{\ln(1+x)e}{6}$$
$$-\frac{\ln(-1+x)d}{18}-\frac{\ln(-1+x)e}{18}+\frac{\ln(x-2)h}{3}+\frac{\ln(1+x)h}{6}+\frac{5\ln(2+x)h}{9}-\frac{\ln(-1+x)h}{18}$$
$$-\frac{\ln(1+x)g}{6}+\frac{\ln(x-2)g}{6}-\frac{\ln(-1+x)g}{18}+\frac{\ln(2+x)g}{18}+\frac{\ln(x-2)d}{48}+\frac{\ln(x-2)e}{24}$$
$$+\frac{13\ln(2+x)e}{72}+\frac{\ln(x-2)f}{12}-\frac{19\ln(2+x)d}{144}+\frac{\ln(1+x)f}{6}-\frac{\ln(-1+x)f}{18}-\frac{7\ln(2+x)f}{36}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^3-2*x^2-x+2)*(h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x)
```

```
[Out] 4/3/(2+x)*h-2/3/(2+x)*g+1/12/(2+x)*d-1/6/(2+x)*e+1/3/(2+x)*f+1/6*
ln(1+x)*d-1/6*ln(1+x)*e-1/18*ln(-1+x)*d-1/18*ln(-1+x)*e+1/3*ln(x-
2)*h+1/6*ln(1+x)*h+5/9*ln(2+x)*h-1/18*ln(-1+x)*h-1/6*ln(1+x)*g+1/
6*ln(x-2)*g-1/18*ln(-1+x)*g+1/18*ln(2+x)*g+1/48*ln(x-2)*d+1/24*ln
(x-2)*e+13/72*ln(2+x)*e+1/12*ln(x-2)*f-19/144*ln(2+x)*d+1/6*ln(1+
x)*f-1/18*ln(-1+x)*f-7/36*ln(2+x)*f
```

Maxima [A] time = 0.694834, size = 124, normalized size = 1.17

$$-\frac{1}{144}\,(19\,d - 26\,e + 28\,f - 8\,g - 80\,h)\log\left(x + 2\right) + \frac{1}{6}\,(d - e + f - g + h)\log\left(x + 1\right) - \frac{1}{18}\,(d + e + f + g + h)\log\left(x - 1\right) + \frac{1}{48}\,(d + 2\,e + 4\,f + 8\,g + 16\,h)\log\left(x - 2\right) + \frac{d - 2\,e + 4\,f - 8\,g + 16\,h}{12\,(x + 2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^4 + g*x^3 + f*x^2 + e*x + d)*(x^3 - 2*x^2 - x + 2)/(x^4 - 5*x^2 + 4)^
```

```
[Out] -1/144*(19*d - 26*e + 28*f - 8*g - 80*h)*log(x + 2) + 1/6*(d - e
+ f - g + h)*log(x + 1) - 1/18*(d + e + f + g + h)*log(x - 1) + 1
/48*(d + 2*e + 4*f + 8*g + 16*h)*log(x - 2) + 1/12*(d - 2*e + 4*f
 - 8*g + 16*h)/(x + 2)
```

Fricas [A] time = 4.17267, size = 221, normalized size = 2.08

$$-\frac{((19\,d - 26\,e + 28\,f - 8\,g - 80\,h)x + 38\,d - 52\,e + 56\,f - 16\,g - 160\,h)\log\left(x + 2\right) - 24\,((d - e + f - g + h)x + 2\,d - 2\,e + 2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^4 + g*x^3 + f*x^2 + e*x + d)*(x^3 - 2*x^2 - x + 2)/(x^4 - 5*x^2 + 4)^
```

```
[Out] -1/144*(((19*d - 26*e + 28*f - 8*g - 80*h)*x + 38*d - 52*e + 56*f
 - 16*g - 160*h)*log(x + 2) - 24*((d - e + f - g + h)*x + 2*d - 2
*e + 2*f - 2*g + 2*h)*log(x + 1) + 8*((d + e + f + g + h)*x + 2*d
 + 2*e + 2*f + 2*g + 2*h)*log(x - 1) - 3*((d + 2*e + 4*f + 8*g +
16*h)*x + 2*d + 4*e + 8*f + 16*g + 32*h)*log(x - 2) - 12*d + 24*e
 - 48*f + 96*g - 192*h)/(x + 2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-2*x**2-x+2)*(h*x**4+g*x**3+f*x**2+e*x+d)/(x**4-5*x**2+4)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.283838, size = 136, normalized size = 1.28

$$-\frac{1}{144}(19d+28f-8g-80h-26e)\ln(|x+2|)+\frac{1}{6}(d+f-g+h-e)\ln(|x+1|)-\frac{1}{18}(d+f+g+h+e)\ln(|x-1|)+\frac{1}{48}(d+4f+8g+16h+2e)\ln(|x-2|)+\frac{d+4f-8g+16h-2e}{12(x+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^4 + g*x^3 + f*x^2 + e*x + d)*(x^3 - 2*x^2 - x + 2)/(x^4 - 5*x^2 + 4)^
```

```
[Out] -1/144*(19*d + 28*f - 8*g - 80*h - 26*e)*ln(abs(x + 2)) + 1/6*(d
+ f - g + h - e)*ln(abs(x + 1)) - 1/18*(d + f + g + h + e)*ln(abs
(x - 1)) + 1/48*(d + 4*f + 8*g + 16*h + 2*e)*ln(abs(x - 2)) + 1/1
2*(d + 4*f - 8*g + 16*h - 2*e)/(x + 2)
```

3.90 $\int \frac{(2-x-2x^2+x^3)(d+ex+fx^2+gx^3+hx^4+ix^5)}{(4-5x^2+x^4)^2} dx$

Optimal. Leaf size=122

$$\frac{d-2e+4f-8g+16h-32i}{12(x+2)} - \frac{1}{18}\log(1-x)(d+e+f+g+h+i) + \frac{1}{48}\log(2-x)(d+2e+4f+8g+16h+32i) + \frac{1}{6}\log(x+1)(d-e+f-g+h-i) - \frac{1}{144}\log(x+2)(19d-26e+28f-8g-80h+352i) + ix$$

```
[Out] i*x + (d - 2*e + 4*f - 8*g + 16*h - 32*i)/(12*(2 + x)) - ((d + e
+ f + g + h + i)*Log[1 - x])/18 + ((d + 2*e + 4*f + 8*g + 16*h +
32*i)*Log[2 - x])/48 + ((d - e + f - g + h - i)*Log[1 + x])/6 - (
(19*d - 26*e + 28*f - 8*g - 80*h + 352*i)*Log[2 + x])/144
```

Rubi [A] time = 0.603296, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 51, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.039

$$\frac{d-2e+4f-8g+16h-32i}{12(x+2)} - \frac{1}{18}\log(1-x)(d+e+f+g+h+i) + \frac{1}{48}\log(2-x)(d+2e+4f+8g+16h+32i) + \frac{1}{6}\log(x+1)(d-e+f-g+h-i) - \frac{1}{144}\log(x+2)(19d-26e+28f-8g-80h+352i) + ix$$

Antiderivative was successfully verified.

```
[In] Int[((2 - x - 2*x^2 + x^3)*(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5))/(4 - 5*x^2 + x^
```

```
[Out] i*x + (d - 2*e + 4*f - 8*g + 16*h - 32*i)/(12*(2 + x)) - ((d + e
+ f + g + h + i)*Log[1 - x])/18 + ((d + 2*e + 4*f + 8*g + 16*h +
32*i)*Log[2 - x])/48 + ((d - e + f - g + h - i)*Log[1 + x])/6 - (
(19*d - 26*e + 28*f - 8*g - 80*h + 352*i)*Log[2 + x])/144
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((x**3-2*x**2-x+2)*(i*x**5+h*x**4+g*x**3+f*x**2+e*x+d)/(x**4-
5*x**2+4)**2,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.138592, size = 118, normalized size = 0.97

$$\frac{1}{144}\left(\frac{12(d-2(e-2f+4g-8h+16i))}{x+2}-8\log(1-x)(d+e+f+g+h+i)\right.$$
$$+3\log(2-x)(d+2e+4(f+2g+4h+8i))+24\log(x+1)(d-e+f-g+h-i)$$
$$\left.+\log(x+2)(-19d+26e-28f+8g+80h-352i)+144ix\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 - x - 2*x^2 + x^3)*(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5))/(4 - 5*x
```

```
[Out] (144*i*x + (12*(d - 2*(e - 2*f + 4*g - 8*h + 16*i)))/(2 + x) - 8*
(d + e + f + g + h + i)*Log[1 - x] + 3*(d + 2*e + 4*(f + 2*g + 4*
h + 8*i))*Log[2 - x] + 24*(d - e + f - g + h - i)*Log[1 + x] + (-
19*d + 26*e - 28*f + 8*g + 80*h - 352*i)*Log[2 + x])/144
```

Maple [A] time = 0.018, size = 221, normalized size = 1.8

$$-\frac{8i}{6+3x}+\frac{4h}{6+3x}-\frac{2g}{6+3x}+\frac{d}{24+12x}-\frac{e}{12+6x}+\frac{f}{6+3x}+\frac{\ln(1+x)d}{6}$$
$$-\frac{\ln(1+x)e}{6}-\frac{\ln(-1+x)d}{18}-\frac{\ln(-1+x)e}{18}+\frac{2\ln(x-2)i}{3}-\frac{\ln(1+x)i}{6}-\frac{\ln(-1+x)i}{18}$$
$$-\frac{22\ln(2+x)i}{9}+\frac{\ln(x-2)h}{3}+\frac{\ln(1+x)h}{6}+\frac{5\ln(2+x)h}{9}-\frac{\ln(-1+x)h}{18}-\frac{\ln(1+x)g}{6}$$
$$+\frac{\ln(x-2)g}{6}-\frac{\ln(-1+x)g}{18}+\frac{\ln(2+x)g}{18}+\frac{\ln(x-2)d}{48}+\frac{\ln(x-2)e}{24}+\frac{13\ln(2+x)e}{72}$$
$$+\frac{\ln(x-2)f}{12}-\frac{19\ln(2+x)d}{144}+\frac{\ln(1+x)f}{6}-\frac{\ln(-1+x)f}{18}-\frac{7\ln(2+x)f}{36}+ix$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-2*x^2-x+2)*(i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x)
```

```
[Out] -8/3/(2+x)*i+4/3/(2+x)*h-2/3/(2+x)*g+1/12/(2+x)*d-1/6/(2+x)*e+1/3
/(2+x)*f+1/6*ln(1+x)*d-1/6*ln(1+x)*e-1/18*ln(-1+x)*d-1/18*ln(-1+x
)*e+2/3*ln(x-2)*i-1/6*ln(1+x)*i-1/18*ln(-1+x)*i-22/9*ln(2+x)*i+1/
3*ln(x-2)*h+1/6*ln(1+x)*h+5/9*ln(2+x)*h-1/18*ln(-1+x)*h-1/6*ln(1+
x)*g+1/6*ln(x-2)*g-1/18*ln(-1+x)*g+1/18*ln(2+x)*g+1/48*ln(x-2)*d+
1/24*ln(x-2)*e+13/72*ln(2+x)*e+1/12*ln(x-2)*f-19/144*ln(2+x)*d+1/
6*ln(1+x)*f-1/18*ln(-1+x)*f-7/36*ln(2+x)*f+i*x
```

Maxima [A] time = 0.715923, size = 146, normalized size = 1.2

$$ix - \frac{1}{144}(19\,d - 26\,e + 28\,f - 8\,g - 80\,h + 352\,i)\log(x+2) + \frac{1}{6}(d - e + f - g + h - i)\log(x+1) - \frac{1}{18}(d + e + f + g + h + i)\log(x-1) + \frac{1}{48}(d + 2\,e + 4\,f + 8\,g + 16\,h + 32\,i)\log(x-2) + \frac{d - 2\,e + 4\,f - 8\,g + 16\,h - 32\,i}{12\,(x+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^5 + h*x^4 + g*x^3 + f*x^2 + e*x + d)*(x^3 - 2*x^2 - x + 2)/(x^4 - 5*x
```

```
[Out] i*x - 1/144*(19*d - 26*e + 28*f - 8*g - 80*h + 352*i)*log(x + 2)
+ 1/6*(d - e + f - g + h - i)*log(x + 1) - 1/18*(d + e + f + g +
h + i)*log(x - 1) + 1/48*(d + 2*e + 4*f + 8*g + 16*h + 32*i)*log(
x - 2) + 1/12*(d - 2*e + 4*f - 8*g + 16*h - 32*i)/(x + 2)
```

Fricas [A] time = 25.4194, size = 270, normalized size = 2.21

$$144\,ix^2 + 288\,ix - ((19\,d - 26\,e + 28\,f - 8\,g - 80\,h + 352\,i)x + 38\,d - 52\,e + 56\,f - 16\,g - 160\,h + 704\,i)\log(x+2) + 24\,((d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^5 + h*x^4 + g*x^3 + f*x^2 + e*x + d)*(x^3 - 2*x^2 - x + 2)/(x^4 - 5*x
```

```
[Out] 1/144*(144*i*x^2 + 288*i*x - ((19*d - 26*e + 28*f - 8*g - 80*h +
352*i)*x + 38*d - 52*e + 56*f - 16*g - 160*h + 704*i)*log(x + 2)
+ 24*((d - e + f - g + h - i)*x + 2*d - 2*e + 2*f - 2*g + 2*h - 2
*i)*log(x + 1) - 8*((d + e + f + g + h + i)*x + 2*d + 2*e + 2*f +
 2*g + 2*h + 2*i)*log(x - 1) + 3*((d + 2*e + 4*f + 8*g + 16*h + 3
2*i)*x + 2*d + 4*e + 8*f + 16*g + 32*h + 64*i)*log(x - 2) + 12*d
- 24*e + 48*f - 96*g + 192*h - 384*i)/(x + 2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-2*x**2-x+2)*(i*x**5+h*x**4+g*x**3+f*x**2+e*x+d)/(x**4-
5*x**2+4)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.289118, size = 158, normalized size = 1.3

$$ix - \frac{1}{144}(19\,d + 28\,f - 8\,g - 80\,h + 352\,i - 26\,e)\ln(|x+2|) + \frac{1}{6}(d + f - g + h - i - e)\ln(|x+1|) - \frac{1}{18}(d + f + g + h + i + e)\ln(|x-1|) + \frac{1}{48}(d + 4\,f + 8\,g + 16\,h + 32\,i + 2\,e)\ln(|x-2|) + \frac{d + 4\,f - 8\,g + 16\,h - 32\,i - 2\,e}{12\,(x+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^5 + h*x^4 + g*x^3 + f*x^2 + e*x + d)*(x^3 - 2*x^2 - x + 2)/(x^4 - 5*x
```

```
[Out] i*x - 1/144*(19*d + 28*f - 8*g - 80*h + 352*i - 26*e)*ln(abs(x +
2)) + 1/6*(d + f - g + h - i - e)*ln(abs(x + 1)) - 1/18*(d + f +
g + h + i + e)*ln(abs(x - 1)) + 1/48*(d + 4*f + 8*g + 16*h + 32*i
 + 2*e)*ln(abs(x - 2)) + 1/12*(d + 4*f - 8*g + 16*h - 32*i - 2*e)
/(x + 2)
```

3.91 $\int \frac{2-3x+x^2}{(4-5x^2+x^4)^2}\,dx$

Optimal. Leaf size=56

$$-\frac{3x+5}{12(x^2+3x+2)}-\frac{1}{36}\log(1-x)+\frac{1}{144}\log(2-x)-\frac{7}{36}\log(x+1)+\frac{31}{144}\log(x+2)$$

```
[Out] -(5 + 3*x)/(12*(2 + 3*x + x^2)) - Log[1 - x]/36 + Log[2 - x]/144
- (7*Log[1 + x])/36 + (31*Log[2 + x])/144
```

Rubi [A] time = 0.131818, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238

$$-\frac{3x+5}{12(x^2+3x+2)}-\frac{1}{36}\log(1-x)+\frac{1}{144}\log(2-x)-\frac{7}{36}\log(x+1)+\frac{31}{144}\log(x+2)$$

Antiderivative was successfully verified.

```
[In]  Int[(2 - 3*x + x^2)/(4 - 5*x^2 + x^4)^2,x]
```

```
[Out] -(5 + 3*x)/(12*(2 + 3*x + x^2)) - Log[1 - x]/36 + Log[2 - x]/144
- (7*Log[1 + x])/36 + (31*Log[2 + x])/144
```

Rubi in Sympy [A] time = 31.265, size = 44, normalized size = 0.79

$$-\frac{18x+30}{72(x^2+3x+2)}-\frac{\log(-x+1)}{36}+\frac{\log(-x+2)}{144}-\frac{7\log(x+1)}{36}+\frac{31\log(x+2)}{144}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**2-3*x+2)/(x**4-5*x**2+4)**2,x)
```

```
[Out] -(18*x + 30)/(72*(x**2 + 3*x + 2)) - log(-x + 1)/36 + log(-x + 2)
/144 - 7*log(x + 1)/36 + 31*log(x + 2)/144
```

Mathematica [A] time = 0.0464615, size = 48, normalized size = 0.86

$$\frac{1}{144}\left(-\frac{12(3x+5)}{x^2+3x+2}-4\log(1-x)+\log(2-x)-28\log(x+1)+31\log(x+2)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(2 - 3*x + x^2)/(4 - 5*x^2 + x^4)^2,x]
```

```
[Out] ((-12*(5 + 3*x))/(2 + 3*x + x^2) - 4*Log[1 - x] + Log[2 - x] - 28
*Log[1 + x] + 31*Log[2 + x])/144
```

Maple [A] time = 0.018, size = 40, normalized size = 0.7

$$-\frac{1}{24+12\,x}+\frac{31\,\ln\left(2+x\right)}{144}-\frac{\ln\left(-1+x\right)}{36}-\frac{1}{6+6\,x}-\frac{7\,\ln\left(1+x\right)}{36}+\frac{\ln\left(x-2\right)}{144}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^2-3*x+2)/(x^4-5*x^2+4)^2,x)
```

```
[Out] -1/12/(2+x)+31/144*ln(2+x)-1/36*ln(-1+x)-1/6/(1+x)-7/36*ln(1+x)+1
/144*ln(x-2)
```

Maxima [A] time = 0.699696, size = 57, normalized size = 1.02

$$-\frac{3\,x+5}{12\,\left(x^{2}+3\,x+2\right)}+\frac{31}{144}\,\log\left(x+2\right)-\frac{7}{36}\,\log\left(x+1\right)-\frac{1}{36}\,\log\left(x-1\right)+\frac{1}{144}\,\log\left(x-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 - 3*x + 2)/(x^4 - 5*x^2 + 4)^2,x, algorithm="maxima")
```

```
[Out] -1/12*(3*x + 5)/(x^2 + 3*x + 2) + 31/144*log(x + 2) - 7/36*log(x
+ 1) - 1/36*log(x - 1) + 1/144*log(x - 2)
```

Fricas [A] time = 0.258591, size = 97, normalized size = 1.73

$$\frac{31\,\left(x^{2}+3\,x+2\right)\,\log\left(x+2\right)-28\,\left(x^{2}+3\,x+2\right)\,\log\left(x+1\right)-4\,\left(x^{2}+3\,x+2\right)\,\log\left(x-1\right)+\left(x^{2}+3\,x+2\right)\,\log\left(x-2\right)-36\,x-}{144\,\left(x^{2}+3\,x+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 - 3*x + 2)/(x^4 - 5*x^2 + 4)^2,x, algorithm="fricas")
```

```
[Out] 1/144*(31*(x^2 + 3*x + 2)*log(x + 2) - 28*(x^2 + 3*x + 2)*log(x +
 1) - 4*(x^2 + 3*x + 2)*log(x - 1) + (x^2 + 3*x + 2)*log(x - 2) -
 36*x - 60)/(x^2 + 3*x + 2)
```

Sympy [A] time = 0.78408, size = 44, normalized size = 0.79

$$-\frac{3x+5}{12x^2+36x+24}+\frac{\log(x-2)}{144}-\frac{\log(x-1)}{36}-\frac{7\log(x+1)}{36}+\frac{31\log(x+2)}{144}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**2-3*x+2)/(x**4-5*x**2+4)**2,x)
```

```
[Out] -(3*x + 5)/(12*x**2 + 36*x + 24) + log(x - 2)/144 - log(x - 1)/36
 - 7*log(x + 1)/36 + 31*log(x + 2)/144
```

GIAC/XCAS [A] time = 0.283228, size = 62, normalized size = 1.11

$$-\frac{3x+5}{12(x+2)(x+1)}+\frac{31}{144}\ln(|x+2|)-\frac{7}{36}\ln(|x+1|)-\frac{1}{36}\ln(|x-1|)+\frac{1}{144}\ln(|x-2|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 - 3*x + 2)/(x^4 - 5*x^2 + 4)^2,x, algorithm="giac")
```

```
[Out] -1/12*(3*x + 5)/((x + 2)*(x + 1)) + 31/144*ln(abs(x + 2)) - 7/36*
ln(abs(x + 1)) - 1/36*ln(abs(x - 1)) + 1/144*ln(abs(x - 2))
```

3.92 $\int \frac{(d+ex)(2-3x+x^2)}{(4-5x^2+x^4)^2}\,dx$

Optimal. Leaf size=89

$$-\frac{x(3d-4e)+5d-6e}{12\left(x^2+3x+2\right)}-\frac{1}{36}(d+e)\log(1-x)+\frac{1}{144}(d+2e)\log(2-x)-\frac{1}{36}(7d-13e)\log(x+1)+\frac{1}{144}(31d-50e)\log(x+2)$$

```
[Out] -(5*d - 6*e + (3*d - 4*e)*x)/(12*(2 + 3*x + x^2)) - ((d + e)*Log[
1 - x])/36 + ((d + 2*e)*Log[2 - x])/144 - ((7*d - 13*e)*Log[1 + x
])/36 + ((31*d - 50*e)*Log[2 + x])/144
```

Rubi [A] time = 0.530986, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192

$$-\frac{x(3d-4e)+5d-6e}{12\left(x^2+3x+2\right)}-\frac{1}{36}(d+e)\log(1-x)+\frac{1}{144}(d+2e)\log(2-x)-\frac{1}{36}(7d-13e)\log(x+1)+\frac{1}{144}(31d-50e)\log(x+2)$$

Antiderivative was successfully verified.

```
[In]  Int[((d + e*x)*(2 - 3*x + x^2))/(4 - 5*x^2 + x^4)^2,x]
```

```
[Out] -(5*d - 6*e + (3*d - 4*e)*x)/(12*(2 + 3*x + x^2)) - ((d + e)*Log[
1 - x])/36 + ((d + 2*e)*Log[2 - x])/144 - ((7*d - 13*e)*Log[1 + x
])/36 + ((31*d - 50*e)*Log[2 + x])/144
```

Rubi in Sympy [A] time = 48.1095, size = 78, normalized size = 0.88

$$\left(\frac{d}{144}+\frac{e}{72}\right)\log\left(-x+2\right)-\left(\frac{d}{36}+\frac{e}{36}\right)\log\left(-x+1\right)-\left(\frac{7d}{36}-\frac{13e}{36}\right)\log\left(x+1\right)+\left(\frac{31d}{144}-\frac{25e}{72}\right)\log\left(x+2\right)-\frac{30d-36e+x\left(18d-24e\right)}{72\left(x^2+3x+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x+d)*(x**2-3*x+2)/(x**4-5*x**2+4)**2,x)
```

```
[Out] (d/144 + e/72)*log(-x + 2) - (d/36 + e/36)*log(-x + 1) - (7*d/36
- 13*e/36)*log(x + 1) + (31*d/144 - 25*e/72)*log(x + 2) - (30*d -
```

```
36*e + x*(18*d - 24*e))/(72*(x**2 + 3*x + 2))
```

Mathematica [A] time = 0.0891434, size = 80, normalized size = 0.9

$$\frac{1}{144}\left(\frac{12(-3dx-5d+4ex+6e)}{x^2+3x+2}-4(d+e)\log(1-x)+(d+2e)\log(2-x)+4(13e-7d)\log(x+1)+(31d-50e)\log(x+2)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[((d + e*x)*(2 - 3*x + x^2))/(4 - 5*x^2 + x^4)^2,x]
```

```
[Out] ((12*(-5*d + 6*e - 3*d*x + 4*e*x))/(2 + 3*x + x^2) - 4*(d + e)*Lo
g[1 - x] + (d + 2*e)*Log[2 - x] + 4*(-7*d + 13*e)*Log[1 + x] + (3
1*d - 50*e)*Log[2 + x])/144
```

Maple [A] time = 0.02, size = 90, normalized size = 1.

$$-\frac{d}{24+12x}+\frac{e}{12+6x}+\frac{31\ln(2+x)d}{144}-\frac{25\ln(2+x)e}{72}-\frac{\ln(-1+x)d}{36}-\frac{\ln(-1+x)e}{36}-\frac{7\ln(1+x)d}{36}+\frac{13\ln(1+x)e}{36}-\frac{d}{6+6x}+\frac{e}{6+6x}+\frac{\ln(x-2)d}{144}+\frac{\ln(x-2)e}{72}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x+d)*(x^2-3*x+2)/(x^4-5*x^2+4)^2,x)
```

```
[Out] -1/12/(2+x)*d+1/6/(2+x)*e+31/144*ln(2+x)*d-25/72*ln(2+x)*e-1/36*l
n(-1+x)*d-1/36*ln(-1+x)*e-7/36*ln(1+x)*d+13/36*ln(1+x)*e-1/6/(1+x
)*d+1/6/(1+x)*e+1/144*ln(x-2)*d+1/72*ln(x-2)*e
```

Maxima [A] time = 0.698071, size = 101, normalized size = 1.13

$$\frac{1}{144}(31d-50e)\log(x+2)-\frac{1}{36}(7d-13e)\log(x+1)-\frac{1}{36}(d+e)\log(x-1)+\frac{1}{144}(d+2e)\log(x-2)-\frac{(3d-4e)x+5d-6e}{12(x^2+3x+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x + d)*(x^2 - 3*x + 2)/(x^4 - 5*x^2 + 4)^2,x, algorithm="maxima")
```

```
[Out] 1/144*(31*d - 50*e)*log(x + 2) - 1/36*(7*d - 13*e)*log(x + 1) - 1
/36*(d + e)*log(x - 1) + 1/144*(d + 2*e)*log(x - 2) - 1/12*((3*d
- 4*e)*x + 5*d - 6*e)/(x^2 + 3*x + 2)
```

Fricas [A] time = 0.294017, size = 207, normalized size = 2.33

$$-\frac{12\,(3\,d-4\,e)x-\left((31\,d-50\,e)x^{2}+3\,(31\,d-50\,e)x+62\,d-100\,e\right)\,\log\left(x+2\right)+4\,\left((7\,d-13\,e)x^{2}+3\,(7\,d-13\,e)x+14\,d-\right.}{144\,(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x + d)*(x^2 - 3*x + 2)/(x^4 - 5*x^2 + 4)^2,x, algorithm="fricas")
```

```
[Out] -1/144*(12*(3*d - 4*e)*x - ((31*d - 50*e)*x^2 + 3*(31*d - 50*e)*x
 + 62*d - 100*e)*log(x + 2) + 4*((7*d - 13*e)*x^2 + 3*(7*d - 13*e
)*x + 14*d - 26*e)*log(x + 1) + 4*((d + e)*x^2 + 3*(d + e)*x + 2*
d + 2*e)*log(x - 1) - ((d + 2*e)*x^2 + 3*(d + 2*e)*x + 2*d + 4*e)
*log(x - 2) + 60*d - 72*e)/(x^2 + 3*x + 2)
```

Sympy [A] time = 17.0033, size = 1255, normalized size = 14.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x+d)*(x**2-3*x+2)/(x**4-5*x**2+4)**2,x)
```

```
[Out] -(d + e)*log(x + (-24383100*d**6 + 187408066*d**5*e + 10439775*d*
*5*(d + e) - 511591980*d**4*e**2 - 94132290*d**4*e*(d + e) + 6672
00*d**4*(d + e)**2 + 469491120*d**3*e**3 + 333672552*d**3*e**2*(d
 + e) - 2703328*d**3*e*(d + e)**2 - 198000*d**3*(d + e)**3 + 3227
78400*d**2*e**4 - 582497712*d**2*e**3*(d + e) + 1752768*d**2*e**2
*(d + e)**2 + 1107552*d**2*e*(d + e)**3 - 863493856*d*e**5 + 5007
76560*d*e**4*(d + e) + 4226944*d*e**3*(d + e)**2 - 1880640*d*e**2
*(d + e)**3 + 429000000*e**6 - 169242912*e**5*(d + e) - 4538112*e
**4*(d + e)**2 + 964224*e**3*(d + e)**3)/(13474125*d**6 - 1028601
75*d**5*e + 274190390*d**4*e**2 - 224142072*d**3*e**3 - 245084096
*d**2*e**4 + 535797456*d*e**5 - 256183200*e**6))/36 + (d + 2*e)*l
og(x + (-24383100*d**6 + 187408066*d**5*e - 10439775*d**5*(d + 2*
e)/4 - 511591980*d**4*e**2 + 47066145*d**4*e*(d + 2*e)/2 + 41700*
d**4*(d + 2*e)**2 + 469491120*d**3*e**3 - 83418138*d**3*e**2*(d +
 2*e) - 168958*d**3*e*(d + 2*e)**2 + 12375*d**3*(d + 2*e)**3/4 +
```

```
322778400*d**2*e**4 + 145624428*d**2*e**3*(d + 2*e) + 109548*d**2
*e**2*(d + 2*e)**2 - 34611*d**2*e*(d + 2*e)**3/2 - 863493856*d*e*
*5 - 125194140*d*e**4*(d + 2*e) + 264184*d*e**3*(d + 2*e)**2 + 29
385*d*e**2*(d + 2*e)**3 + 429000000*e**6 + 42310728*e**5*(d + 2*e
) - 283632*e**4*(d + 2*e)**2 - 15066*e**3*(d + 2*e)**3)/(13474125
*d**6 - 102860175*d**5*e + 274190390*d**4*e**2 - 224142072*d**3*e
**3 - 245084096*d**2*e**4 + 535797456*d*e**5 - 256183200*e**6))/1
44 - (7*d - 13*e)*log(x + (-24383100*d**6 + 187408066*d**5*e + 10
439775*d**5*(7*d - 13*e) - 511591980*d**4*e**2 - 94132290*d**4*e*
(7*d - 13*e) + 667200*d**4*(7*d - 13*e)**2 + 469491120*d**3*e**3
+ 333672552*d**3*e**2*(7*d - 13*e) - 2703328*d**3*e*(7*d - 13*e)*
*2 - 198000*d**3*(7*d - 13*e)**3 + 322778400*d**2*e**4 - 58249771
2*d**2*e**3*(7*d - 13*e) + 1752768*d**2*e**2*(7*d - 13*e)**2 + 11
07552*d**2*e*(7*d - 13*e)**3 - 863493856*d*e**5 + 500776560*d*e**
4*(7*d - 13*e) + 4226944*d*e**3*(7*d - 13*e)**2 - 1880640*d*e**2*
(7*d - 13*e)**3 + 429000000*e**6 - 169242912*e**5*(7*d - 13*e) -
4538112*e**4*(7*d - 13*e)**2 + 964224*e**3*(7*d - 13*e)**3)/(1347
4125*d**6 - 102860175*d**5*e + 274190390*d**4*e**2 - 224142072*d*
*3*e**3 - 245084096*d**2*e**4 + 535797456*d*e**5 - 256183200*e**6
))/36 + (31*d - 50*e)*log(x + (-24383100*d**6 + 187408066*d**5*e
- 10439775*d**5*(31*d - 50*e)/4 - 511591980*d**4*e**2 + 47066145*
d**4*e*(31*d - 50*e)/2 + 41700*d**4*(31*d - 50*e)**2 + 469491120*
d**3*e**3 - 83418138*d**3*e**2*(31*d - 50*e) - 168958*d**3*e*(31*
d - 50*e)**2 + 12375*d**3*(31*d - 50*e)**3/4 + 322778400*d**2*e**
4 + 145624428*d**2*e**3*(31*d - 50*e) + 109548*d**2*e**2*(31*d -
50*e)**2 - 34611*d**2*e*(31*d - 50*e)**3/2 - 863493856*d*e**5 - 1
25194140*d*e**4*(31*d - 50*e) + 264184*d*e**3*(31*d - 50*e)**2 +
29385*d*e**2*(31*d - 50*e)**3 + 429000000*e**6 + 42310728*e**5*(3
1*d - 50*e) - 283632*e**4*(31*d - 50*e)**2 - 15066*e**3*(31*d - 5
0*e)**3)/(13474125*d**6 - 102860175*d**5*e + 274190390*d**4*e**2
- 224142072*d**3*e**3 - 245084096*d**2*e**4 + 535797456*d*e**5 -
256183200*e**6))/144 - (5*d - 6*e + x*(3*d - 4*e))/(12*x**2 + 36*
x + 24)
```

GIAC/XCAS [A] time = 0.286233, size = 115, normalized size = 1.29

$$\frac{1}{144}(31\,d-50\,e)\ln\left(|x+2|\right)-\frac{1}{36}(7\,d-13\,e)\ln\left(|x+1|\right)-\frac{1}{36}(d+e)\ln\left(|x-1|\right)+\frac{1}{144}(d+2\,e)\ln\left(|x-2|\right)-\frac{(3\,d-4\,e)x+5\,d-6\,e}{12\,(x+2)(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + d)*(x^2 - 3*x + 2)/(x^4 - 5*x^2 + 4)^2,x, algorithm="giac")
```

```
[Out] 1/144*(31*d - 50*e)*ln(abs(x + 2)) - 1/36*(7*d - 13*e)*ln(abs(x +
 1)) - 1/36*(d + e)*ln(abs(x - 1)) + 1/144*(d + 2*e)*ln(abs(x - 2
)) - 1/12*((3*d - 4*e)*x + 5*d - 6*e)/((x + 2)*(x + 1))
```

3.93 $\int \frac{(2-3x+x^2)(d+ex+fx^2)}{(4-5x^2+x^4)^2}\,dx$

Optimal. Leaf size=105

$$-\frac{x(3d-4e+6f)+5d-6e+8f}{12(x^2+3x+2)}-\frac{1}{36}\log(1-x)(d+e+f)+\frac{1}{144}\log(2-x)(d+2e+4f)-\frac{1}{36}\log(x+1)(7d-13e+19f)+\frac{1}{144}\log(x+2)(31d-50e+76f)$$

```
[Out] -(5*d - 6*e + 8*f + (3*d - 4*e + 6*f)*x)/(12*(2 + 3*x + x^2)) - (
(d + e + f)*Log[1 - x])/36 + ((d + 2*e + 4*f)*Log[2 - x])/144 - (
(7*d - 13*e + 19*f)*Log[1 + x])/36 + ((31*d - 50*e + 76*f)*Log[2
+ x])/144
```

Rubi [A] time = 0.639148, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161

$$-\frac{x(3d-4e+6f)+5d-6e+8f}{12(x^2+3x+2)}-\frac{1}{36}\log(1-x)(d+e+f)+\frac{1}{144}\log(2-x)(d+2e+4f)-\frac{1}{36}\log(x+1)(7d-13e+19f)+\frac{1}{144}\log(x+2)(31d-50e+76f)$$

Antiderivative was successfully verified.

```
[In]  Int[((2 - 3*x + x^2)*(d + e*x + f*x^2))/(4 - 5*x^2 + x^4)^2,x]
```

```
[Out] -(5*d - 6*e + 8*f + (3*d - 4*e + 6*f)*x)/(12*(2 + 3*x + x^2)) - (
(d + e + f)*Log[1 - x])/36 + ((d + 2*e + 4*f)*Log[2 - x])/144 - (
(7*d - 13*e + 19*f)*Log[1 + x])/36 + ((31*d - 50*e + 76*f)*Log[2
+ x])/144
```

Rubi in Sympy [A] time = 63.1557, size = 102, normalized size = 0.97

$$\left(\frac{d}{144}+\frac{e}{72}+\frac{f}{36}\right)\log(-x+2)-\left(\frac{d}{36}+\frac{e}{36}+\frac{f}{36}\right)\log(-x+1)-\left(\frac{7d}{36}-\frac{13e}{36}+\frac{19f}{36}\right)\log(x+1)+\left(\frac{31d}{144}-\frac{25e}{72}+\frac{19f}{36}\right)\log(x+2)-\frac{30d-36e+48f+x(18d-24e+36f)}{72(x^2+3x+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**2-3*x+2)*(f*x**2+e*x+d)/(x**4-5*x**2+4)**2,x)
```

```
[Out] (d/144 + e/72 + f/36)*log(-x + 2) - (d/36 + e/36 + f/36)*log(-x +
 1) - (7*d/36 - 13*e/36 + 19*f/36)*log(x + 1) + (31*d/144 - 25*e/
72 + 19*f/36)*log(x + 2) - (30*d - 36*e + 48*f + x*(18*d - 24*e +
 36*f))/(72*(x**2 + 3*x + 2))
```

Mathematica [A] time = 0.144277, size = 97, normalized size = 0.92

$$\frac{1}{144}\left(-\frac{12(d(3x+5)-4ex-6e+6fx+8f)}{x^2+3x+2}-4\log(1-x)(d+e+f)\right.$$
$$\left.+\log(2-x)(d+2e+4f)-4\log(x+1)(7d-13e+19f)+\log(x+2)(31d-50e+76f)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[((2 - 3*x + x^2)*(d + e*x + f*x^2))/(4 - 5*x^2 + x^4)^2,x]
```

```
[Out] ((-12*(-6*e + 8*f - 4*e*x + 6*f*x + d*(5 + 3*x)))/(2 + 3*x + x^2)
 - 4*(d + e + f)*Log[1 - x] + (d + 2*e + 4*f)*Log[2 - x] - 4*(7*d
 - 13*e + 19*f)*Log[1 + x] + (31*d - 50*e + 76*f)*Log[2 + x])/144
```

Maple [A] time = 0.02, size = 134, normalized size = 1.3

$$-\frac{d}{24+12x}+\frac{e}{12+6x}-\frac{f}{6+3x}+\frac{31\ln(2+x)d}{144}-\frac{25\ln(2+x)e}{72}+\frac{19\ln(2+x)f}{36}$$
$$-\frac{\ln(-1+x)d}{36}-\frac{\ln(-1+x)e}{36}-\frac{\ln(-1+x)f}{36}-\frac{7\ln(1+x)d}{36}+\frac{13\ln(1+x)e}{36}$$
$$-\frac{19\ln(1+x)f}{36}-\frac{d}{6+6x}+\frac{e}{6+6x}-\frac{f}{6+6x}+\frac{\ln(x-2)d}{144}+\frac{\ln(x-2)e}{72}+\frac{\ln(x-2)f}{36}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^2-3*x+2)*(f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x)
```

```
[Out] -1/12/(2+x)*d+1/6/(2+x)*e-1/3/(2+x)*f+31/144*ln(2+x)*d-25/72*ln(2
+x)*e+19/36*ln(2+x)*f-1/36*ln(-1+x)*d-1/36*ln(-1+x)*e-1/36*ln(-1+
x)*f-7/36*ln(1+x)*d+13/36*ln(1+x)*e-19/36*ln(1+x)*f-1/6/(1+x)*d+1
/6/(1+x)*e-1/6/(1+x)*f+1/144*ln(x-2)*d+1/72*ln(x-2)*e+1/36*ln(x-2
)*f
```

Maxima [A] time = 0.696648, size = 123, normalized size = 1.17

$$\frac{1}{144}\,(31\,d - 50\,e + 76\,f)\log\left(x + 2\right) - \frac{1}{36}\,(7\,d - 13\,e + 19\,f)\log\left(x + 1\right) - \frac{1}{36}\,(d + e + f)\log\left(x - 1\right) + \frac{1}{144}\,(d + 2\,e + 4\,f)\log\left(x - 2\right) - \frac{(3\,d - 4\,e + 6\,f)x + 5\,d - 6\,e + 8\,f}{12\,(x^2 + 3\,x + 2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2 + e*x + d)*(x^2 - 3*x + 2)/(x^4 - 5*x^2 + 4)^2,x, algorithm="maxi
```

```
[Out] 1/144*(31*d - 50*e + 76*f)*log(x + 2) - 1/36*(7*d - 13*e + 19*f)*
log(x + 1) - 1/36*(d + e + f)*log(x - 1) + 1/144*(d + 2*e + 4*f)*
log(x - 2) - 1/12*((3*d - 4*e + 6*f)*x + 5*d - 6*e + 8*f)/(x^2 +
3*x + 2)
```

Fricas [A] time = 0.354541, size = 258, normalized size = 2.46

$$-\frac{12\,(3\,d - 4\,e + 6\,f)x - \left((31\,d - 50\,e + 76\,f)x^2 + 3\,(31\,d - 50\,e + 76\,f)x + 62\,d - 100\,e + 152\,f\right)\,\log\left(x + 2\right) + 4\,\left((7\,d - 13\,e\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2 + e*x + d)*(x^2 - 3*x + 2)/(x^4 - 5*x^2 + 4)^2,x, algorithm="fric
```

```
[Out] -1/144*(12*(3*d - 4*e + 6*f)*x - ((31*d - 50*e + 76*f)*x^2 + 3*(3
1*d - 50*e + 76*f)*x + 62*d - 100*e + 152*f)*log(x + 2) + 4*((7*d
 - 13*e + 19*f)*x^2 + 3*(7*d - 13*e + 19*f)*x + 14*d - 26*e + 38*
f)*log(x + 1) + 4*((d + e + f)*x^2 + 3*(d + e + f)*x + 2*d + 2*e
+ 2*f)*log(x - 1) - ((d + 2*e + 4*f)*x^2 + 3*(d + 2*e + 4*f)*x +
2*d + 4*e + 8*f)*log(x - 2) + 60*d - 72*e + 96*f)/(x^2 + 3*x + 2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**2-3*x+2)*(f*x**2+e*x+d)/(x**4-5*x**2+4)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.285846, size = 136, normalized size = 1.3

$$\frac{1}{144}\left(31\,d + 76\,f - 50\,e\right)\ln\left(|x+2|\right) - \frac{1}{36}\left(7\,d + 19\,f - 13\,e\right)\ln\left(|x+1|\right) - \frac{1}{36}\left(d + f + e\right)\ln\left(|x-1|\right) + \frac{1}{144}\left(d + 4\,f + 2\,e\right)\ln\left(|x-2|\right) - \frac{\left(3\,d + 6\,f - 4\,e\right)x + 5\,d + 8\,f - 6\,e}{12\left(x+2\right)\left(x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2 + e*x + d)*(x^2 - 3*x + 2)/(x^4 - 5*x^2 + 4)^2,x, algorithm="giac
```

```
[Out] 1/144*(31*d + 76*f - 50*e)*ln(abs(x + 2)) - 1/36*(7*d + 19*f - 13
*e)*ln(abs(x + 1)) - 1/36*(d + f + e)*ln(abs(x - 1)) + 1/144*(d +
 4*f + 2*e)*ln(abs(x - 2)) - 1/12*((3*d + 6*f - 4*e)*x + 5*d + 8*
f - 6*e)/((x + 2)*(x + 1))
```

3.94 $\int \frac{(2-3x+x^2)(d+ex+fx^2+gx^3)}{(4-5x^2+x^4)^2}\,dx$

Optimal. Leaf size=117

$$-\frac{d-2e+4f-8g}{12(x+2)}-\frac{d-e+f-g}{6(x+1)}-\frac{1}{36}\log(1-x)(d+e+f+g)+\frac{1}{144}\log(2-x)(d+2e+4f+8g)-\frac{1}{36}\log(x+1)(7d-13e+19f-25g)+\frac{1}{144}\log(x+2)(31d-50e+76f-104g)$$

```
[Out] -(d - e + f - g)/(6*(1 + x)) - (d - 2*e + 4*f - 8*g)/(12*(2 + x))
 - ((d + e + f + g)*Log[1 - x])/36 + ((d + 2*e + 4*f + 8*g)*Log[2
 - x])/144 - ((7*d - 13*e + 19*f - 25*g)*Log[1 + x])/36 + ((31*d
- 50*e + 76*f - 104*g)*Log[2 + x])/144
```

Rubi [A] time = 0.486432, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$

$$-\frac{d-2e+4f-8g}{12(x+2)}-\frac{d-e+f-g}{6(x+1)}-\frac{1}{36}\log(1-x)(d+e+f+g)+\frac{1}{144}\log(2-x)(d+2e+4f+8g)-\frac{1}{36}\log(x+1)(7d-13e+19f-25g)+\frac{1}{144}\log(x+2)(31d-50e+76f-104g)$$

Antiderivative was successfully verified.

```
[In] Int[((2 - 3*x + x^2)*(d + e*x + f*x^2 + g*x^3))/(4 - 5*x^2 + x^4)^2,x]
```

```
[Out] -(d - e + f - g)/(6*(1 + x)) - (d - 2*e + 4*f - 8*g)/(12*(2 + x))
 - ((d + e + f + g)*Log[1 - x])/36 + ((d + 2*e + 4*f + 8*g)*Log[2
 - x])/144 - ((7*d - 13*e + 19*f - 25*g)*Log[1 + x])/36 + ((31*d
- 50*e + 76*f - 104*g)*Log[2 + x])/144
```

Rubi in Sympy [A] time = 105.42, size = 122, normalized size = 1.04

$$\left(\frac{d}{144}+\frac{e}{72}+\frac{f}{36}+\frac{g}{18}\right)\log(-x+2)-\left(\frac{d}{36}+\frac{e}{36}+\frac{f}{36}+\frac{g}{36}\right)\log(-x+1)-\left(\frac{7d}{36}-\frac{13e}{36}+\frac{19f}{36}-\frac{25g}{36}\right)\log(x+1)+\left(\frac{31d}{144}-\frac{25e}{72}+\frac{19f}{36}-\frac{13g}{18}\right)\log(x+2)-\frac{\frac{d}{12}-\frac{e}{6}+\frac{f}{3}-\frac{2g}{3}}{x+2}-\frac{\frac{d}{6}-\frac{e}{6}+\frac{f}{6}-\frac{g}{6}}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((x**2-3*x+2)*(g*x**3+f*x**2+e*x+d)/(x**4-5*x**2+4)**2,x)
```

```
[Out] (d/144 + e/72 + f/36 + g/18)*log(-x + 2) - (d/36 + e/36 + f/36 +
g/36)*log(-x + 1) - (7*d/36 - 13*e/36 + 19*f/36 - 25*g/36)*log(x
+ 1) + (31*d/144 - 25*e/72 + 19*f/36 - 13*g/18)*log(x + 2) - (d/1
2 - e/6 + f/3 - 2*g/3)/(x + 2) - (d/6 - e/6 + f/6 - g/6)/(x + 1)
```

Mathematica [A] time = 0.113982, size = 114, normalized size = 0.97

$$\frac{1}{144}\left(\frac{12(-3dx-5d+4ex+6e-6fx-8f+10gx+12g)}{x^2+3x+2}-4\log(1-x)(d+e+f+g)\right.$$
$$\left.+\log(2-x)(d+2e+4f+8g)+4\log(x+1)(-7d+13e-19f+25g)+\log(x+2)(31d-50e+76f-104g)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 - 3*x + x^2)*(d + e*x + f*x^2 + g*x^3))/(4 - 5*x^2 + x^4)^2,x]
```

```
[Out] ((12*(-5*d + 6*e - 8*f + 12*g - 3*d*x + 4*e*x - 6*f*x + 10*g*x))/
(2 + 3*x + x^2) - 4*(d + e + f + g)*Log[1 - x] + (d + 2*e + 4*f +
 8*g)*Log[2 - x] + 4*(-7*d + 13*e - 19*f + 25*g)*Log[1 + x] + (31
*d - 50*e + 76*f - 104*g)*Log[2 + x])/144
```

Maple [A] time = 0.021, size = 178, normalized size = 1.5

$$-\frac{d}{24+12x}+\frac{e}{12+6x}-\frac{f}{6+3x}+\frac{2g}{6+3x}+\frac{31\ln(2+x)d}{144}-\frac{25\ln(2+x)e}{72}+\frac{19\ln(2+x)f}{36}$$
$$-\frac{13\ln(2+x)g}{18}-\frac{\ln(-1+x)d}{36}-\frac{\ln(-1+x)e}{36}-\frac{\ln(-1+x)f}{36}-\frac{\ln(-1+x)g}{36}$$
$$-\frac{7\ln(1+x)d}{36}+\frac{13\ln(1+x)e}{36}-\frac{19\ln(1+x)f}{36}+\frac{25\ln(1+x)g}{36}-\frac{d}{6+6x}$$
$$+\frac{e}{6+6x}-\frac{f}{6+6x}+\frac{g}{6+6x}+\frac{\ln(x-2)d}{144}+\frac{\ln(x-2)e}{72}+\frac{\ln(x-2)f}{36}+\frac{\ln(x-2)g}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^2-3*x+2)*(g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x)
```

```
[Out] -1/12/(2+x)*d+1/6/(2+x)*e-1/3/(2+x)*f+2/3/(2+x)*g+31/144*ln(2+x)*
d-25/72*ln(2+x)*e+19/36*ln(2+x)*f-13/18*ln(2+x)*g-1/36*ln(-1+x)*d
-1/36*ln(-1+x)*e-1/36*ln(-1+x)*f-1/36*ln(-1+x)*g-7/36*ln(1+x)*d+1
3/36*ln(1+x)*e-19/36*ln(1+x)*f+25/36*ln(1+x)*g-1/6/(1+x)*d+1/6/(1
+x)*e-1/6/(1+x)*f+1/6/(1+x)*g+1/144*ln(x-2)*d+1/72*ln(x-2)*e+1/36
```

```
*ln(x-2)*f+1/18*ln(x-2)*g
```

Maxima [A] time = 0.706376, size = 144, normalized size = 1.23

$$\frac{1}{144}\left(31\,d-50\,e+76\,f-104\,g\right)\log\left(x+2\right)-\frac{1}{36}\left(7\,d-13\,e+19\,f-25\,g\right)\log\left(x+1\right)-\frac{1}{36}\left(d+e+f+g\right)\log\left(x-1\right)+\frac{1}{144}\left(d+2\,e+4\,f+8\,g\right)\log\left(x-2\right)-\frac{(3\,d-4\,e+6\,f-10\,g)x+5\,d-6\,e+8\,f-12\,g}{12\left(x^2+3\,x+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3 + f*x^2 + e*x + d)*(x^2 - 3*x + 2)/(x^4 - 5*x^2 + 4)^2,x, algorith
```

```
[Out] 1/144*(31*d - 50*e + 76*f - 104*g)*log(x + 2) - 1/36*(7*d - 13*e
+ 19*f - 25*g)*log(x + 1) - 1/36*(d + e + f + g)*log(x - 1) + 1/1
44*(d + 2*e + 4*f + 8*g)*log(x - 2) - 1/12*((3*d - 4*e + 6*f - 10
*g)*x + 5*d - 6*e + 8*f - 12*g)/(x^2 + 3*x + 2)
```

Fricas [A] time = 0.898926, size = 309, normalized size = 2.64

$$-\frac{12\left(3\,d-4\,e+6\,f-10\,g\right)x-\left(\left(31\,d-50\,e+76\,f-104\,g\right)x^2+3\left(31\,d-50\,e+76\,f-104\,g\right)x+62\,d-100\,e+152\,f-208}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3 + f*x^2 + e*x + d)*(x^2 - 3*x + 2)/(x^4 - 5*x^2 + 4)^2,x, algorith
```

```
[Out] -1/144*(12*(3*d - 4*e + 6*f - 10*g)*x - ((31*d - 50*e + 76*f - 10
4*g)*x^2 + 3*(31*d - 50*e + 76*f - 104*g)*x + 62*d - 100*e + 152*
f - 208*g)*log(x + 2) + 4*((7*d - 13*e + 19*f - 25*g)*x^2 + 3*(7*
d - 13*e + 19*f - 25*g)*x + 14*d - 26*e + 38*f - 50*g)*log(x + 1)
 + 4*((d + e + f + g)*x^2 + 3*(d + e + f + g)*x + 2*d + 2*e + 2*f
 + 2*g)*log(x - 1) - ((d + 2*e + 4*f + 8*g)*x^2 + 3*(d + 2*e + 4*
f + 8*g)*x + 2*d + 4*e + 8*f + 16*g)*log(x - 2) + 60*d - 72*e + 9
6*f - 144*g)/(x^2 + 3*x + 2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**2-3*x+2)*(g*x**3+f*x**2+e*x+d)/(x**4-5*x**2+4)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.282431, size = 158, normalized size = 1.35

$$\frac{1}{144}\,(31\,d + 76\,f - 104\,g - 50\,e)\ln\left(|x + 2|\right) - \frac{1}{36}\,(7\,d + 19\,f - 25\,g - 13\,e)\ln\left(|x + 1|\right) - \frac{1}{36}\,(d + f + g + e)\ln\left(|x - 1|\right) + \frac{1}{144}\,(d + 4\,f + 8\,g + 2\,e)\ln\left(|x - 2|\right) - \frac{(3\,d + 6\,f - 10\,g - 4\,e)x + 5\,d + 8\,f - 12\,g - 6\,e}{12\,(x + 2)(x + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3 + f*x^2 + e*x + d)*(x^2 - 3*x + 2)/(x^4 - 5*x^2 + 4)^2,x, algorith
```

```
[Out] 1/144*(31*d + 76*f - 104*g - 50*e)*ln(abs(x + 2)) - 1/36*(7*d + 1
9*f - 25*g - 13*e)*ln(abs(x + 1)) - 1/36*(d + f + g + e)*ln(abs(x
 - 1)) + 1/144*(d + 4*f + 8*g + 2*e)*ln(abs(x - 2)) - 1/12*((3*d
+ 6*f - 10*g - 4*e)*x + 5*d + 8*f - 12*g - 6*e)/((x + 2)*(x + 1))
```

3.95 $\int \frac{(2-3x+x^2)(d+ex+fx^2+gx^3+hx^4)}{(4-5x^2+x^4)^2}\,dx$

Optimal. Leaf size=131

$$-\frac{d-e+f-g+h}{6(x+1)}-\frac{d-2e+4f-8g+16h}{12(x+2)}-\frac{1}{36}\log(1-x)(d+e+f+g+h)+\frac{1}{144}\log(2-x)(d+2e+4f+8g+16h)-\frac{1}{36}\log(x+1)(7d-13e+19f-25g+31h)+\frac{1}{144}\log(x+2)(31d-50e+76f-104g+112h)$$

```
[Out] -(d - e + f - g + h)/(6*(1 + x)) - (d - 2*e + 4*f - 8*g + 16*h)/(
12*(2 + x)) - ((d + e + f + g + h)*Log[1 - x])/36 + ((d + 2*e + 4
*f + 8*g + 16*h)*Log[2 - x])/144 - ((7*d - 13*e + 19*f - 25*g + 3
1*h)*Log[1 + x])/36 + ((31*d - 50*e + 76*f - 104*g + 112*h)*Log[2
 + x])/144
```

Rubi [A] time = 0.559654, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.049

$$-\frac{d-e+f-g+h}{6(x+1)}-\frac{d-2e+4f-8g+16h}{12(x+2)}-\frac{1}{36}\log(1-x)(d+e+f+g+h)+\frac{1}{144}\log(2-x)(d+2e+4f+8g+16h)-\frac{1}{36}\log(x+1)(7d-13e+19f-25g+31h)+\frac{1}{144}\log(x+2)(31d-50e+76f-104g+112h)$$

Antiderivative was successfully verified.

```
[In]  Int[((2 - 3*x + x^2)*(d + e*x + f*x^2 + g*x^3 + h*x^4))/(4 - 5*x^2 + x^4)^2,x]
```


```
[Out] -(d - e + f - g + h)/(6*(1 + x)) - (d - 2*e + 4*f - 8*g + 16*h)/(
12*(2 + x)) - ((d + e + f + g + h)*Log[1 - x])/36 + ((d + 2*e + 4
*f + 8*g + 16*h)*Log[2 - x])/144 - ((7*d - 13*e + 19*f - 25*g + 3
1*h)*Log[1 + x])/36 + ((31*d - 50*e + 76*f - 104*g + 112*h)*Log[2
 + x])/144
```

Rubi in Sympy [A] time = 137.068, size = 148, normalized size = 1.13

$$\left(\frac{d}{144}+\frac{e}{72}+\frac{f}{36}+\frac{g}{18}+\frac{h}{9}\right)\log(-x+2)-\left(\frac{d}{36}+\frac{e}{36}+\frac{f}{36}+\frac{g}{36}+\frac{h}{36}\right)\log(-x+1)$$
$$-\left(\frac{7d}{36}-\frac{13e}{36}+\frac{19f}{36}-\frac{25g}{36}+\frac{31h}{36}\right)\log(x+1)$$
$$+\left(\frac{31d}{144}-\frac{25e}{72}+\frac{19f}{36}-\frac{13g}{18}+\frac{7h}{9}\right)\log(x+2)-\frac{\frac{d}{12}-\frac{e}{6}+\frac{f}{3}-\frac{2g}{3}+\frac{4h}{3}}{x+2}-\frac{\frac{d}{6}-\frac{e}{6}+\frac{f}{6}-\frac{g}{6}+\frac{h}{6}}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((x**2-3*x+2)*(h*x**4+g*x**3+f*x**2+e*x+d)/(x**4-5*x**2+4)**2,x)
```

```
[Out] (d/144 + e/72 + f/36 + g/18 + h/9)*log(-x + 2) - (d/36 + e/36 + f
/36 + g/36 + h/36)*log(-x + 1) - (7*d/36 - 13*e/36 + 19*f/36 - 25
*g/36 + 31*h/36)*log(x + 1) + (31*d/144 - 25*e/72 + 19*f/36 - 13*
g/18 + 7*h/9)*log(x + 2) - (d/12 - e/6 + f/3 - 2*g/3 + 4*h/3)/(x
+ 2) - (d/6 - e/6 + f/6 - g/6 + h/6)/(x + 1)
```

Mathematica [A] time = 0.12316, size = 136, normalized size = 1.04

$$\frac{1}{144}\left(-\frac{12(d(3x+5)+2(-e(2x+3)+3fx+4f-5gx-6g+9hx+10h))}{x^2+3x+2}\right.$$
$$-4\log(1-x)(d+e+f+g+h)+\log(2-x)(d+2(e+2f+4g+8h))$$
$$\left.-4\log(x+1)(7d-13e+19f-25g+31h)+\log(x+2)(31d-50e+76f-104g+112h)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 - 3*x + x^2)*(d + e*x + f*x^2 + g*x^3 + h*x^4))/(4 - 5*x^2 + x^4)^2,x
```

```
[Out] ((-12*(d*(5 + 3*x) + 2*(4*f - 6*g + 10*h + 3*f*x - 5*g*x + 9*h*x
- e*(3 + 2*x))))/(2 + 3*x + x^2) - 4*(d + e + f + g + h)*Log[1 -
x] + (d + 2*(e + 2*f + 4*g + 8*h))*Log[2 - x] - 4*(7*d - 13*e + 1
9*f - 25*g + 31*h)*Log[1 + x] + (31*d - 50*e + 76*f - 104*g + 112
*h)*Log[2 + x])/144
```

Maple [A] time = 0.021, size = 222, normalized size = 1.7

$$-\frac{h}{6+6x}-\frac{4h}{6+3x}+\frac{g}{6+6x}+\frac{2g}{6+3x}-\frac{f}{6+6x}-\frac{d}{6+6x}+\frac{e}{6+6x}-\frac{d}{24+12x}+\frac{e}{12+6x}-\frac{f}{6+3x}-\frac{7\ln(1+x)d}{36}+\frac{13\ln(1+x)e}{36}-\frac{\ln(-1+x)d}{36}-\frac{\ln(-1+x)e}{36}+\frac{\ln(x-2)h}{9}-\frac{31\ln(1+x)h}{36}+\frac{7\ln(2+x)h}{9}-\frac{\ln(-1+x)h}{36}+\frac{25\ln(1+x)g}{36}+\frac{\ln(x-2)g}{18}-\frac{\ln(-1+x)g}{36}-\frac{13\ln(2+x)g}{18}+\frac{\ln(x-2)d}{144}+\frac{\ln(x-2)e}{72}-\frac{25\ln(2+x)e}{72}+\frac{\ln(x-2)f}{36}+\frac{31\ln(2+x)d}{144}-\frac{19\ln(1+x)f}{36}-\frac{\ln(-1+x)f}{36}+\frac{19\ln(2+x)f}{36}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int((x^2-3*x+2)*(h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x)
```

```
[Out] -1/6/(1+x)*h-4/3/(2+x)*h+1/6/(1+x)*g+2/3/(2+x)*g-1/6/(1+x)*f-1/6/
(1+x)*d+1/6/(1+x)*e-1/12/(2+x)*d+1/6/(2+x)*e-1/3/(2+x)*f-7/36*ln(
1+x)*d+13/36*ln(1+x)*e-1/36*ln(-1+x)*d-1/36*ln(-1+x)*e+1/9*ln(x-2
)*h-31/36*ln(1+x)*h+7/9*ln(2+x)*h-1/36*ln(-1+x)*h+25/36*ln(1+x)*g
+1/18*ln(x-2)*g-1/36*ln(-1+x)*g-13/18*ln(2+x)*g+1/144*ln(x-2)*d+1
/72*ln(x-2)*e-25/72*ln(2+x)*e+1/36*ln(x-2)*f+31/144*ln(2+x)*d-19/
36*ln(1+x)*f-1/36*ln(-1+x)*f+19/36*ln(2+x)*f
```

Maxima [A] time = 0.698622, size = 166, normalized size = 1.27

$$\frac{1}{144}(31d-50e+76f-104g+112h)\log(x+2)-\frac{1}{36}(7d-13e+19f-25g+31h)\log(x+1)-\frac{1}{36}(d+e+f+g+h)\log(x-1)+\frac{1}{144}(d+2e+4f+8g+16h)\log(x-2)-\frac{(3d-4e+6f-10g+18h)x+5d-6e+8f-12g+20h}{12(x^2+3x+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^4 + g*x^3 + f*x^2 + e*x + d)*(x^2 - 3*x + 2)/(x^4 - 5*x^2 + 4)^2,x, a
```

```
[Out] 1/144*(31*d - 50*e + 76*f - 104*g + 112*h)*log(x + 2) - 1/36*(7*d
 - 13*e + 19*f - 25*g + 31*h)*log(x + 1) - 1/36*(d + e + f + g +
h)*log(x - 1) + 1/144*(d + 2*e + 4*f + 8*g + 16*h)*log(x - 2) - 1
/12*((3*d - 4*e + 6*f - 10*g + 18*h)*x + 5*d - 6*e + 8*f - 12*g +
 20*h)/(x^2 + 3*x + 2)
```

Fricas [A] time = 4.62455, size = 360, normalized size = 2.75

$$-\frac{12\,(3\,d-4\,e+6\,f-10\,g+18\,h)x-\left((31\,d-50\,e+76\,f-104\,g+112\,h)x^2+3\,(31\,d-50\,e+76\,f-104\,g+112\,h)x+62\,d\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^4 + g*x^3 + f*x^2 + e*x + d)*(x^2 - 3*x + 2)/(x^4 - 5*x^2 + 4)^2,x, a
```

```
[Out] -1/144*(12*(3*d - 4*e + 6*f - 10*g + 18*h)*x - ((31*d - 50*e + 76
*f - 104*g + 112*h)*x^2 + 3*(31*d - 50*e + 76*f - 104*g + 112*h)*
x + 62*d - 100*e + 152*f - 208*g + 224*h)*log(x + 2) + 4*((7*d -
13*e + 19*f - 25*g + 31*h)*x^2 + 3*(7*d - 13*e + 19*f - 25*g + 31
*h)*x + 14*d - 26*e + 38*f - 50*g + 62*h)*log(x + 1) + 4*((d + e
+ f + g + h)*x^2 + 3*(d + e + f + g + h)*x + 2*d + 2*e + 2*f + 2*
g + 2*h)*log(x - 1) - ((d + 2*e + 4*f + 8*g + 16*h)*x^2 + 3*(d +
2*e + 4*f + 8*g + 16*h)*x + 2*d + 4*e + 8*f + 16*g + 32*h)*log(x
- 2) + 60*d - 72*e + 96*f - 144*g + 240*h)/(x^2 + 3*x + 2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-3*x+2)*(h*x**4+g*x**3+f*x**2+e*x+d)/(x**4-5*x**2+4)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.283873, size = 180, normalized size = 1.37

$$\frac{1}{144}\,(31\,d+76\,f-104\,g+112\,h-50\,e)\ln\left(|x+2|\right)-\frac{1}{36}\,(7\,d+19\,f-25\,g+31\,h-13\,e)\ln\left(|x+1|\right)-\frac{1}{36}\,(d+f+g+h+e)\ln\left(|x-1|\right)+\frac{1}{144}\,(d+4\,f+8\,g+16\,h+2\,e)\ln\left(|x-2|\right)-\frac{(3\,d+6\,f-10\,g+18\,h-4\,e)x+5\,d+8\,f-12\,g+20\,h-6\,e}{12\,(x+2)(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^4 + g*x^3 + f*x^2 + e*x + d)*(x^2 - 3*x + 2)/(x^4 - 5*x^2 + 4)^2,x, a
```

```
[Out] 1/144*(31*d + 76*f - 104*g + 112*h - 50*e)*ln(abs(x + 2)) - 1/36*
(7*d + 19*f - 25*g + 31*h - 13*e)*ln(abs(x + 1)) - 1/36*(d + f +
g + h + e)*ln(abs(x - 1)) + 1/144*(d + 4*f + 8*g + 16*h + 2*e)*ln
(abs(x - 2)) - 1/12*((3*d + 6*f - 10*g + 18*h - 4*e)*x + 5*d + 8*
f - 12*g + 20*h - 6*e)/((x + 2)*(x + 1))
```

3.96 $\int \frac{(2-3x+x^2)(d+ex+fx^2+gx^3+hx^4+ix^5)}{(4-5x^2+x^4)^2}\,dx$

Optimal. Leaf size=147

$$-\frac{d-2e+4f-8g+16h-32i}{12(x+2)}-\frac{d-e+f-g+h-i}{6(x+1)}-\frac{1}{36}\log(1-x)(d+e+f+g+h+i)+\frac{1}{144}\log(2-x)(d+2e+4f+8g+16h+32i)-\frac{1}{36}\log(x+1)(7d-13e+19f-25g+31h-37i)+\frac{1}{144}\log(x+2)(31d-50e+76f-104g+112h-32i)$$

```
[Out] -(d - e + f - g + h - i)/(6*(1 + x)) - (d - 2*e + 4*f - 8*g + 16*
h - 32*i)/(12*(2 + x)) - ((d + e + f + g + h + i)*Log[1 - x])/36
+ ((d + 2*e + 4*f + 8*g + 16*h + 32*i)*Log[2 - x])/144 - ((7*d -
13*e + 19*f - 25*g + 31*h - 37*i)*Log[1 + x])/36 + ((31*d - 50*e
+ 76*f - 104*g + 112*h - 32*i)*Log[2 + x])/144
```

Rubi [A] time = 0.637047, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043

$$-\frac{d-2e+4f-8g+16h-32i}{12(x+2)}-\frac{d-e+f-g+h-i}{6(x+1)}-\frac{1}{36}\log(1-x)(d+e+f+g+h+i)+\frac{1}{144}\log(2-x)(d+2e+4f+8g+16h+32i)-\frac{1}{36}\log(x+1)(7d-13e+19f-25g+31h-37i)+\frac{1}{144}\log(x+2)(31d-50e+76f-104g+112h-32i)$$

Antiderivative was successfully verified.

```
[In] Int[((2 - 3*x + x^2)*(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5))/(4 - 5*x^2 + x^4)^2,
```

```
[Out] -(d - e + f - g + h - i)/(6*(1 + x)) - (d - 2*e + 4*f - 8*g + 16*
h - 32*i)/(12*(2 + x)) - ((d + e + f + g + h + i)*Log[1 - x])/36
+ ((d + 2*e + 4*f + 8*g + 16*h + 32*i)*Log[2 - x])/144 - ((7*d -
13*e + 19*f - 25*g + 31*h - 37*i)*Log[1 + x])/36 + ((31*d - 50*e
+ 76*f - 104*g + 112*h - 32*i)*Log[2 + x])/144
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((x**2-3*x+2)*(i*x**5+h*x**4+g*x**3+f*x**2+e*x+d)/(x**4-
5*x**2+4)**2,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.195181, size = 153, normalized size = 1.04

$$\frac{1}{144}\left(\frac{12(2(e(2x+3)-3fx-4f+5gx+6g-9hx-10h+17ix+18i)-d(3x+5))}{x^2+3x+2} - 4\log(1-x)(d+e+f+g+h+i)+\log(2-x)(d+2e+4(f+2g+4h+8i)) + 4\log(x+1)(-7d+13e-19f+25g-31h+37i)+\log(x+2)(31d-50e+76f-104g+112h-32i)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 - 3*x + x^2)*(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5))/(4 - 5*x^2 + x
```

```
[Out] ((12*(-(d*(5 + 3*x)) + 2*(-4*f + 6*g - 10*h + 18*i - 3*f*x + 5*g*
x - 9*h*x + 17*i*x + e*(3 + 2*x))))/(2 + 3*x + x^2) - 4*(d + e +
f + g + h + i)*Log[1 - x] + (d + 2*e + 4*(f + 2*g + 4*h + 8*i))*L
og[2 - x] + 4*(-7*d + 13*e - 19*f + 25*g - 31*h + 37*i)*Log[1 + x
] + (31*d - 50*e + 76*f - 104*g + 112*h - 32*i)*Log[2 + x])/144
```

Maple [A] time = 0.022, size = 266, normalized size = 1.8

$$\frac{i}{6+6x}+\frac{8i}{6+3x}-\frac{h}{6+6x}-\frac{4h}{6+3x}+\frac{g}{6+6x}+\frac{2g}{6+3x}-\frac{f}{6+6x}-\frac{d}{6+6x}+\frac{e}{6+6x} -\frac{d}{24+12x}+\frac{e}{12+6x}-\frac{f}{6+3x}-\frac{7\ln(1+x)d}{36}+\frac{13\ln(1+x)e}{36}-\frac{\ln(-1+x)d}{36} -\frac{\ln(-1+x)e}{36}+\frac{2\ln(x-2)i}{9}+\frac{37\ln(1+x)i}{36}-\frac{\ln(-1+x)i}{36}-\frac{2\ln(2+x)i}{9} +\frac{\ln(x-2)h}{9}-\frac{31\ln(1+x)h}{36}+\frac{7\ln(2+x)h}{9}-\frac{\ln(-1+x)h}{36}+\frac{25\ln(1+x)g}{36} +\frac{\ln(x-2)g}{18}-\frac{\ln(-1+x)g}{36}-\frac{13\ln(2+x)g}{18}+\frac{\ln(x-2)d}{144}+\frac{\ln(x-2)e}{72}-\frac{25\ln(2+x)e}{72} +\frac{\ln(x-2)f}{36}+\frac{31\ln(2+x)d}{144}-\frac{19\ln(1+x)f}{36}-\frac{\ln(-1+x)f}{36}+\frac{19\ln(2+x)f}{36}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-3*x+2)*(i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x)
```

```
[Out] 1/6/(1+x)*i+8/3/(2+x)*i-1/6/(1+x)*h-4/3/(2+x)*h+1/6/(1+x)*g+2/3/(
2+x)*g-1/6/(1+x)*f-1/6/(1+x)*d+1/6/(1+x)*e-1/12/(2+x)*d+1/6/(2+x)
*e-1/3/(2+x)*f-7/36*ln(1+x)*d+13/36*ln(1+x)*e-1/36*ln(-1+x)*d-1/3
6*ln(-1+x)*e+2/9*ln(x-2)*i+37/36*ln(1+x)*i-1/36*ln(-1+x)*i-2/9*ln
(2+x)*i+1/9*ln(x-2)*h-31/36*ln(1+x)*h+7/9*ln(2+x)*h-1/36*ln(-1+x)
*h+25/36*ln(1+x)*g+1/18*ln(x-2)*g-1/36*ln(-1+x)*g-13/18*ln(2+x)*g
+1/144*ln(x-2)*d+1/72*ln(x-2)*e-25/72*ln(2+x)*e+1/36*ln(x-2)*f+31
/144*ln(2+x)*d-19/36*ln(1+x)*f-1/36*ln(-1+x)*f+19/36*ln(2+x)*f
```

Maxima [A] time = 0.708799, size = 188, normalized size = 1.28

$$\frac{1}{144}(31\,d-50\,e+76\,f-104\,g+112\,h-32\,i)\log(x+2)-\frac{1}{36}(7\,d-13\,e+19\,f-25\,g+31\,h-37\,i)\log(x+1)-\frac{1}{36}(d+e+f+g+h+i)\log(x-1)+\frac{1}{144}(d+2\,e+4\,f+8\,g+16\,h+32\,i)\log(x-2)-\frac{(3\,d-4\,e+6\,f-10\,g+18\,h-34\,i)x+5\,d-6\,e+8\,f-12\,g+20\,h-36\,i}{12\,(x^2+3\,x+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^5 + h*x^4 + g*x^3 + f*x^2 + e*x + d)*(x^2 - 3*x + 2)/(x^4 - 5*x^2 + 4
```

```
[Out] 1/144*(31*d - 50*e + 76*f - 104*g + 112*h - 32*i)*log(x + 2) - 1/
36*(7*d - 13*e + 19*f - 25*g + 31*h - 37*i)*log(x + 1) - 1/36*(d
+ e + f + g + h + i)*log(x - 1) + 1/144*(d + 2*e + 4*f + 8*g + 16
*h + 32*i)*log(x - 2) - 1/12*((3*d - 4*e + 6*f - 10*g + 18*h - 34
*i)*x + 5*d - 6*e + 8*f - 12*g + 20*h - 36*i)/(x^2 + 3*x + 2)
```

Fricas [A] time = 27.1214, size = 412, normalized size = 2.8

$$-\frac{12\,(3\,d-4\,e+6\,f-10\,g+18\,h-34\,i)x-\left((31\,d-50\,e+76\,f-104\,g+112\,h-32\,i)x^2+3\,(31\,d-50\,e+76\,f-104\,g+1\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^5 + h*x^4 + g*x^3 + f*x^2 + e*x + d)*(x^2 - 3*x + 2)/(x^4 - 5*x^2 + 4
```

```
[Out] -1/144*(12*(3*d - 4*e + 6*f - 10*g + 18*h - 34*i)*x - ((31*d - 50
*e + 76*f - 104*g + 112*h - 32*i)*x^2 + 3*(31*d - 50*e + 76*f - 1
04*g + 112*h - 32*i)*x + 62*d - 100*e + 152*f - 208*g + 224*h - 6
4*i)*log(x + 2) + 4*((7*d - 13*e + 19*f - 25*g + 31*h - 37*i)*x^2
 + 3*(7*d - 13*e + 19*f - 25*g + 31*h - 37*i)*x + 14*d - 26*e + 3
```

```
8*f - 50*g + 62*h - 74*i)*log(x + 1) + 4*((d + e + f + g + h + i)
*x^2 + 3*(d + e + f + g + h + i)*x + 2*d + 2*e + 2*f + 2*g + 2*h
+ 2*i)*log(x - 1) - ((d + 2*e + 4*f + 8*g + 16*h + 32*i)*x^2 + 3*
(d + 2*e + 4*f + 8*g + 16*h + 32*i)*x + 2*d + 4*e + 8*f + 16*g +
32*h + 64*i)*log(x - 2) + 60*d - 72*e + 96*f - 144*g + 240*h - 43
2*i)/(x^2 + 3*x + 2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**2-3*x+2)*(i*x**5+h*x**4+g*x**3+f*x**2+e*x+d)/(x**4-
5*x**2+4)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.28695, size = 201, normalized size = 1.37

$$\frac{1}{144}\left(31\,d+76\,f-104\,g+112\,h-32\,i-50\,e\right)\ln\left(|x+2|\right)-\frac{1}{36}\left(7\,d+19\,f-25\,g+31\,h-37\,i-13\,e\right)\ln\left(|x+1|\right)-\frac{1}{36}\left(d+f+g+h+i+e\right)\ln\left(|x-1|\right)+\frac{1}{144}\left(d+4\,f+8\,g+16\,h+32\,i+2\,e\right)\ln\left(|x-2|\right)-\frac{\left(3\,d+6\,f-10\,g+18\,h-34\,i-4\,e\right)x+5\,d+8\,f-12\,g+20\,h-36\,i-6\,e}{12\left(x+2\right)\left(x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^5 + h*x^4 + g*x^3 + f*x^2 + e*x + d)*(x^2 - 3*x + 2)/(x^4 - 5*x^2 + 4
```

```
[Out] 1/144*(31*d + 76*f - 104*g + 112*h - 32*i - 50*e)*ln(abs(x + 2))
- 1/36*(7*d + 19*f - 25*g + 31*h - 37*i - 13*e)*ln(abs(x + 1)) -
1/36*(d + f + g + h + i + e)*ln(abs(x - 1)) + 1/144*(d + 4*f + 8*
g + 16*h + 32*i + 2*e)*ln(abs(x - 2)) - 1/12*((3*d + 6*f - 10*g +
 18*h - 34*i - 4*e)*x + 5*d + 8*f - 12*g + 20*h - 36*i - 6*e)/((x
 + 2)*(x + 1))
```

3.97 $\int \frac{2+x}{(4-5x^2+x^4)^2}\,dx$

Optimal. Leaf size=68

$$\frac{1}{12(1-x)}+\frac{1}{36(2-x)}-\frac{1}{36(x+1)}+\frac{1}{18}\log(1-x)-\frac{35}{432}\log(2-x)+\frac{1}{54}\log(x+1)+\frac{1}{144}\log(x+2)$$

```
[Out] 1/(12*(1 - x)) + 1/(36*(2 - x)) - 1/(36*(1 + x)) + Log[1 - x]/18
- (35*Log[2 - x])/432 + Log[1 + x]/54 + Log[2 + x]/144
```

Rubi [A] time = 0.109365, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$\frac{1}{12(1-x)}+\frac{1}{36(2-x)}-\frac{1}{36(x+1)}+\frac{1}{18}\log(1-x)-\frac{35}{432}\log(2-x)+\frac{1}{54}\log(x+1)+\frac{1}{144}\log(x+2)$$

Antiderivative was successfully verified.

```
[In]  Int[(2 + x)/(4 - 5*x^2 + x^4)^2,x]
```

```
[Out] 1/(12*(1 - x)) + 1/(36*(2 - x)) - 1/(36*(1 + x)) + Log[1 - x]/18
- (35*Log[2 - x])/432 + Log[1 + x]/54 + Log[2 + x]/144
```

Rubi in Sympy [A] time = 21.859, size = 48, normalized size = 0.71

$$\frac{\log(-x+1)}{18}-\frac{35\log(-x+2)}{432}+\frac{\log(x+1)}{54}+\frac{\log(x+2)}{144}-\frac{1}{36(x+1)}+\frac{1}{36(-x+2)}+\frac{1}{12(-x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((2+x)/(x**4-5*x**2+4)**2,x)
```

```
[Out] log(-x + 1)/18 - 35*log(-x + 2)/432 + log(x + 1)/54 + log(x + 2)/
144 - 1/(36*(x + 1)) + 1/(36*(-x + 2)) + 1/(12*(-x + 1))
```

Mathematica [A] time = 0.0548921, size = 60, normalized size = 0.88

$$\frac{1}{432}\left(\frac{12\left(-5x^2+6x+5\right)}{x^3-2x^2-x+2}+24\log(1-x)-35\log(2-x)+8\log(x+1)+3\log(x+2)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(2 + x)/(4 - 5*x^2 + x^4)^2,x]
```

```
[Out] ((12*(5 + 6*x - 5*x^2))/(2 - x - 2*x^2 + x^3) + 24*Log[1 - x] - 3
5*Log[2 - x] + 8*Log[1 + x] + 3*Log[2 + x])/432
```

Maple [A] time = 0.021, size = 47, normalized size = 0.7

$$\frac{\ln(2+x)}{144} - \frac{1}{-12+12x} + \frac{\ln(-1+x)}{18} - \frac{1}{36+36x} + \frac{\ln(1+x)}{54} - \frac{1}{36x-72} - \frac{35\ \ln(x-2)}{432}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((2+x)/(x^4-5*x^2+4)^2,x)
```

```
[Out] 1/144*ln(2+x)-1/12/(-1+x)+1/18*ln(-1+x)-1/36/(1+x)+1/54*ln(1+x)-1
/36/(x-2)-35/432*ln(x-2)
```

Maxima [A] time = 0.706691, size = 70, normalized size = 1.03

$$-\frac{5x^2-6x-5}{36(x^3-2x^2-x+2)} + \frac{1}{144}\log(x+2) + \frac{1}{54}\log(x+1) + \frac{1}{18}\log(x-1) - \frac{35}{432}\log(x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + 2)/(x^4 - 5*x^2 + 4)^2,x, algorithm="maxima")
```

```
[Out] -1/36*(5*x^2 - 6*x - 5)/(x^3 - 2*x^2 - x + 2) + 1/144*log(x + 2)
+ 1/54*log(x + 1) + 1/18*log(x - 1) - 35/432*log(x - 2)
```

Fricas [A] time = 0.254751, size = 139, normalized size = 2.04

$$-\frac{60x^2 - 3\left(x^3-2x^2-x+2\right)\log(x+2) - 8\left(x^3-2x^2-x+2\right)\log(x+1) - 24\left(x^3-2x^2-x+2\right)\log(x-1) + 35\left(x^3 - \right.}{432(x^3-2x^2-x+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + 2)/(x^4 - 5*x^2 + 4)^2,x, algorithm="fricas")
```

```
[Out] -1/432*(60*x^2 - 3*(x^3 - 2*x^2 - x + 2)*log(x + 2) - 8*(x^3 - 2*
x^2 - x + 2)*log(x + 1) - 24*(x^3 - 2*x^2 - x + 2)*log(x - 1) + 3
5*(x^3 - 2*x^2 - x + 2)*log(x - 2) - 72*x - 60)/(x^3 - 2*x^2 - x
+ 2)
```

Sympy [A] time = 0.846249, size = 53, normalized size = 0.78

$$-\frac{5x^2-6x-5}{36x^3-72x^2-36x+72}-\frac{35\log(x-2)}{432}+\frac{\log(x-1)}{18}+\frac{\log(x+1)}{54}+\frac{\log(x+2)}{144}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2+x)/(x**4-5*x**2+4)**2,x)
```

```
[Out] -(5*x**2 - 6*x - 5)/(36*x**3 - 72*x**2 - 36*x + 72) - 35*log(x -
2)/432 + log(x - 1)/18 + log(x + 1)/54 + log(x + 2)/144
```

GIAC/XCAS [A] time = 0.287628, size = 76, normalized size = 1.12

$$-\frac{5\,x^2-6\,x-5}{36\,(x+1)(x-1)(x-2)}+\frac{1}{144}\ln(|x+2|)+\frac{1}{54}\ln(|x+1|)+\frac{1}{18}\ln(|x-1|)-\frac{35}{432}\ln(|x-2|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + 2)/(x^4 - 5*x^2 + 4)^2,x, algorithm="giac")
```

```
[Out] -1/36*(5*x^2 - 6*x - 5)/((x + 1)*(x - 1)*(x - 2)) + 1/144*ln(abs(
x + 2)) + 1/54*ln(abs(x + 1)) + 1/18*ln(abs(x - 1)) - 35/432*ln(a
bs(x - 2))
```

3.98 $\int \frac{(2+x)(d+ex)}{(4-5x^2+x^4)^2}\,dx$

Optimal. Leaf size=105

$$-\frac{d-e}{36(x+1)}+\frac{d+e}{12(1-x)}+\frac{d+2e}{36(2-x)}+\frac{1}{36}(2d+5e)\log(1-x)$$
$$-\frac{1}{432}(35d+58e)\log(2-x)+\frac{1}{108}(2d+e)\log(x+1)+\frac{1}{144}(d-2e)\log(x+2)$$

```
[Out] (d + e)/(12*(1 - x)) + (d + 2*e)/(36*(2 - x)) - (d - e)/(36*(1 +
x)) + ((2*d + 5*e)*Log[1 - x])/36 - ((35*d + 58*e)*Log[2 - x])/43
2 + ((2*d + e)*Log[1 + x])/108 + ((d - 2*e)*Log[2 + x])/144
```

Rubi [A] time = 0.386524, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095

$$-\frac{d-e}{36(x+1)}+\frac{d+e}{12(1-x)}+\frac{d+2e}{36(2-x)}+\frac{1}{36}(2d+5e)\log(1-x)$$
$$-\frac{1}{432}(35d+58e)\log(2-x)+\frac{1}{108}(2d+e)\log(x+1)+\frac{1}{144}(d-2e)\log(x+2)$$

Antiderivative was successfully verified.

```
[In]  Int[((2 + x)*(d + e*x))/(4 - 5*x^2 + x^4)^2,x]
```

```
[Out] (d + e)/(12*(1 - x)) + (d + 2*e)/(36*(2 - x)) - (d - e)/(36*(1 +
x)) + ((2*d + 5*e)*Log[1 - x])/36 - ((35*d + 58*e)*Log[2 - x])/43
2 + ((2*d + e)*Log[1 + x])/108 + ((d - 2*e)*Log[2 + x])/144
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((2+x)*(e*x+d)/(x**4-5*x**2+4)**2,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.165186, size = 97, normalized size = 0.92

$$\frac{1}{432}\left(\frac{12\left(d\left(-5x^2+6x+5\right)+2e\left(5-2x^2\right)\right)}{x^3-2x^2-x+2}+12(2d+5e)\log(1-x)-(35d+58e)\log(2-x)+4(2d+e)\log(x+1)+3(d-2e)\log(x+2)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[((2 + x)*(d + e*x))/(4 - 5*x^2 + x^4)^2,x]
```

```
[Out] ((12*(d*(5 + 6*x - 5*x^2) + 2*e*(5 - 2*x^2)))/(2 - x - 2*x^2 + x^
3) + 12*(2*d + 5*e)*Log[1 - x] - (35*d + 58*e)*Log[2 - x] + 4*(2*
d + e)*Log[1 + x] + 3*(d - 2*e)*Log[2 + x])/432
```

Maple [A] time = 0.022, size = 106, normalized size = 1.

$$\frac{\ln(2+x)\,d}{144}-\frac{\ln(2+x)\,e}{72}-\frac{d}{-12+12\,x}-\frac{e}{-12+12\,x}+\frac{\ln(-1+x)\,d}{18}+\frac{5\,\ln(-1+x)\,e}{36}-\frac{d}{36+36\,x}+\frac{e}{36+36\,x}+\frac{\ln(1+x)\,d}{54}+\frac{\ln(1+x)\,e}{108}-\frac{35\,\ln(x-2)\,d}{432}-\frac{29\,\ln(x-2)\,e}{216}-\frac{d}{36\,x-72}-\frac{e}{18\,x-36}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((2+x)*(e*x+d)/(x^4-5*x^2+4)^2,x)
```

```
[Out] 1/144*ln(2+x)*d-1/72*ln(2+x)*e-1/12/(-1+x)*d-1/12/(-1+x)*e+1/18*l
n(-1+x)*d+5/36*ln(-1+x)*e-1/36/(1+x)*d+1/36/(1+x)*e+1/54*ln(1+x)*
d+1/108*ln(1+x)*e-35/432*ln(x-2)*d-29/216*ln(x-2)*e-1/36/(x-2)*d-
1/18/(x-2)*e
```

Maxima [A] time = 0.700365, size = 119, normalized size = 1.13

$$\frac{1}{144}\,(d-2\,e)\log(x+2)+\frac{1}{108}\,(2\,d+e)\log(x+1)+\frac{1}{36}\,(2\,d+5\,e)\log(x-1)-\frac{1}{432}\,(35\,d+58\,e)\log(x-2)-\frac{(5\,d+4\,e)x^2-6\,dx-5\,d-10\,e}{36\,(x^3-2\,x^2-x+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + d)*(x + 2)/(x^4 - 5*x^2 + 4)^2,x, algorithm="maxima")
```

```
[Out] 1/144*(d - 2*e)*log(x + 2) + 1/108*(2*d + e)*log(x + 1) + 1/36*(2
*d + 5*e)*log(x - 1) - 1/432*(35*d + 58*e)*log(x - 2) - 1/36*((5*
d + 4*e)*x^2 - 6*d*x - 5*d - 10*e)/(x^3 - 2*x^2 - x + 2)
```

Fricas [A] time = 0.277462, size = 285, normalized size = 2.71

$$-\frac{12\,(5\,d+4\,e)x^2-72\,dx-3\,\big((d-2\,e)x^3-2\,(d-2\,e)x^2-(d-2\,e)x+2\,d-4\,e\big)\,\log\,(x+2)-4\,\big((2\,d+e)x^3-2\,(2\,d+e)x^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + d)*(x + 2)/(x^4 - 5*x^2 + 4)^2,x, algorithm="fricas")
```

```
[Out] -1/432*(12*(5*d + 4*e)*x^2 - 72*d*x - 3*((d - 2*e)*x^3 - 2*(d - 2
*e)*x^2 - (d - 2*e)*x + 2*d - 4*e)*log(x + 2) - 4*((2*d + e)*x^3
- 2*(2*d + e)*x^2 - (2*d + e)*x + 4*d + 2*e)*log(x + 1) - 12*((2*
d + 5*e)*x^3 - 2*(2*d + 5*e)*x^2 - (2*d + 5*e)*x + 4*d + 10*e)*lo
g(x - 1) + ((35*d + 58*e)*x^3 - 2*(35*d + 58*e)*x^2 - (35*d + 58*
e)*x + 70*d + 116*e)*log(x - 2) - 60*d - 120*e)/(x^3 - 2*x^2 - x
+ 2)
```

Sympy [A] time = 16.4521, size = 1032, normalized size = 9.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)*(e*x+d)/(x**4-5*x**2+4)**2,x)
```

```
[Out] (d - 2*e)*log(x + (8710660*d**5 + 91884504*d**4*e - 7579779*d**4*
(d - 2*e)/4 + 364910432*d**3*e**2 - 18128055*d**3*e*(d - 2*e) - 8
3772*d**3*(d - 2*e)**2 + 686697536*d**2*e**3 - 60296868*d**2*e**2
*(d - 2*e) - 597816*d**2*e*(d - 2*e)**2 + 65907*d**2*(d - 2*e)**3
/4 + 614357568*d*e**4 - 85949220*d*e**3*(d - 2*e) - 1500048*d*e**
2*(d - 2*e)**2 + 105840*d*e*(d - 2*e)**3 + 208470400*e**5 - 45136
356*e**4*(d - 2*e) - 1196064*e**3*(d - 2*e)**2 + 128277*e**2*(d -
 2*e)**3)/(3374210*d**5 + 38645295*d**4*e + 170558380*d**3*e**2 +
 362061760*d**2*e**3 + 370298160*d*e**4 + 146466320*e**5))/144 +
(2*d + e)*log(x + (8710660*d**5 + 91884504*d**4*e - 2526593*d**4*
(2*d + e) + 364910432*d**3*e**2 - 24170740*d**3*e*(2*d + e) - 148
928*d**3*(2*d + e)**2 + 686697536*d**2*e**3 - 80395824*d**2*e**2*
(2*d + e) - 1062784*d**2*e*(2*d + e)**2 + 39056*d**2*(2*d + e)**3
 + 614357568*d*e**4 - 114598960*d*e**3*(2*d + e) - 2666752*d*e**2
*(2*d + e)**2 + 250880*d*e*(2*d + e)**3 + 208470400*e**5 - 601818
```

```
08*e**4*(2*d + e) - 2126336*e**3*(2*d + e)**2 + 304064*e**2*(2*d
+ e)**3)/(3374210*d**5 + 38645295*d**4*e + 170558380*d**3*e**2 +
362061760*d**2*e**3 + 370298160*d*e**4 + 146466320*e**5))/108 + (
2*d + 5*e)*log(x + (8710660*d**5 + 91884504*d**4*e - 7579779*d**4
*(2*d + 5*e) + 364910432*d**3*e**2 - 72512220*d**3*e*(2*d + 5*e)
- 1340352*d**3*(2*d + 5*e)**2 + 686697536*d**2*e**3 - 241187472*d
**2*e**2*(2*d + 5*e) - 9565056*d**2*e*(2*d + 5*e)**2 + 1054512*d*
*2*(2*d + 5*e)**3 + 614357568*d*e**4 - 343796880*d*e**3*(2*d + 5*
e) - 24000768*d*e**2*(2*d + 5*e)**2 + 6773760*d*e*(2*d + 5*e)**3
+ 208470400*e**5 - 180545424*e**4*(2*d + 5*e) - 19137024*e**3*(2*
d + 5*e)**2 + 8209728*e**2*(2*d + 5*e)**3)/(3374210*d**5 + 386452
95*d**4*e + 170558380*d**3*e**2 + 362061760*d**2*e**3 + 370298160
*d*e**4 + 146466320*e**5))/36 - (35*d + 58*e)*log(x + (8710660*d*
*5 + 91884504*d**4*e + 2526593*d**4*(35*d + 58*e)/4 + 364910432*d
**3*e**2 + 6042685*d**3*e*(35*d + 58*e) - 9308*d**3*(35*d + 58*e)
**2 + 686697536*d**2*e**3 + 20098956*d**2*e**2*(35*d + 58*e) - 66
424*d**2*e*(35*d + 58*e)**2 - 2441*d**2*(35*d + 58*e)**3/4 + 6143
57568*d*e**4 + 28649740*d*e**3*(35*d + 58*e) - 166672*d*e**2*(35*
d + 58*e)**2 - 3920*d*e*(35*d + 58*e)**3 + 208470400*e**5 + 15045
452*e**4*(35*d + 58*e) - 132896*e**3*(35*d + 58*e)**2 - 4751*e**2
*(35*d + 58*e)**3)/(3374210*d**5 + 38645295*d**4*e + 170558380*d*
*3*e**2 + 362061760*d**2*e**3 + 370298160*d*e**4 + 146466320*e**5
))/432 - (-6*d*x - 5*d - 10*e + x**2*(5*d + 4*e))/(36*x**3 - 72*x
**2 - 36*x + 72)
```

GIAC/XCAS [A] time = 0.286671, size = 132, normalized size = 1.26

$$\frac{1}{144}(d-2e)\ln(|x+2|)+\frac{1}{108}(2d+e)\ln(|x+1|)+\frac{1}{36}(2d+5e)\ln(|x-1|)-\frac{1}{432}(35d+58e)\ln(|x-2|)-\frac{(5d+4e)x^2-6dx-5d-10e}{36(x+1)(x-1)(x-2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x + d)*(x + 2)/(x^4 - 5*x^2 + 4)^2,x, algorithm="giac")
```

```
[Out] 1/144*(d - 2*e)*ln(abs(x + 2)) + 1/108*(2*d + e)*ln(abs(x + 1)) +
 1/36*(2*d + 5*e)*ln(abs(x - 1)) - 1/432*(35*d + 58*e)*ln(abs(x -
 2)) - 1/36*((5*d + 4*e)*x^2 - 6*d*x - 5*d - 10*e)/((x + 1)*(x -
1)*(x - 2))
```

3.99 $\int \frac{(2+x)(d+ex+fx^2)}{(4-5x^2+x^4)^2}\,dx$

Optimal. Leaf size=122

$$-\frac{d-e+f}{36(x+1)}+\frac{d+e+f}{12(1-x)}+\frac{d+2e+4f}{36(2-x)}+\frac{1}{36}\log(1-x)(2d+5e+8f)$$
$$-\frac{1}{432}\log(2-x)(35d+58e+92f)+\frac{1}{108}\log(x+1)(2d+e-4f)+\frac{1}{144}\log(x+2)(d-2e+4f)$$

```
[Out] (d + e + f)/(12*(1 - x)) + (d + 2*e + 4*f)/(36*(2 - x)) - (d - e
+ f)/(36*(1 + x)) + ((2*d + 5*e + 8*f)*Log[1 - x])/36 - ((35*d +
58*e + 92*f)*Log[2 - x])/432 + ((2*d + e - 4*f)*Log[1 + x])/108 +
 ((d - 2*e + 4*f)*Log[2 + x])/144
```

Rubi [A] time = 0.439293, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$

$$-\frac{d-e+f}{36(x+1)}+\frac{d+e+f}{12(1-x)}+\frac{d+2e+4f}{36(2-x)}+\frac{1}{36}\log(1-x)(2d+5e+8f)$$
$$-\frac{1}{432}\log(2-x)(35d+58e+92f)+\frac{1}{108}\log(x+1)(2d+e-4f)+\frac{1}{144}\log(x+2)(d-2e+4f)$$

Antiderivative was successfully verified.

```
[In]  Int[((2 + x)*(d + e*x + f*x^2))/(4 - 5*x^2 + x^4)^2,x]
```

```
[Out] (d + e + f)/(12*(1 - x)) + (d + 2*e + 4*f)/(36*(2 - x)) - (d - e
+ f)/(36*(1 + x)) + ((2*d + 5*e + 8*f)*Log[1 - x])/36 - ((35*d +
58*e + 92*f)*Log[2 - x])/432 + ((2*d + e - 4*f)*Log[1 + x])/108 +
 ((d - 2*e + 4*f)*Log[2 + x])/144
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((2+x)*(f*x**2+e*x+d)/(x**4-5*x**2+4)**2,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.0951911, size = 121, normalized size = 0.99

$$\frac{1}{432}\left(\frac{12\left(d\left(-5x^2+6x+5\right)+e\left(10-4x^2\right)+2f\left(-4x^2+3x+4\right)\right)}{x^3-2x^2-x+2}+12\log(1-x)(2d+5e+8f)\right.$$
$$\left.-\log(2-x)(35d+58e+92f)+4\log(x+1)(2d+e-4f)+3\log(x+2)(d-2e+4f)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[((2 + x)*(d + e*x + f*x^2))/(4 - 5*x^2 + x^4)^2,x]
```

```
[Out] ((12*(d*(5 + 6*x - 5*x^2) + e*(10 - 4*x^2) + 2*f*(4 + 3*x - 4*x^2
)))/(2 - x - 2*x^2 + x^3) + 12*(2*d + 5*e + 8*f)*Log[1 - x] - (35
*d + 58*e + 92*f)*Log[2 - x] + 4*(2*d + e - 4*f)*Log[1 + x] + 3*(
d - 2*e + 4*f)*Log[2 + x])/432
```

Maple [A] time = 0.022, size = 158, normalized size = 1.3

$$\frac{\ln(2+x)\,d}{144}-\frac{\ln(2+x)\,e}{72}+\frac{\ln(2+x)\,f}{36}-\frac{d}{-12+12\,x}-\frac{e}{-12+12\,x}-\frac{f}{-12+12\,x}+\frac{\ln(-1+x)\,d}{18}$$
$$+\frac{5\,\ln(-1+x)\,e}{36}+\frac{2\,\ln(-1+x)\,f}{9}-\frac{d}{36+36\,x}+\frac{e}{36+36\,x}-\frac{f}{36+36\,x}+\frac{\ln(1+x)\,d}{54}+\frac{\ln(1+x)\,e}{108}$$
$$-\frac{\ln(1+x)\,f}{27}-\frac{35\,\ln(x-2)\,d}{432}-\frac{29\,\ln(x-2)\,e}{216}-\frac{23\,\ln(x-2)\,f}{108}-\frac{d}{36\,x-72}-\frac{e}{18\,x-36}-\frac{f}{9\,x-18}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((2+x)*(f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x)
```

```
[Out] 1/144*ln(2+x)*d-1/72*ln(2+x)*e+1/36*ln(2+x)*f-1/12/(-1+x)*d-1/12/
(-1+x)*e-1/12/(-1+x)*f+1/18*ln(-1+x)*d+5/36*ln(-1+x)*e+2/9*ln(-1+
x)*f-1/36/(1+x)*d+1/36/(1+x)*e-1/36/(1+x)*f+1/54*ln(1+x)*d+1/108*
ln(1+x)*e-1/27*ln(1+x)*f-35/432*ln(x-2)*d-29/216*ln(x-2)*e-23/108
*ln(x-2)*f-1/36/(x-2)*d-1/18/(x-2)*e-1/9/(x-2)*f
```

Maxima [A] time = 0.705089, size = 146, normalized size = 1.2

$$\frac{1}{144}\,(d-2\,e+4\,f)\log(x+2)+\frac{1}{108}\,(2\,d+e-4\,f)\log(x+1)+\frac{1}{36}\,(2\,d+5\,e+8\,f)\log(x-1)$$
$$-\frac{1}{432}\,(35\,d+58\,e+92\,f)\log(x-2)-\frac{(5\,d+4\,e+8\,f)x^2-6\,(d+f)x-5\,d-10\,e-8\,f}{36\,(x^3-2\,x^2-x+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^2 + e*x + d)*(x + 2)/(x^4 - 5*x^2 + 4)^2,x, algorithm="maxima")
```

```
[Out] 1/144*(d - 2*e + 4*f)*log(x + 2) + 1/108*(2*d + e - 4*f)*log(x +
1) + 1/36*(2*d + 5*e + 8*f)*log(x - 1) - 1/432*(35*d + 58*e + 92*
f)*log(x - 2) - 1/36*((5*d + 4*e + 8*f)*x^2 - 6*(d + f)*x - 5*d -
 10*e - 8*f)/(x^3 - 2*x^2 - x + 2)
```

Fricas [A] time = 0.367125, size = 360, normalized size = 2.95

$$-\frac{12\,(5\,d+4\,e+8\,f)x^2-72\,(d+f)x-3\left((d-2\,e+4\,f)x^3-2\,(d-2\,e+4\,f)x^2-(d-2\,e+4\,f)x+2\,d-4\,e+8\,f\right)\log\left(x+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^2 + e*x + d)*(x + 2)/(x^4 - 5*x^2 + 4)^2,x, algorithm="fricas")
```

```
[Out] -1/432*(12*(5*d + 4*e + 8*f)*x^2 - 72*(d + f)*x - 3*((d - 2*e + 4
*f)*x^3 - 2*(d - 2*e + 4*f)*x^2 - (d - 2*e + 4*f)*x + 2*d - 4*e +
 8*f)*log(x + 2) - 4*((2*d + e - 4*f)*x^3 - 2*(2*d + e - 4*f)*x^2
 - (2*d + e - 4*f)*x + 4*d + 2*e - 8*f)*log(x + 1) - 12*((2*d + 5
*e + 8*f)*x^3 - 2*(2*d + 5*e + 8*f)*x^2 - (2*d + 5*e + 8*f)*x + 4
*d + 10*e + 16*f)*log(x - 1) + ((35*d + 58*e + 92*f)*x^3 - 2*(35*
d + 58*e + 92*f)*x^2 - (35*d + 58*e + 92*f)*x + 70*d + 116*e + 18
4*f)*log(x - 2) - 60*d - 120*e - 96*f)/(x^3 - 2*x^2 - x + 2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2+x)*(f*x**2+e*x+d)/(x**4-5*x**2+4)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.286379, size = 159, normalized size = 1.3

$$\frac{1}{144}(d+4f-2e)\ln(|x+2|)+\frac{1}{108}(2d-4f+e)\ln(|x+1|)+\frac{1}{36}(2d+8f+5e)\ln(|x-1|)-\frac{1}{432}(35d+92f+58e)\ln(|x-2|)-\frac{(5d+8f+4e)x^2-6(d+f)x-5d-8f-10e}{36(x+1)(x-1)(x-2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2 + e*x + d)*(x + 2)/(x^4 - 5*x^2 + 4)^2,x, algorithm="giac")
```

```
[Out] 1/144*(d + 4*f - 2*e)*ln(abs(x + 2)) + 1/108*(2*d - 4*f + e)*ln(a
bs(x + 1)) + 1/36*(2*d + 8*f + 5*e)*ln(abs(x - 1)) - 1/432*(35*d
+ 92*f + 58*e)*ln(abs(x - 2)) - 1/36*((5*d + 8*f + 4*e)*x^2 - 6*(
d + f)*x - 5*d - 8*f - 10*e)/((x + 1)*(x - 1)*(x - 2))
```

3.100 $\int \frac{(2+x)(d+ex+fx^2+gx^3)}{(4-5x^2+x^4)^2}\,dx$

Optimal. Leaf size=141

$$-\frac{d-e+f-g}{36(x+1)}+\frac{d+e+f+g}{12(1-x)}+\frac{d+2e+4f+8g}{36(2-x)}+\frac{1}{36}\log(1-x)(2d+5e+8f+11g)$$
$$-\frac{1}{432}\log(2-x)(35d+58e+92f+136g)+\frac{1}{108}\log(x+1)(2d+e-4f+7g)+\frac{1}{144}\log(x+2)(d-2e+4f-8g)$$

```
[Out] (d + e + f + g)/(12*(1 - x)) + (d + 2*e + 4*f + 8*g)/(36*(2 - x))
 - (d - e + f - g)/(36*(1 + x)) + ((2*d + 5*e + 8*f + 11*g)*Log[1
 - x])/36 - ((35*d + 58*e + 92*f + 136*g)*Log[2 - x])/432 + ((2*d
 + e - 4*f + 7*g)*Log[1 + x])/108 + ((d - 2*e + 4*f - 8*g)*Log[2
+ x])/144
```

Rubi [A] time = 0.508023, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065

$$-\frac{d-e+f-g}{36(x+1)}+\frac{d+e+f+g}{12(1-x)}+\frac{d+2e+4f+8g}{36(2-x)}+\frac{1}{36}\log(1-x)(2d+5e+8f+11g)$$
$$-\frac{1}{432}\log(2-x)(35d+58e+92f+136g)+\frac{1}{108}\log(x+1)(2d+e-4f+7g)+\frac{1}{144}\log(x+2)(d-2e+4f-8g)$$

Antiderivative was successfully verified.

```
[In]  Int[((2 + x)*(d + e*x + f*x^2 + g*x^3))/(4 - 5*x^2 + x^4)^2,x]
```

```
[Out] (d + e + f + g)/(12*(1 - x)) + (d + 2*e + 4*f + 8*g)/(36*(2 - x))
 - (d - e + f - g)/(36*(1 + x)) + ((2*d + 5*e + 8*f + 11*g)*Log[1
 - x])/36 - ((35*d + 58*e + 92*f + 136*g)*Log[2 - x])/432 + ((2*d
 + e - 4*f + 7*g)*Log[1 + x])/108 + ((d - 2*e + 4*f - 8*g)*Log[2
+ x])/144
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((2+x)*(g*x**3+f*x**2+e*x+d)/(x**4-5*x**2+4)**2,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.150077, size = 144, normalized size = 1.02

$$\frac{1}{432}\left(\frac{12\left(d\left(-5x^2+6x+5\right)+2\left(e\left(5-2x^2\right)+f\left(-4x^2+3x+4\right)+g\left(8-5x^2\right)\right)\right)}{x^3-2x^2-x+2}\right.$$
$$+12\log(1-x)(2d+5e+8f+11g)-\log(2-x)(35d+58e+92f+136g)$$
$$\left.+4\log(x+1)(2d+e-4f+7g)+3\log(x+2)(d-2e+4f-8g)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[((2 + x)*(d + e*x + f*x^2 + g*x^3))/(4 - 5*x^2 + x^4)^2,x]
```

```
[Out] ((12*(d*(5 + 6*x - 5*x^2) + 2*(g*(8 - 5*x^2) + f*(4 + 3*x - 4*x^2
) + e*(5 - 2*x^2))))/(2 - x - 2*x^2 + x^3) + 12*(2*d + 5*e + 8*f
+ 11*g)*Log[1 - x] - (35*d + 58*e + 92*f + 136*g)*Log[2 - x] + 4*
(2*d + e - 4*f + 7*g)*Log[1 + x] + 3*(d - 2*e + 4*f - 8*g)*Log[2
+ x])/432
```

Maple [A] time = 0.024, size = 210, normalized size = 1.5

$$\frac{g}{36+36x}-\frac{g}{-12+12x}-\frac{2g}{9x-18}-\frac{f}{36+36x}-\frac{d}{36+36x}+\frac{e}{36+36x}-\frac{d}{36x-72}$$
$$-\frac{e}{18x-36}-\frac{f}{9x-18}-\frac{f}{-12+12x}-\frac{d}{-12+12x}-\frac{e}{-12+12x}+\frac{\ln(1+x)d}{54}$$
$$+\frac{\ln(1+x)e}{108}+\frac{\ln(-1+x)d}{18}+\frac{5\ln(-1+x)e}{36}+\frac{7\ln(1+x)g}{108}-\frac{17\ln(x-2)g}{54}$$
$$+\frac{11\ln(-1+x)g}{36}-\frac{\ln(2+x)g}{18}-\frac{35\ln(x-2)d}{432}-\frac{29\ln(x-2)e}{216}-\frac{\ln(2+x)e}{72}$$
$$-\frac{23\ln(x-2)f}{108}+\frac{\ln(2+x)d}{144}-\frac{\ln(1+x)f}{27}+\frac{2\ln(-1+x)f}{9}+\frac{\ln(2+x)f}{36}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((2+x)*(g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x)
```

```
[Out] 1/36/(1+x)*g-1/12/(-1+x)*g-2/9/(x-2)*g-1/36/(1+x)*f-1/36/(1+x)*d+
1/36/(1+x)*e-1/36/(x-2)*d-1/18/(x-2)*e-1/9/(x-2)*f-1/12/(-1+x)*f-
1/12/(-1+x)*d-1/12/(-1+x)*e+1/54*ln(1+x)*d+1/108*ln(1+x)*e+1/18*l
n(-1+x)*d+5/36*ln(-1+x)*e+7/108*ln(1+x)*g-17/54*ln(x-2)*g+11/36*l
n(-1+x)*g-1/18*ln(2+x)*g-35/432*ln(x-2)*d-29/216*ln(x-2)*e-1/72*l
n(2+x)*e-23/108*ln(x-2)*f+1/144*ln(2+x)*d-1/27*ln(1+x)*f+2/9*ln(-
```

```
1+x)*f+1/36*ln(2+x)*f
```

Maxima [A] time = 0.715162, size = 170, normalized size = 1.21

$$\frac{1}{144}\,(d-2\,e+4\,f-8\,g)\log\left(x+2\right)+\frac{1}{108}\,(2\,d+e-4\,f+7\,g)\log\left(x+1\right)+\frac{1}{36}\,(2\,d+5\,e+8\,f+11\,g)\log\left(x-1\right)-\frac{1}{432}\,(35\,d+58\,e+92\,f+136\,g)\log\left(x-2\right)-\frac{(5\,d+4\,e+8\,f+10\,g)x^2-6\,(d+f)x-5\,d-10\,e-8\,f-16\,g}{36\,(x^3-2\,x^2-x+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3 + f*x^2 + e*x + d)*(x + 2)/(x^4 - 5*x^2 + 4)^2,x, algorithm="maxi
```

```
[Out] 1/144*(d - 2*e + 4*f - 8*g)*log(x + 2) + 1/108*(2*d + e - 4*f + 7
*g)*log(x + 1) + 1/36*(2*d + 5*e + 8*f + 11*g)*log(x - 1) - 1/432
*(35*d + 58*e + 92*f + 136*g)*log(x - 2) - 1/36*((5*d + 4*e + 8*f
 + 10*g)*x^2 - 6*(d + f)*x - 5*d - 10*e - 8*f - 16*g)/(x^3 - 2*x^
2 - x + 2)
```

Fricas [A] time = 0.913155, size = 433, normalized size = 3.07

$$-\frac{12\,(5\,d+4\,e+8\,f+10\,g)x^2-72\,(d+f)x-3\,\left((d-2\,e+4\,f-8\,g)x^3-2\,(d-2\,e+4\,f-8\,g)x^2-(d-2\,e+4\,f-8\,g)x+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3 + f*x^2 + e*x + d)*(x + 2)/(x^4 - 5*x^2 + 4)^2,x, algorithm="fric
```

```
[Out] -1/432*(12*(5*d + 4*e + 8*f + 10*g)*x^2 - 72*(d + f)*x - 3*((d -
2*e + 4*f - 8*g)*x^3 - 2*(d - 2*e + 4*f - 8*g)*x^2 - (d - 2*e + 4
*f - 8*g)*x + 2*d - 4*e + 8*f - 16*g)*log(x + 2) - 4*((2*d + e -
4*f + 7*g)*x^3 - 2*(2*d + e - 4*f + 7*g)*x^2 - (2*d + e - 4*f + 7
*g)*x + 4*d + 2*e - 8*f + 14*g)*log(x + 1) - 12*((2*d + 5*e + 8*f
 + 11*g)*x^3 - 2*(2*d + 5*e + 8*f + 11*g)*x^2 - (2*d + 5*e + 8*f
+ 11*g)*x + 4*d + 10*e + 16*f + 22*g)*log(x - 1) + ((35*d + 58*e
+ 92*f + 136*g)*x^3 - 2*(35*d + 58*e + 92*f + 136*g)*x^2 - (35*d
+ 58*e + 92*f + 136*g)*x + 70*d + 116*e + 184*f + 272*g)*log(x -
2) - 60*d - 120*e - 96*f - 192*g)/(x^3 - 2*x^2 - x + 2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2+x)*(g*x**3+f*x**2+e*x+d)/(x**4-5*x**2+4)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.28452, size = 184, normalized size = 1.3

$$\frac{1}{144}\,(d+4\,f-8\,g-2\,e)\ln\left(|x+2|\right)+\frac{1}{108}\,(2\,d-4\,f+7\,g+e)\ln\left(|x+1|\right)+\frac{1}{36}\,(2\,d+8\,f+11\,g+5\,e)\ln\left(|x-1|\right)-\frac{1}{432}\,(35\,d+92\,f+136\,g+58\,e)\ln\left(|x-2|\right)-\frac{(5\,d+8\,f+10\,g+4\,e)x^2-6\,(d+f)x-5\,d-8\,f-16\,g-10\,e}{36\,(x+1)(x-1)(x-2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((g*x^3 + f*x^2 + e*x + d)*(x + 2)/(x^4 - 5*x^2 + 4)^2,x, algorithm="giac
```

```
[Out] 1/144*(d + 4*f - 8*g - 2*e)*ln(abs(x + 2)) + 1/108*(2*d - 4*f + 7
*g + e)*ln(abs(x + 1)) + 1/36*(2*d + 8*f + 11*g + 5*e)*ln(abs(x -
 1)) - 1/432*(35*d + 92*f + 136*g + 58*e)*ln(abs(x - 2)) - 1/36*(
(5*d + 8*f + 10*g + 4*e)*x^2 - 6*(d + f)*x - 5*d - 8*f - 16*g - 1
0*e)/((x + 1)*(x - 1)*(x - 2))
```

3.101 $\int \frac{(2+x)\left(d+ex+fx^2+gx^3+hx^4\right)}{\left(4-5x^2+x^4\right)^2}\,dx$

Optimal. Leaf size=158

$$-\frac{d-e+f-g+h}{36(x+1)}+\frac{d+e+f+g+h}{12(1-x)}+\frac{d+2e+4f+8g+16h}{36(2-x)}$$
$$+\frac{1}{36}\log(1-x)(2d+5e+8f+11g+14h)-\frac{1}{432}\log(2-x)(35d+58e+92f+136g+176h)$$
$$+\frac{1}{108}\log(x+1)(2d+e-4f+7g-10h)+\frac{1}{144}\log(x+2)(d-2e+4f-8g+16h)$$

```
[Out] (d + e + f + g + h)/(12*(1 - x)) + (d + 2*e + 4*f + 8*g + 16*h)/(
36*(2 - x)) - (d - e + f - g + h)/(36*(1 + x)) + ((2*d + 5*e + 8*
f + 11*g + 14*h)*Log[1 - x])/36 - ((35*d + 58*e + 92*f + 136*g +
176*h)*Log[2 - x])/432 + ((2*d + e - 4*f + 7*g - 10*h)*Log[1 + x]
)/108 + ((d - 2*e + 4*f - 8*g + 16*h)*Log[2 + x])/144
```

Rubi [A] time = 0.605783, antiderivative size = 158, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056

$$-\frac{d-e+f-g+h}{36(x+1)}+\frac{d+e+f+g+h}{12(1-x)}+\frac{d+2e+4f+8g+16h}{36(2-x)}$$
$$+\frac{1}{36}\log(1-x)(2d+5e+8f+11g+14h)-\frac{1}{432}\log(2-x)(35d+58e+92f+136g+176h)$$
$$+\frac{1}{108}\log(x+1)(2d+e-4f+7g-10h)+\frac{1}{144}\log(x+2)(d-2e+4f-8g+16h)$$

Antiderivative was successfully verified.

```
[In] Int[((2 + x)*(d + e*x + f*x^2 + g*x^3 + h*x^4))/(4 - 5*x^2 + x^4)^2,x]
```

```
[Out] (d + e + f + g + h)/(12*(1 - x)) + (d + 2*e + 4*f + 8*g + 16*h)/(
36*(2 - x)) - (d - e + f - g + h)/(36*(1 + x)) + ((2*d + 5*e + 8*
f + 11*g + 14*h)*Log[1 - x])/36 - ((35*d + 58*e + 92*f + 136*g +
176*h)*Log[2 - x])/432 + ((2*d + e - 4*f + 7*g - 10*h)*Log[1 + x]
)/108 + ((d - 2*e + 4*f - 8*g + 16*h)*Log[2 + x])/144
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((2+x)*(h*x**4+g*x**3+f*x**2+e*x+d)/(x**4-5*x**2+4)**2,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.204148, size = 169, normalized size = 1.07

$$\frac{1}{432}\left(\frac{12\left(d\left(-5x^2+6x+5\right)+2\left(e\left(5-2x^2\right)+f\left(-4x^2+3x+4\right)-5gx^2+8g-10hx^2+3hx+10h\right)\right)}{x^3-2x^2-x+2}+12\log(1-x)(2d+5e+8f+11g+14h)-\log(2-x)(35d+58e+92f+136g+176h)+4\log(x+1)(2d+e-4f+7g-10h)+3\log(x+2)(d-2e+4f-8g+16h)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + x)*(d + e*x + f*x^2 + g*x^3 + h*x^4))/(4 - 5*x^2 + x^4)^2,x]
```

```
[Out] ((12*(d*(5 + 6*x - 5*x^2) + 2*(8*g + 10*h + 3*h*x - 5*g*x^2 - 10*
h*x^2 + f*(4 + 3*x - 4*x^2) + e*(5 - 2*x^2))))/(2 - x - 2*x^2 + x
^3) + 12*(2*d + 5*e + 8*f + 11*g + 14*h)*Log[1 - x] - (35*d + 58*
e + 92*f + 136*g + 176*h)*Log[2 - x] + 4*(2*d + e - 4*f + 7*g - 1
0*h)*Log[1 + x] + 3*(d - 2*e + 4*f - 8*g + 16*h)*Log[2 + x])/432
```

Maple [A] time = 0.024, size = 262, normalized size = 1.7

$$-\frac{4h}{9x-18}-\frac{h}{36+36x}-\frac{h}{-12+12x}+\frac{g}{36+36x}-\frac{g}{-12+12x}-\frac{2g}{9x-18}-\frac{f}{36+36x}-\frac{d}{36+36x}+\frac{e}{36+36x}-\frac{d}{36x-72}-\frac{e}{18x-36}-\frac{f}{9x-18}-\frac{f}{-12+12x}-\frac{d}{-12+12x}-\frac{e}{-12+12x}+\frac{\ln(1+x)d}{54}+\frac{\ln(1+x)e}{108}+\frac{\ln(-1+x)d}{18}+\frac{5\ln(-1+x)e}{36}-\frac{11\ln(x-2)h}{27}-\frac{5\ln(1+x)h}{54}+\frac{\ln(2+x)h}{9}+\frac{7\ln(-1+x)h}{18}+\frac{7\ln(1+x)g}{108}-\frac{17\ln(x-2)g}{54}+\frac{11\ln(-1+x)g}{36}-\frac{\ln(2+x)g}{18}-\frac{35\ln(x-2)d}{432}-\frac{29\ln(x-2)e}{216}-\frac{\ln(2+x)e}{72}-\frac{23\ln(x-2)f}{108}+\frac{\ln(2+x)d}{144}-\frac{\ln(1+x)f}{27}+\frac{2\ln(-1+x)f}{9}+\frac{\ln(2+x)f}{36}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+x)*(h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x)
```

```
[Out] -4/9/(x-2)*h-1/36/(1+x)*h-1/12/(-1+x)*h+1/36/(1+x)*g-1/12/(-1+x)*
g-2/9/(x-2)*g-1/36/(1+x)*f-1/36/(1+x)*d+1/36/(1+x)*e-1/36/(x-2)*d
```

```
-1/18/(x-2)*e-1/9/(x-2)*f-1/12/(-1+x)*f-1/12/(-1+x)*d-1/12/(-1+x)
*e+1/54*ln(1+x)*d+1/108*ln(1+x)*e+1/18*ln(-1+x)*d+5/36*ln(-1+x)*e
-11/27*ln(x-2)*h-5/54*ln(1+x)*h+1/9*ln(2+x)*h+7/18*ln(-1+x)*h+7/1
08*ln(1+x)*g-17/54*ln(x-2)*g+11/36*ln(-1+x)*g-1/18*ln(2+x)*g-35/4
32*ln(x-2)*d-29/216*ln(x-2)*e-1/72*ln(2+x)*e-23/108*ln(x-2)*f+1/1
44*ln(2+x)*d-1/27*ln(1+x)*f+2/9*ln(-1+x)*f+1/36*ln(2+x)*f
```

Maxima [A] time = 0.709866, size = 196, normalized size = 1.24

$$\frac{1}{144}(d-2e+4f-8g+16h)\log(x+2)+\frac{1}{108}(2d+e-4f+7g-10h)\log(x+1)+\frac{1}{36}(2d+5e+8f+11g+14h)\log(x-1)-\frac{1}{432}(35d+58e+92f+136g+176h)\log(x-2)-\frac{(5d+4e+8f+10g+20h)x^2-6(d+f+h)x-5d-10e-8f-16g-20h}{36(x^3-2x^2-x+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^4 + g*x^3 + f*x^2 + e*x + d)*(x + 2)/(x^4 - 5*x^2 + 4)^2,x, algorith
```

```
[Out] 1/144*(d - 2*e + 4*f - 8*g + 16*h)*log(x + 2) + 1/108*(2*d + e -
4*f + 7*g - 10*h)*log(x + 1) + 1/36*(2*d + 5*e + 8*f + 11*g + 14*
h)*log(x - 1) - 1/432*(35*d + 58*e + 92*f + 136*g + 176*h)*log(x
- 2) - 1/36*((5*d + 4*e + 8*f + 10*g + 20*h)*x^2 - 6*(d + f + h)*
x - 5*d - 10*e - 8*f - 16*g - 20*h)/(x^3 - 2*x^2 - x + 2)
```

Fricas [A] time = 4.58709, size = 508, normalized size = 3.22

$$-\frac{12(5d+4e+8f+10g+20h)x^2-72(d+f+h)x-3\left((d-2e+4f-8g+16h)x^3-2(d-2e+4f-8g+16h)x^2-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^4 + g*x^3 + f*x^2 + e*x + d)*(x + 2)/(x^4 - 5*x^2 + 4)^2,x, algorith
```

```
[Out] -1/432*(12*(5*d + 4*e + 8*f + 10*g + 20*h)*x^2 - 72*(d + f + h)*x
 - 3*((d - 2*e + 4*f - 8*g + 16*h)*x^3 - 2*(d - 2*e + 4*f - 8*g +
 16*h)*x^2 - (d - 2*e + 4*f - 8*g + 16*h)*x + 2*d - 4*e + 8*f - 1
6*g + 32*h)*log(x + 2) - 4*((2*d + e - 4*f + 7*g - 10*h)*x^3 - 2*
(2*d + e - 4*f + 7*g - 10*h)*x^2 - (2*d + e - 4*f + 7*g - 10*h)*x
 + 4*d + 2*e - 8*f + 14*g - 20*h)*log(x + 1) - 12*((2*d + 5*e + 8
*f + 11*g + 14*h)*x^3 - 2*(2*d + 5*e + 8*f + 11*g + 14*h)*x^2 - (
2*d + 5*e + 8*f + 11*g + 14*h)*x + 4*d + 10*e + 16*f + 22*g + 28*
h)*log(x - 1) + ((35*d + 58*e + 92*f + 136*g + 176*h)*x^3 - 2*(35
```

```
*d + 58*e + 92*f + 136*g + 176*h)*x^2 - (35*d + 58*e + 92*f + 136
*g + 176*h)*x + 70*d + 116*e + 184*f + 272*g + 352*h)*log(x - 2)
- 60*d - 120*e - 96*f - 192*g - 240*h)/(x^3 - 2*x^2 - x + 2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2+x)*(h*x**4+g*x**3+f*x**2+e*x+d)/(x**4-5*x**2+4)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.287094, size = 209, normalized size = 1.32

$$\frac{1}{144}(d + 4f - 8g + 16h - 2e)\ln(|x + 2|) + \frac{1}{108}(2d - 4f + 7g - 10h + e)\ln(|x + 1|) + \frac{1}{36}(2d + 8f + 11g + 14h + 5e)\ln(|x - 1|) - \frac{1}{432}(35d + 92f + 136g + 176h + 58e)\ln(|x - 2|) - \frac{(5d + 8f + 10g + 20h + 4e)x^2 - 6(d + f + h)x - 5d - 8f - 16g - 20h - 10e}{36(x + 1)(x - 1)(x - 2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^4 + g*x^3 + f*x^2 + e*x + d)*(x + 2)/(x^4 - 5*x^2 + 4)^2,x, algorith
```

```
[Out] 1/144*(d + 4*f - 8*g + 16*h - 2*e)*ln(abs(x + 2)) + 1/108*(2*d -
4*f + 7*g - 10*h + e)*ln(abs(x + 1)) + 1/36*(2*d + 8*f + 11*g + 1
4*h + 5*e)*ln(abs(x - 1)) - 1/432*(35*d + 92*f + 136*g + 176*h +
58*e)*ln(abs(x - 2)) - 1/36*((5*d + 8*f + 10*g + 20*h + 4*e)*x^2
- 6*(d + f + h)*x - 5*d - 8*f - 16*g - 20*h - 10*e)/((x + 1)*(x -
 1)*(x - 2))
```

3.102 $\int \frac{(2+x)\left(d+ex+fx^2+gx^3+hx^4+ix^5\right)}{(4-5x^2+x^4)^2}\,dx$

Optimal. Leaf size=177

$$-\frac{d-e+f-g+h-i}{36(x+1)}+\frac{d+e+f+g+h+i}{12(1-x)}+\frac{d+2e+4f+8g+16h+32i}{36(2-x)}+\frac{1}{36}\log(1-x)(2d+5e+8f+11g+14h+17i)-\frac{1}{432}\log(2-x)(35d+58e+92f+136g+176h+160i)+\frac{1}{108}\log(x+1)(2d+e-4f+7g-10h+13i)+\frac{1}{144}\log(x+2)(d-2e+4f-8g+16h-32i)$$

```
[Out] (d + e + f + g + h + i)/(12*(1 - x)) + (d + 2*e + 4*f + 8*g + 16*
h + 32*i)/(36*(2 - x)) - (d - e + f - g + h - i)/(36*(1 + x)) + (
(2*d + 5*e + 8*f + 11*g + 14*h + 17*i)*Log[1 - x])/36 - ((35*d +
58*e + 92*f + 136*g + 176*h + 160*i)*Log[2 - x])/432 + ((2*d + e
- 4*f + 7*g - 10*h + 13*i)*Log[1 + x])/108 + ((d - 2*e + 4*f - 8*
g + 16*h - 32*i)*Log[2 + x])/144
```

Rubi [A] time = 0.724607, antiderivative size = 177, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.049$

$$-\frac{d-e+f-g+h-i}{36(x+1)}+\frac{d+e+f+g+h+i}{12(1-x)}+\frac{d+2e+4f+8g+16h+32i}{36(2-x)}+\frac{1}{36}\log(1-x)(2d+5e+8f+11g+14h+17i)-\frac{1}{432}\log(2-x)(35d+58e+92f+136g+176h+160i)+\frac{1}{108}\log(x+1)(2d+e-4f+7g-10h+13i)+\frac{1}{144}\log(x+2)(d-2e+4f-8g+16h-32i)$$

Antiderivative was successfully verified.

```
[In] Int[((2 + x)*(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5))/(4 - 5*x^2 + x^4)^2,x]
```

```
[Out] (d + e + f + g + h + i)/(12*(1 - x)) + (d + 2*e + 4*f + 8*g + 16*
h + 32*i)/(36*(2 - x)) - (d - e + f - g + h - i)/(36*(1 + x)) + (
(2*d + 5*e + 8*f + 11*g + 14*h + 17*i)*Log[1 - x])/36 - ((35*d +
58*e + 92*f + 136*g + 176*h + 160*i)*Log[2 - x])/432 + ((2*d + e
- 4*f + 7*g - 10*h + 13*i)*Log[1 + x])/108 + ((d - 2*e + 4*f - 8*
g + 16*h - 32*i)*Log[2 + x])/144
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((2+x)*(i*x**5+h*x**4+g*x**3+f*x**2+e*x+d)/(x**4-5*x**2+4)**2,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.225792, size = 195, normalized size = 1.1

$$\frac{-5dx^2+6dx+5d-4ex^2+10e-8fx^2+6fx+8f-10gx^2+16g-20hx^2+6hx+20h-34ix^2+40i}{36\left(x^3-2x^2-x+2\right)} + \frac{1}{36}\log(1-x)(2d+5e+8f+11g+14h+17i)+\frac{1}{432}\log(2-x)(-35d-58e-92f-136g-176h-160i) + \frac{1}{108}\log(x+1)(2d+e-4f+7g-10h+13i)+\frac{1}{144}\log(x+2)(d-2e+4f-8g+16h-32i)$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + x)*(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5))/(4 - 5*x^2 + x^4)^2,x
```

```
[Out] (5*d + 10*e + 8*f + 16*g + 20*h + 40*i + 6*d*x + 6*f*x + 6*h*x -
5*d*x^2 - 4*e*x^2 - 8*f*x^2 - 10*g*x^2 - 20*h*x^2 - 34*i*x^2)/(36
*(2 - x - 2*x^2 + x^3)) + ((2*d + 5*e + 8*f + 11*g + 14*h + 17*i)
*Log[1 - x])/36 + ((-35*d - 58*e - 92*f - 136*g - 176*h - 160*i)*
Log[2 - x])/432 + ((2*d + e - 4*f + 7*g - 10*h + 13*i)*Log[1 + x]
)/108 + ((d - 2*e + 4*f - 8*g + 16*h - 32*i)*Log[2 + x])/144
```

Maple [A] time = 0.025, size = 314, normalized size = 1.8

$$-\frac{8i}{9x-18}+\frac{i}{36+36x}-\frac{i}{-12+12x}-\frac{4h}{9x-18}-\frac{h}{36+36x}-\frac{h}{-12+12x}+\frac{g}{36+36x} -\frac{g}{-12+12x}-\frac{2g}{9x-18}-\frac{f}{36+36x}-\frac{d}{36+36x}+\frac{e}{36+36x}-\frac{d}{36x-72}-\frac{e}{18x-36} -\frac{f}{9x-18}-\frac{f}{-12+12x}-\frac{d}{-12+12x}-\frac{e}{-12+12x}+\frac{\ln(1+x)d}{54}+\frac{\ln(1+x)e}{108}+\frac{\ln(-1+x)d}{18} +\frac{5\ln(-1+x)e}{36}-\frac{10\ln(x-2)i}{27}+\frac{13\ln(1+x)i}{108}+\frac{17\ln(-1+x)i}{36}-\frac{2\ln(2+x)i}{9} -\frac{11\ln(x-2)h}{27}-\frac{5\ln(1+x)h}{54}+\frac{\ln(2+x)h}{9}+\frac{7\ln(-1+x)h}{18}+\frac{7\ln(1+x)g}{108} -\frac{17\ln(x-2)g}{54}+\frac{11\ln(-1+x)g}{36}-\frac{\ln(2+x)g}{18}-\frac{35\ln(x-2)d}{432}-\frac{29\ln(x-2)e}{216} -\frac{\ln(2+x)e}{72}-\frac{23\ln(x-2)f}{108}+\frac{\ln(2+x)d}{144}-\frac{\ln(1+x)f}{27}+\frac{2\ln(-1+x)f}{9}+\frac{\ln(2+x)f}{36}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((2+x)*(i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x)
```

```
[Out] -8/9/(x-2)*i+1/36/(1+x)*i-1/12/(-1+x)*i-4/9/(x-2)*h-1/36/(1+x)*h-
1/12/(-1+x)*h+1/36/(1+x)*g-1/12/(-1+x)*g-2/9/(x-2)*g-1/36/(1+x)*f
-1/36/(1+x)*d+1/36/(1+x)*e-1/36/(x-2)*d-1/18/(x-2)*e-1/9/(x-2)*f-
1/12/(-1+x)*f-1/12/(-1+x)*d-1/12/(-1+x)*e+1/54*ln(1+x)*d+1/108*ln
(1+x)*e+1/18*ln(-1+x)*d+5/36*ln(-1+x)*e-10/27*ln(x-2)*i+13/108*ln
(1+x)*i+17/36*ln(-1+x)*i-2/9*ln(2+x)*i-11/27*ln(x-2)*h-5/54*ln(1+
x)*h+1/9*ln(2+x)*h+7/18*ln(-1+x)*h+7/108*ln(1+x)*g-17/54*ln(x-2)*
g+11/36*ln(-1+x)*g-1/18*ln(2+x)*g-35/432*ln(x-2)*d-29/216*ln(x-2)
*e-1/72*ln(2+x)*e-23/108*ln(x-2)*f+1/144*ln(2+x)*d-1/27*ln(1+x)*f
+2/9*ln(-1+x)*f+1/36*ln(2+x)*f
```

Maxima [A] time = 0.706743, size = 220, normalized size = 1.24

$$\frac{1}{144}\left(d-2e+4f-8g+16h-32i\right)\log\left(x+2\right)+\frac{1}{108}\left(2d+e-4f+7g-10h+13i\right)\log\left(x+1\right)+\frac{1}{36}\left(2d+5e+8f+11g+14h+17i\right)\log\left(x-1\right)-\frac{1}{432}\left(35d+58e+92f+136g+176h+160i\right)\log\left(x-2\right)-\frac{(5d+4e+8f+10g+20h+34i)x^2-6(d+f+h)x-5d-10e-8f-16g-20h-40i}{36\left(x^3-2x^2-x+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^5 + h*x^4 + g*x^3 + f*x^2 + e*x + d)*(x + 2)/(x^4 - 5*x^2 + 4)^2,x, a
```

```
[Out] 1/144*(d - 2*e + 4*f - 8*g + 16*h - 32*i)*log(x + 2) + 1/108*(2*d
 + e - 4*f + 7*g - 10*h + 13*i)*log(x + 1) + 1/36*(2*d + 5*e + 8*
f + 11*g + 14*h + 17*i)*log(x - 1) - 1/432*(35*d + 58*e + 92*f +
136*g + 176*h + 160*i)*log(x - 2) - 1/36*((5*d + 4*e + 8*f + 10*g
 + 20*h + 34*i)*x^2 - 6*(d + f + h)*x - 5*d - 10*e - 8*f - 16*g -
 20*h - 40*i)/(x^3 - 2*x^2 - x + 2)
```

Fricas [A] time = 26.7788, size = 581, normalized size = 3.28

$$-\frac{12\left(5d+4e+8f+10g+20h+34i\right)x^2-72\left(d+f+h\right)x-3\left(\left(d-2e+4f-8g+16h-32i\right)x^3-2\left(d-2e+4f-8g\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^5 + h*x^4 + g*x^3 + f*x^2 + e*x + d)*(x + 2)/(x^4 - 5*x^2 + 4)^2,x, a
```

```
[Out] -1/432*(12*(5*d + 4*e + 8*f + 10*g + 20*h + 34*i)*x^2 - 72*(d + f
 + h)*x - 3*((d - 2*e + 4*f - 8*g + 16*h - 32*i)*x^3 - 2*(d - 2*e
 + 4*f - 8*g + 16*h - 32*i)*x^2 - (d - 2*e + 4*f - 8*g + 16*h - 3
2*i)*x + 2*d - 4*e + 8*f - 16*g + 32*h - 64*i)*log(x + 2) - 4*((2
*d + e - 4*f + 7*g - 10*h + 13*i)*x^3 - 2*(2*d + e - 4*f + 7*g -
10*h + 13*i)*x^2 - (2*d + e - 4*f + 7*g - 10*h + 13*i)*x + 4*d +
2*e - 8*f + 14*g - 20*h + 26*i)*log(x + 1) - 12*((2*d + 5*e + 8*f
 + 11*g + 14*h + 17*i)*x^3 - 2*(2*d + 5*e + 8*f + 11*g + 14*h + 1
7*i)*x^2 - (2*d + 5*e + 8*f + 11*g + 14*h + 17*i)*x + 4*d + 10*e
+ 16*f + 22*g + 28*h + 34*i)*log(x - 1) + ((35*d + 58*e + 92*f +
136*g + 176*h + 160*i)*x^3 - 2*(35*d + 58*e + 92*f + 136*g + 176*
h + 160*i)*x^2 - (35*d + 58*e + 92*f + 136*g + 176*h + 160*i)*x +
 70*d + 116*e + 184*f + 272*g + 352*h + 320*i)*log(x - 2) - 60*d
- 120*e - 96*f - 192*g - 240*h - 480*i)/(x^3 - 2*x^2 - x + 2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)*(i*x**5+h*x**4+g*x**3+f*x**2+e*x+d)/(x**4-5*x**2+4)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.28586, size = 234, normalized size = 1.32

$$\frac{1}{144}\left(d+4\,f-8\,g+16\,h-32\,i-2\,e\right)\ln\left(|x+2|\right)+\frac{1}{108}\left(2\,d-4\,f+7\,g-10\,h+13\,i+e\right)\ln\left(|x+1|\right)+\frac{1}{36}\left(2\,d+8\,f+11\,g+14\,h+17\,i+5\,e\right)\ln\left(|x-1|\right)-\frac{1}{432}\left(35\,d+92\,f+136\,g+176\,h+160\,i+58\,e\right)\ln\left(|x-2|\right)-\frac{\left(5\,d+8\,f+10\,g+20\,h+34\,i+4\,e\right)x^2-6\left(d+f+h\right)x-5\,d-8\,f-16\,g-20\,h-40\,i-10\,e}{36\left(x+1\right)\left(x-1\right)\left(x-2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^5 + h*x^4 + g*x^3 + f*x^2 + e*x + d)*(x + 2)/(x^4 - 5*x^2 + 4)^2,x, a
```

```
[Out] 1/144*(d + 4*f - 8*g + 16*h - 32*i - 2*e)*ln(abs(x + 2)) + 1/108*
(2*d - 4*f + 7*g - 10*h + 13*i + e)*ln(abs(x + 1)) + 1/36*(2*d +
8*f + 11*g + 14*h + 17*i + 5*e)*ln(abs(x - 1)) - 1/432*(35*d + 92
*f + 136*g + 176*h + 160*i + 58*e)*ln(abs(x - 2)) - 1/36*((5*d +
```

```
8*f + 10*g + 20*h + 34*i + 4*e)*x^2 - 6*(d + f + h)*x - 5*d - 8*f
 - 16*g - 20*h - 40*i - 10*e)/((x + 1)*(x - 1)*(x - 2))
```

3.103 $\int \left(d+ex+fx^2+gx^3\right)\left(a+bx^2+cx^4\right)^{3/2}\,dx$

Optimal. Leaf size=717

$$
\begin{aligned}
&-\frac{x\sqrt{a+bx^2+cx^4}\left(-84a^2c^2f+57ab^2cf-144abc^2d-8b^4f+18b^3cd\right)}{315c^{5/2}\left(\sqrt{a}+\sqrt{c}x^2\right)} \\
&-\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(-84a^2c^2f+57ab^2cf+\sqrt{a}\sqrt{c}\left(24abcf-180ac^2d-4b^3f+9b^2cd\right)-144abc^2d-8b^4f+1\right.}{630c^{11/4}\sqrt{a+bx^2+cx^4}} \\
&+\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(-84a^2c^2f+57ab^2cf-144abc^2d-8b^4f+18b^3cd\right)E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{315c^{11/4}\sqrt{a+bx^2+cx^4}} \\
&+\frac{3\left(b^2-4ac\right)^2(2ce-bg)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{512c^{7/2}}-\frac{3\left(b^2-4ac\right)\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}(2ce-bg)}{256c^3} \\
&+\frac{x\sqrt{a+bx^2+cx^4}\left(3cx^2\left(14acf-4b^2f+9bcd\right)+9abcf+90ac^2d-4b^3f+9b^2cd\right)}{315c^2} \\
&+\frac{\left(b+2cx^2\right)\left(a+bx^2+cx^4\right)^{3/2}(2ce-bg)}{32c^2} \\
&+\frac{x\left(a+bx^2+cx^4\right)^{3/2}\left(3(bf+3cd)+7cfx^2\right)}{63c}+\frac{g\left(a+bx^2+cx^4\right)^{5/2}}{10c}
\end{aligned}
$$

```
[Out] -((18*b^3*c*d - 144*a*b*c^2*d - 8*b^4*f + 57*a*b^2*c*f - 84*a^2*c
^2*f)*x*Sqrt[a + b*x^2 + c*x^4])/(315*c^(5/2)*(Sqrt[a] + Sqrt[c]*
x^2)) - (3*(b^2 - 4*a*c)*(2*c*e - b*g)*(b + 2*c*x^2)*Sqrt[a + b*x
^2 + c*x^4])/(256*c^3) + (x*(9*b^2*c*d + 90*a*c^2*d - 4*b^3*f + 9
*a*b*c*f + 3*c*(9*b*c*d - 4*b^2*f + 14*a*c*f)*x^2)*Sqrt[a + b*x^2
 + c*x^4])/(315*c^2) + ((2*c*e - b*g)*(b + 2*c*x^2)*(a + b*x^2 +
c*x^4)^(3/2))/(32*c^2) + (x*(3*(3*c*d + b*f) + 7*c*f*x^2)*(a + b*
x^2 + c*x^4)^(3/2))/(63*c) + (g*(a + b*x^2 + c*x^4)^(5/2))/(10*c)
 + (3*(b^2 - 4*a*c)^2*(2*c*e - b*g)*ArcTanh[(b + 2*c*x^2)/(2*Sqrt
[c]*Sqrt[a + b*x^2 + c*x^4])])/(512*c^(7/2)) + (a^(1/4)*(18*b^3*c
*d - 144*a*b*c^2*d - 8*b^4*f + 57*a*b^2*c*f - 84*a^2*c^2*f)*(Sqrt
[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^
2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sq
rt[c]))/4])/(315*c^(11/4)*Sqrt[a + b*x^2 + c*x^4]) - (a^(1/4)*(18
*b^3*c*d - 144*a*b*c^2*d - 8*b^4*f + 57*a*b^2*c*f - 84*a^2*c^2*f
+ Sqrt[a]*Sqrt[c]*(9*b^2*c*d - 180*a*c^2*d - 4*b^3*f + 24*a*b*c*f
))*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sq
rt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sq
rt[a]*Sqrt[c]))/4])/(630*c^(11/4)*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] time = 1.39402, antiderivative size = 717, normalized size of antiderivative = 1., number of

steps used = 12, number of rules used = 10, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$

$$-\frac{x\sqrt{a+bx^2+cx^4}\left(-84a^2c^2f+57ab^2cf-144abc^2d-8b^4f+18b^3cd\right)}{315c^{5/2}\left(\sqrt{a}+\sqrt{c}x^2\right)}$$
$$-\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(-84a^2c^2f+57ab^2cf+\sqrt{a}\sqrt{c}\left(24abcf-180ac^2d-4b^3f+9b^2cd\right)-144abc^2d-8b^4f+1\right.}{630c^{11/4}\sqrt{a+bx^2+cx^4}}$$
$$+\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(-84a^2c^2f+57ab^2cf-144abc^2d-8b^4f+18b^3cd\right)E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{315c^{11/4}\sqrt{a+bx^2+cx^4}}$$
$$+\frac{3\left(b^2-4ac\right)^2(2ce-bg)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{512c^{7/2}}-\frac{3\left(b^2-4ac\right)\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}(2ce-bg)}{256c^3}$$
$$+\frac{x\sqrt{a+bx^2+cx^4}\left(3cx^2\left(14acf-4b^2f+9bcd\right)+9abcf+90ac^2d-4b^3f+9b^2cd\right)}{315c^2}$$
$$+\frac{\left(b+2cx^2\right)\left(a+bx^2+cx^4\right)^{3/2}(2ce-bg)}{32c^2}$$
$$+\frac{x\left(a+bx^2+cx^4\right)^{3/2}\left(3(bf+3cd)+7cfx^2\right)}{63c}+\frac{g\left(a+bx^2+cx^4\right)^{5/2}}{10c}$$

Warning: Unable to verify antiderivative.

```
[In]  Int[(d + e*x + f*x^2 + g*x^3)*(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] -((18*b^3*c*d - 144*a*b*c^2*d - 8*b^4*f + 57*a*b^2*c*f - 84*a^2*c
^2*f)*x*Sqrt[a + b*x^2 + c*x^4])/(315*c^(5/2)*(Sqrt[a] + Sqrt[c]*
x^2)) - (3*(b^2 - 4*a*c)*(2*c*e - b*g)*(b + 2*c*x^2)*Sqrt[a + b*x
^2 + c*x^4])/(256*c^3) + (x*(9*b^2*c*d + 90*a*c^2*d - 4*b^3*f + 9
*a*b*c*f + 3*c*(9*b*c*d - 4*b^2*f + 14*a*c*f)*x^2)*Sqrt[a + b*x^2
 + c*x^4])/(315*c^2) + ((2*c*e - b*g)*(b + 2*c*x^2)*(a + b*x^2 +
c*x^4)^(3/2))/(32*c^2) + (x*(3*(3*c*d + b*f) + 7*c*f*x^2)*(a + b*
x^2 + c*x^4)^(3/2))/(63*c) + (g*(a + b*x^2 + c*x^4)^(5/2))/(10*c)
 + (3*(b^2 - 4*a*c)^2*(2*c*e - b*g)*ArcTanh[(b + 2*c*x^2)/(2*Sqrt
[c]*Sqrt[a + b*x^2 + c*x^4])])/(512*c^(7/2)) + (a^(1/4)*(18*b^3*c
*d - 144*a*b*c^2*d - 8*b^4*f + 57*a*b^2*c*f - 84*a^2*c^2*f)*(Sqrt
[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^
2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sq
rt[c]))/4])/(315*c^(11/4)*Sqrt[a + b*x^2 + c*x^4]) - (a^(1/4)*(18
*b^3*c*d - 144*a*b*c^2*d - 8*b^4*f + 57*a*b^2*c*f - 84*a^2*c^2*f
+ Sqrt[a]*Sqrt[c]*(9*b^2*c*d - 180*a*c^2*d - 4*b^3*f + 24*a*b*c*f
))*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sq
rt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sq
rt[a]*Sqrt[c]))/4])/(630*c^(11/4)*Sqrt[a + b*x^2 + c*x^4])
```

Rubi in Sympy [A] time = 132.275, size = 709, normalized size = 0.99

$$-\frac{\sqrt[4]{a}\sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\left(\sqrt{a}+\sqrt{c}x^2\right)\left(84a^2c^2f-57ab^2cf+144abc^2d+8b^4f-18b^3cd\right)E\left(2\,\mathrm{atan}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}-\frac{b}{4\sqrt{a}\sqrt{c}}\right)}{315c^{\frac{11}{4}}\sqrt{a+bx^2+cx^4}}$$

$$+\frac{\sqrt[4]{a}\sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\left(\sqrt{a}+\sqrt{c}x^2\right)\left(\sqrt{a}\sqrt{c}\left(-24abcf+180ac^2d+4b^3f-9b^2cd\right)+84a^2c^2f-57ab^2cf+144abc^2d+8b^4f-18\right.}{630c^{\frac{11}{4}}\sqrt{a+bx^2+cx^4}}$$

$$+\frac{g\left(a+bx^2+cx^4\right)^{\frac{5}{2}}}{10c}+\frac{x\left(a+bx^2+cx^4\right)^{\frac{3}{2}}\left(3bf+9cd+7cfx^2\right)}{63c}$$

$$-\frac{x\sqrt{a+bx^2+cx^4}\left(-9abcf-90ac^2d+4b^3f-9b^2cd+3cx^2\left(-14acf+4b^2f-9bcd\right)\right)}{315c^2}$$

$$-\frac{\left(b+2cx^2\right)(bg-2ce)\left(a+bx^2+cx^4\right)^{\frac{3}{2}}}{32c^2}+\frac{3\left(b+2cx^2\right)\left(-4ac+b^2\right)(bg-2ce)\sqrt{a+bx^2+cx^4}}{256c^3}$$

$$+\frac{x\sqrt{a+bx^2+cx^4}\left(84a^2c^2f-57ab^2cf+144abc^2d+8b^4f-18b^3cd\right)}{315c^{\frac{5}{2}}\left(\sqrt{a}+\sqrt{c}x^2\right)}$$

$$-\frac{3\left(-4ac+b^2\right)^2(bg-2ce)\,\mathrm{atanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{512c^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((g*x**3+f*x**2+e*x+d)*(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] -a**(1/4)*sqrt((a + b*x**2 + c*x**4)/(sqrt(a) + sqrt(c)*x**2)**2)
*(sqrt(a) + sqrt(c)*x**2)*(84*a**2*c**2*f - 57*a*b**2*c*f + 144*a
*b*c**2*d + 8*b**4*f - 18*b**3*c*d)*elliptic_e(2*atan(c**(1/4)*x/
a**(1/4)), 1/2 - b/(4*sqrt(a)*sqrt(c)))/(315*c**(11/4)*sqrt(a + b
*x**2 + c*x**4)) + a**(1/4)*sqrt((a + b*x**2 + c*x**4)/(sqrt(a) +
 sqrt(c)*x**2)**2)*(sqrt(a) + sqrt(c)*x**2)*(sqrt(a)*sqrt(c)*(-24
*a*b*c*f + 180*a*c**2*d + 4*b**3*f - 9*b**2*c*d) + 84*a**2*c**2*f
 - 57*a*b**2*c*f + 144*a*b*c**2*d + 8*b**4*f - 18*b**3*c*d)*ellip
tic_f(2*atan(c**(1/4)*x/a**(1/4)), 1/2 - b/(4*sqrt(a)*sqrt(c)))/(
630*c**(11/4)*sqrt(a + b*x**2 + c*x**4)) + g*(a + b*x**2 + c*x**4
)**(5/2)/(10*c) + x*(a + b*x**2 + c*x**4)**(3/2)*(3*b*f + 9*c*d +
 7*c*f*x**2)/(63*c) - x*sqrt(a + b*x**2 + c*x**4)*(-9*a*b*c*f - 9
0*a*c**2*d + 4*b**3*f - 9*b**2*c*d + 3*c*x**2*(-14*a*c*f + 4*b**2
*f - 9*b*c*d))/(315*c**2) - (b + 2*c*x**2)*(b*g - 2*c*e)*(a + b*x
**2 + c*x**4)**(3/2)/(32*c**2) + 3*(b + 2*c*x**2)*(-4*a*c + b**2)
*(b*g - 2*c*e)*sqrt(a + b*x**2 + c*x**4)/(256*c**3) + x*sqrt(a +
b*x**2 + c*x**4)*(84*a**2*c**2*f - 57*a*b**2*c*f + 144*a*b*c**2*d
 + 8*b**4*f - 18*b**3*c*d)/(315*c**(5/2)*(sqrt(a) + sqrt(c)*x**2)
) - 3*(-4*a*c + b**2)**2*(b*g - 2*c*e)*atanh((b + 2*c*x**2)/(2*sq
rt(c)*sqrt(a + b*x**2 + c*x**4)))/(512*c**(7/2))
```

Mathematica [C] time = 5.37154, size = 2588, normalized size = 3.61

Result too large to show

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x + f*x^2 + g*x^3)*(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (-2*Sqrt[c]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*(a + b*x^2 + c*x^4)*(
-945*b^4*g + 2*b^3*c*(945*e + x*(512*f + 315*g*x)) - 12*b^2*c*(-5
25*a*g + c*x*(192*d + 105*e*x + 64*f*x^2 + 42*g*x^3)) - 8*b*c^2*(
3*a*(525*e + 256*f*x + 147*g*x^2) + 2*c*x^3*(1152*d + 945*e*x + 8
00*f*x^2 + 693*g*x^3)) - 16*c^2*(504*a^2*g + 2*c^2*x^5*(360*d + 7
*x*(45*e + 40*f*x + 36*g*x^2)) + a*c*x*(2160*d + 7*x*(225*e + 16*
x*(11*f + 9*g*x))))) + (2304*I)*Sqrt[2]*b^3*c^(3/2)*(b - Sqrt[b^2
 - 4*a*c])*d*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2
 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*(Elliptic
E[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt
[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] - EllipticF[I*ArcSinh[Sqr
t[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/
(b - Sqrt[b^2 - 4*a*c])]) + (18432*I)*Sqrt[2]*a*b*c^(5/2)*(-b + S
qrt[b^2 - 4*a*c])*d*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + S
qrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*(E
llipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b
 + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] - EllipticF[I*ArcS
inh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4
*a*c])/(b - Sqrt[b^2 - 4*a*c])]) + (7296*I)*Sqrt[2]*a*b^2*c^(3/2)
*(b - Sqrt[b^2 - 4*a*c])*f*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)
/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*
c])]*(EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]
*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] - EllipticF
[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[
b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])]) + (1024*I)*Sqrt[2]*b^4*Sq
rt[c]*(-b + Sqrt[b^2 - 4*a*c])*f*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*
c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2
- 4*a*c])]*(EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*
a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] - Ell
ipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b +
 Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])]) + (10752*I)*Sqrt[2]
*a^2*c^(5/2)*(-b + Sqrt[b^2 - 4*a*c])*f*Sqrt[(b + Sqrt[b^2 - 4*a*
c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b - Sq
rt[b^2 - 4*a*c])]*(EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b
^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])
] - EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x
], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])]) + (2304*I)*S
qrt[2]*a*b^2*c^(5/2)*d*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b
+ Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]
*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x],
(b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] - (46080*I)*Sqrt
[2]*a^2*c^(7/2)*d*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqr
t[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Elli
pticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b +
```

```
Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] - (1024*I)*Sqrt[2]*a*
b^3*c^(3/2)*f*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^
2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Elliptic
F[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt
[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] + (6144*I)*Sqrt[2]*a^2*b*
c^(5/2)*f*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 -
4*a*c])]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticF[I*
ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2
 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] + 1890*b^4*c*Sqrt[c/(b + Sqrt
[b^2 - 4*a*c])]*e*Sqrt[a + b*x^2 + c*x^4]*Log[b + 2*c*x^2 + 2*Sqr
t[c]*Sqrt[a + b*x^2 + c*x^4]] - 15120*a*b^2*c^2*Sqrt[c/(b + Sqrt[
b^2 - 4*a*c])]*e*Sqrt[a + b*x^2 + c*x^4]*Log[b + 2*c*x^2 + 2*Sqrt
[c]*Sqrt[a + b*x^2 + c*x^4]] + 30240*a^2*c^3*Sqrt[c/(b + Sqrt[b^2
 - 4*a*c])]*e*Sqrt[a + b*x^2 + c*x^4]*Log[b + 2*c*x^2 + 2*Sqrt[c]
*Sqrt[a + b*x^2 + c*x^4]] - 945*b^5*Sqrt[c/(b + Sqrt[b^2 - 4*a*c]
)]*g*Sqrt[a + b*x^2 + c*x^4]*Log[b + 2*c*x^2 + 2*Sqrt[c]*Sqrt[a +
 b*x^2 + c*x^4]] + 7560*a*b^3*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*g
*Sqrt[a + b*x^2 + c*x^4]*Log[b + 2*c*x^2 + 2*Sqrt[c]*Sqrt[a + b*x
^2 + c*x^4]] - 15120*a^2*b*c^2*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*g*
Sqrt[a + b*x^2 + c*x^4]*Log[b + 2*c*x^2 + 2*Sqrt[c]*Sqrt[a + b*x^
2 + c*x^4]])/(161280*c^(7/2)*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*Sqrt
[a + b*x^2 + c*x^4])
```

Maple [B] time = 0.013, size = 3038, normalized size = 4.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((g*x^3+f*x^2+e*x+d)*(c*x^4+b*x^2+a)^(3/2),x)
```

```
[Out] 1/8*e*c*x^6*(c*x^4+b*x^2+a)^(1/2)+3/16*e*b*x^4*(c*x^4+b*x^2+a)^(1
/2)+3/16*e*a^2*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))/c^
(1/2)+3/256*e/c^(5/2)*b^4*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a
)^(1/2))-3/128*e/c^2*b^3*(c*x^4+b*x^2+a)^(1/2)+1/10*g*a^2/c*(c*x^
4+b*x^2+a)^(1/2)+5/16*e*a*x^2*(c*x^4+b*x^2+a)^(1/2)+1/10*g*c*x^8*
(c*x^4+b*x^2+a)^(1/2)+11/80*g*b*x^6*(c*x^4+b*x^2+a)^(1/2)-2/15*f*
a^3*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^
2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(
c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*EllipticF(1/2*x*2^(1/
2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^
(1/2))/a/c)^(1/2))+2/15*f*a^3*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)
^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+
b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1
/2))*EllipticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/
2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))+1/7*d*2^(1/2)/((-b+(
-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(
1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/
2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*
```

```
(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))*a^2+10/63*f*b*x^5*(c*x
^4+b*x^2+a)^(1/2)+1/9*f*c*x^7*(c*x^4+b*x^2+a)^(1/2)+11/45*f*x^3*(
c*x^4+b*x^2+a)^(1/2)*a+7/160*g/c*b*a*x^2*(c*x^4+b*x^2+a)^(1/2)+8/
105*f/c*x*(c*x^4+b*x^2+a)^(1/2)*a*b+5/32*e/c*b*a*(c*x^4+b*x^2+a)^
(1/2)+3/7*d*x*(c*x^4+b*x^2+a)^(1/2)*a+8/35*d*a^2*2^(1/2)/((-b+(-4
*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/
2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)
/(b+(-4*a*c+b^2)^(1/2))*b*EllipticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^
2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))
-2/105*f/c*a^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b
+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x
^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*
a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^
(1/2))*b+1/315*f/c^2*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*
(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1
/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*(
(-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2
))/a/c)^(1/2))*b^3-1/140*d*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1
/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2
)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/
2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^
(1/2))/a/c)^(1/2))/c*a*b^2-8/35*d*a^2*2^(1/2)/((-b+(-4*a*c+b^2)^(
1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+
(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c
+b^2)^(1/2))*b*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a
)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))+1/5*g*a*x^
4*(c*x^4+b*x^2+a)^(1/2)+4/315*f*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2)
)/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*
a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2
)^(1/2))*b^4/c^2*EllipticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))
/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))+19/210*f
*a^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b
^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/
(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))/c*b^2*EllipticF(1/2*
x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*
c+b^2)^(1/2))/a/c)^(1/2))-1/35*d*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2
))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4
*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^
2)^(1/2))*b^3/c*EllipticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/
a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-19/210*f*
a^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^
2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(
c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))/c*b^2*EllipticE(1/2*x
*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c
+b^2)^(1/2))/a/c)^(1/2))-4/315*f*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2
))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4
*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^
2)^(1/2))*b^4/c^2*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2)
)/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))+1/35*d*
a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)
^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*
x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*b^3/c*EllipticF(1/2*x*2
^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b
^2)^(1/2))/a/c)^(1/2))+1/64*e/c*b^2*x^2*(c*x^4+b*x^2+a)^(1/2)+1/7
```

```
*d*c*x^5*(c*x^4+b*x^2+a)^(1/2)+8/35*d*b*x^3*(c*x^4+b*x^2+a)^(1/2)
+3/256*g/c^3*b^4*(c*x^4+b*x^2+a)^(1/2)-3/512*g/c^(7/2)*b^5*ln((1/
2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))+1/105*f/c*x^3*(c*x^4+b*
x^2+a)^(1/2)*b^2+3/64*g/c^(5/2)*b^3*a*ln((1/2*b+c*x^2)/c^(1/2)+(c
*x^4+b*x^2+a)^(1/2))-3/32*g*a^2*b/c^(3/2)*ln((1/2*b+c*x^2)/c^(1/2
)+(c*x^4+b*x^2+a)^(1/2))+1/160*g/c*b^2*x^4*(c*x^4+b*x^2+a)^(1/2)-
1/128*g/c^2*b^3*x^2*(c*x^4+b*x^2+a)^(1/2)+1/35*d/c*x*(c*x^4+b*x^2
+a)^(1/2)*b^2-5/64*g/c^2*b^2*a*(c*x^4+b*x^2+a)^(1/2)-3/32*e/c^(3/
2)*b^2*a*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))-4/315*f/
c^2*x*(c*x^4+b*x^2+a)^(1/2)*b^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^4+bx^2+a\right)^{\frac{3}{2}}\left(gx^3+fx^2+ex+d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2 + a)^(3/2)*(g*x^3 + f*x^2 + e*x + d),x, algorithm="maxima
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)*(g*x^3 + f*x^2 + e*x + d), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(cgx^7+cfx^6+(ce+bg)x^5+(cd+bf)x^4+(be+ag)x^3+aex+(bd+af)x^2+ad\right)\sqrt{cx^4+bx^2+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2 + a)^(3/2)*(g*x^3 + f*x^2 + e*x + d),x, algorithm="fricas
```

```
[Out] integral((c*g*x^7 + c*f*x^6 + (c*e + b*g)*x^5 + (c*d + b*f)*x^4 +
 (b*e + a*g)*x^3 + a*e*x + (b*d + a*f)*x^2 + a*d)*sqrt(c*x^4 + b*
x^2 + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a+bx^2+cx^4\right)^{\frac{3}{2}}\left(d+ex+fx^2+gx^3\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((g*x**3+f*x**2+e*x+d)*(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Integral((a + b*x**2 + c*x**4)**(3/2)*(d + e*x + f*x**2 + g*x**3)
, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^4 + bx^2 + a\right)^{\frac{3}{2}} \left(gx^3 + fx^2 + ex + d\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^(3/2)*(g*x^3 + f*x^2 + e*x + d),x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)*(g*x^3 + f*x^2 + e*x + d), x)
```

3.104 $\int (d+ex+fx^2+gx^3)\sqrt{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=505

$$\frac{x\sqrt{a+bx^2+cx^4}\left(6acf-2b^2f+5bcd\right)}{15c^{3/2}\left(\sqrt{a}+\sqrt{c}x^2\right)}-\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(6acf-2b^2f+5bcd\right)E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{15c^{7/4}\sqrt{a+bx^2+cx^4}}-\frac{\left(b^2-4ac\right)(2ce-bg)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{32c^{5/2}}+\frac{\sqrt[4]{a}\left(2\sqrt{a}\sqrt{c}+b\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(3\sqrt{a}\sqrt{c}f-2bf+5cd\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{30c^{7/4}\sqrt{a+bx^2+cx^4}}+\frac{\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}(2ce-bg)}{16c^2}+\frac{x\sqrt{a+bx^2+cx^4}\left(bf+5cd+3cfx^2\right)}{15c}+\frac{g\left(a+bx^2+cx^4\right)^{3/2}}{6c}$$

```
[Out] ((5*b*c*d - 2*b^2*f + 6*a*c*f)*x*Sqrt[a + b*x^2 + c*x^4])/(15*c^(
3/2)*(Sqrt[a] + Sqrt[c]*x^2)) + ((2*c*e - b*g)*(b + 2*c*x^2)*Sqrt
[a + b*x^2 + c*x^4])/(16*c^2) + (x*(5*c*d + b*f + 3*c*f*x^2)*Sqrt
[a + b*x^2 + c*x^4])/(15*c) + (g*(a + b*x^2 + c*x^4)^(3/2))/(6*c)
 - ((b^2 - 4*a*c)*(2*c*e - b*g)*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*
Sqrt[a + b*x^2 + c*x^4])])/(32*c^(5/2)) - (a^(1/4)*(5*b*c*d - 2*b
^2*f + 6*a*c*f)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/
(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)
], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(15*c^(7/4)*Sqrt[a + b*x^2 + c*x
^4]) + (a^(1/4)*(b + 2*Sqrt[a]*Sqrt[c])*(5*c*d - 2*b*f + 3*Sqrt[a
]*Sqrt[c]*f)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sq
rt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)],
(2 - b/(Sqrt[a]*Sqrt[c]))/4])/(30*c^(7/4)*Sqrt[a + b*x^2 + c*x^4]
)
```

Rubi [A] time = 0.771819, antiderivative size = 505, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$

$$\frac{x\sqrt{a+bx^2+cx^4}\left(6acf-2b^2f+5bcd\right)}{15c^{3/2}\left(\sqrt{a}+\sqrt{c}x^2\right)} - \frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(6acf-2b^2f+5bcd\right)E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{15c^{7/4}\sqrt{a+bx^2+cx^4}} - \frac{\left(b^2-4ac\right)(2ce-bg)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{32c^{5/2}} + \frac{\sqrt[4]{a}\left(2\sqrt{a}\sqrt{c}+b\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(3\sqrt{a}\sqrt{c}f-2bf+5cd\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{30c^{7/4}\sqrt{a+bx^2+cx^4}} + \frac{\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}(2ce-bg)}{16c^2} + \frac{x\sqrt{a+bx^2+cx^4}\left(bf+5cd+3cfx^2\right)}{15c} + \frac{g\left(a+bx^2+cx^4\right)^{3/2}}{6c}$$

Warning: Unable to verify antiderivative.

```
[In]  Int[(d + e*x + f*x^2 + g*x^3)*Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] ((5*b*c*d - 2*b^2*f + 6*a*c*f)*x*Sqrt[a + b*x^2 + c*x^4])/(15*c^(
3/2)*(Sqrt[a] + Sqrt[c]*x^2)) + ((2*c*e - b*g)*(b + 2*c*x^2)*Sqrt
[a + b*x^2 + c*x^4])/(16*c^2) + (x*(5*c*d + b*f + 3*c*f*x^2)*Sqrt
[a + b*x^2 + c*x^4])/(15*c) + (g*(a + b*x^2 + c*x^4)^(3/2))/(6*c)
 - ((b^2 - 4*a*c)*(2*c*e - b*g)*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*
Sqrt[a + b*x^2 + c*x^4])])/(32*c^(5/2)) - (a^(1/4)*(5*b*c*d - 2*b
^2*f + 6*a*c*f)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/
(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)
], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(15*c^(7/4)*Sqrt[a + b*x^2 + c*x
^4]) + (a^(1/4)*(b + 2*Sqrt[a]*Sqrt[c])*(5*c*d - 2*b*f + 3*Sqrt[a
]*Sqrt[c]*f)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sq
rt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)],
(2 - b/(Sqrt[a]*Sqrt[c]))/4])/(30*c^(7/4)*Sqrt[a + b*x^2 + c*x^4]
)
```

Rubi in Sympy [A] time = 85.1694, size = 474, normalized size = 0.94

$$\frac{\sqrt[4]{a}\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}+\sqrt{c}x^2\right)\left(-6acf+2b^2f-5bcd\right)E\left(2\operatorname{atan}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}-\frac{b}{4\sqrt{a}\sqrt{c}}\right)}{15c^{\frac{7}{4}}\sqrt{a+bx^2+cx^4}}$$
$$-\frac{\sqrt[4]{a}\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}+\sqrt{c}x^2\right)\left(\sqrt{a}\sqrt{c}\left(bf-10cd\right)-6acf+2b^2f-5bcd\right)F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}-\frac{b}{4\sqrt{a}\sqrt{c}}\right)}{30c^{\frac{7}{4}}\sqrt{a+bx^2+cx^4}}$$
$$+\frac{g\left(a+bx^2+cx^4\right)^{\frac{3}{2}}}{6c}+\frac{x\sqrt{a+bx^2+cx^4}\left(bf+5cd+3cfx^2\right)}{15c}-\frac{\left(b+2cx^2\right)\left(bg-2ce\right)\sqrt{a+bx^2+cx^4}}{16c^2}$$
$$-\frac{x\sqrt{a+bx^2+cx^4}\left(-6acf+2b^2f-5bcd\right)}{15c^{\frac{3}{2}}\left(\sqrt{a}+\sqrt{c}x^2\right)}+\frac{\left(-4ac+b^2\right)\left(bg-2ce\right)\operatorname{atanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{32c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((g*x**3+f*x**2+e*x+d)*(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] a**(1/4)*sqrt((a + b*x**2 + c*x**4)/(sqrt(a) + sqrt(c)*x**2)**2)*
(sqrt(a) + sqrt(c)*x**2)*(-6*a*c*f + 2*b**2*f - 5*b*c*d)*elliptic
_e(2*atan(c**(1/4)*x/a**(1/4)), 1/2 - b/(4*sqrt(a)*sqrt(c)))/(15*
c**(7/4)*sqrt(a + b*x**2 + c*x**4)) - a**(1/4)*sqrt((a + b*x**2 +
 c*x**4)/(sqrt(a) + sqrt(c)*x**2)**2)*(sqrt(a) + sqrt(c)*x**2)*(s
qrt(a)*sqrt(c)*(b*f - 10*c*d) - 6*a*c*f + 2*b**2*f - 5*b*c*d)*ell
iptic_f(2*atan(c**(1/4)*x/a**(1/4)), 1/2 - b/(4*sqrt(a)*sqrt(c)))
/(30*c**(7/4)*sqrt(a + b*x**2 + c*x**4)) + g*(a + b*x**2 + c*x**4
)**(3/2)/(6*c) + x*sqrt(a + b*x**2 + c*x**4)*(b*f + 5*c*d + 3*c*f
*x**2)/(15*c) - (b + 2*c*x**2)*(b*g - 2*c*e)*sqrt(a + b*x**2 + c*
x**4)/(16*c**2) - x*sqrt(a + b*x**2 + c*x**4)*(-6*a*c*f + 2*b**2*
f - 5*b*c*d)/(15*c**(3/2)*(sqrt(a) + sqrt(c)*x**2)) + (-4*a*c + b
**2)*(b*g - 2*c*e)*atanh((b + 2*c*x**2)/(2*sqrt(c)*sqrt(a + b*x**
2 + c*x**4)))/(32*c**(5/2))
```

Mathematica [C] time = 6.34107, size = 1534, normalized size = 3.04

$$\sqrt{cx^4+bx^2+a}\left(\frac{gx^4}{6}+\frac{fx^3}{5}+\frac{(6ce+bg)x^2}{24c}+\frac{(5cd+bf)x}{15c}+\frac{-3gb^2+6ceb+8acg}{48c^2}\right)$$
$$+\frac{\frac{15g\log\left(2cx^2+b+2\sqrt{c}\sqrt{cx^4+bx^2+a}\right)b^3}{2\sqrt{c}}-\frac{8i\sqrt{2}\left(\sqrt{b^2-4ac}-b\right)f\sqrt{1-\frac{2cx^2}{-b-\sqrt{b^2-4ac}}}\sqrt{1-\frac{2cx^2}{\sqrt{b^2-4ac}-b}}\left(E\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{-\frac{c}{-b-\sqrt{b^2-4ac}}}x\right)\middle|\frac{-b-\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}-b}\right)-F\left(i\sinh^{-}\right.\right.}{\sqrt{-\frac{c}{-b-\sqrt{b^2-4ac}}}\sqrt{cx^4+bx^2+a}}}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x + f*x^2 + g*x^3)*Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] Sqrt[a + b*x^2 + c*x^4]*((6*b*c*e - 3*b^2*g + 8*a*c*g)/(48*c^2) +
 ((5*c*d + b*f)*x)/(15*c) + ((6*c*e + b*g)*x^2)/(24*c) + (f*x^3)/
5 + (g*x^4)/6) + (((20*I)*Sqrt[2]*b*c*(-b + Sqrt[b^2 - 4*a*c])*d*
Sqrt[1 - (2*c*x^2)/(-b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 - (2*c*x^2)/(
-b + Sqrt[b^2 - 4*a*c])]*(EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[-(c/(-
b - Sqrt[b^2 - 4*a*c]))]*x], (-b - Sqrt[b^2 - 4*a*c])/(-b + Sqrt[
b^2 - 4*a*c])] - EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[-(c/(-b - Sqrt[
b^2 - 4*a*c]))]*x], (-b - Sqrt[b^2 - 4*a*c])/(-b + Sqrt[b^2 - 4*a
*c])]))/(Sqrt[-(c/(-b - Sqrt[b^2 - 4*a*c]))]*Sqrt[a + b*x^2 + c*x
^4]) - ((8*I)*Sqrt[2]*b^2*(-b + Sqrt[b^2 - 4*a*c])*f*Sqrt[1 - (2*
c*x^2)/(-b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 - (2*c*x^2)/(-b + Sqrt[b^
2 - 4*a*c])]*(EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[-(c/(-b - Sqrt[b^2
 - 4*a*c]))]*x], (-b - Sqrt[b^2 - 4*a*c])/(-b + Sqrt[b^2 - 4*a*c]
)] - EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[-(c/(-b - Sqrt[b^2 - 4*a*c]
))]*x], (-b - Sqrt[b^2 - 4*a*c])/(-b + Sqrt[b^2 - 4*a*c])]))/(Sqr
t[-(c/(-b - Sqrt[b^2 - 4*a*c]))]*Sqrt[a + b*x^2 + c*x^4]) + ((24*
I)*Sqrt[2]*a*c*(-b + Sqrt[b^2 - 4*a*c])*f*Sqrt[1 - (2*c*x^2)/(-b
- Sqrt[b^2 - 4*a*c])]*Sqrt[1 - (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])
]*(EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[-(c/(-b - Sqrt[b^2 - 4*a*c]))
]*x], (-b - Sqrt[b^2 - 4*a*c])/(-b + Sqrt[b^2 - 4*a*c])] - Ellipt
icF[I*ArcSinh[Sqrt[2]*Sqrt[-(c/(-b - Sqrt[b^2 - 4*a*c]))]*x], (-b
 - Sqrt[b^2 - 4*a*c])/(-b + Sqrt[b^2 - 4*a*c])]))/(Sqrt[-(c/(-b -
 Sqrt[b^2 - 4*a*c]))]*Sqrt[a + b*x^2 + c*x^4]) - ((80*I)*Sqrt[2]*
a*c^2*d*Sqrt[1 - (2*c*x^2)/(-b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 - (2*
c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt
[-(c/(-b - Sqrt[b^2 - 4*a*c]))]*x], (-b - Sqrt[b^2 - 4*a*c])/(-b
+ Sqrt[b^2 - 4*a*c])])/(Sqrt[-(c/(-b - Sqrt[b^2 - 4*a*c]))]*Sqrt[
a + b*x^2 + c*x^4]) + ((8*I)*Sqrt[2]*a*b*c*f*Sqrt[1 - (2*c*x^2)/(
-b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 - (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*
c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[-(c/(-b - Sqrt[b^2 - 4*a*c]
))]*x], (-b - Sqrt[b^2 - 4*a*c])/(-b + Sqrt[b^2 - 4*a*c])])/(Sqrt
[-(c/(-b - Sqrt[b^2 - 4*a*c]))]*Sqrt[a + b*x^2 + c*x^4]) - 15*b^2
*Sqrt[c]*e*Log[b + 2*c*x^2 + 2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4]] +
 60*a*c^(3/2)*e*Log[b + 2*c*x^2 + 2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^
4]] + (15*b^3*g*Log[b + 2*c*x^2 + 2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^
4]])/(2*Sqrt[c]) - 30*a*b*Sqrt[c]*g*Log[b + 2*c*x^2 + 2*Sqrt[c]*S
qrt[a + b*x^2 + c*x^4]])/(240*c^2)
```

Maple [B] time = 0.01, size = 1585, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((g*x^3+f*x^2+e*x+d)*(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] 1/3*x*d*(c*x^4+b*x^2+a)^(1/2)+1/6*d*a*2^(1/2)/((-b+(-4*a*c+b^2)^(
1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+
(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(
```

```
1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-
4*a*c+b^2)^(1/2))/a/c)^(1/2))-1/6*d*b*a*2^(1/2)/((-b+(-4*a*c+b^2)
^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(
b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a
*c+b^2)^(1/2))*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a
)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))+1/6*d*b*a*
2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(
1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^
4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*EllipticE(1/2*x*2^(1/2)*(
(-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2
))/a/c)^(1/2))+1/4*e*(c*x^4+b*x^2+a)^(1/2)*x^2+1/8*e/c*(c*x^4+b*x
^2+a)^(1/2)*b+1/4*e/c^(1/2)*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2
+a)^(1/2))*a-1/16*e/c^(3/2)*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2
+a)^(1/2))*b^2+1/5*f*x^3*(c*x^4+b*x^2+a)^(1/2)+1/15*f*b/c*x*(c*x^
4+b*x^2+a)^(1/2)-1/60*f*b/c*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)
^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+
b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^
(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^
2)^(1/2))/a/c)^(1/2))-1/5*f*a^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/
a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*
c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^
(1/2))*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),
1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))+1/5*f*a^2*2^(1/2)/
((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*
x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+
a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*EllipticE(1/2*x*2^(1/2)*((-b+(-4*
a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^
(1/2))+1/15*f*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-
b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*
x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*b^2/c*Ell
ipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*
b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-1/15*f*a*2^(1/2)/((-b+(-4*a*
c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*
(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b
+(-4*a*c+b^2)^(1/2))*b^2/c*EllipticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b
^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2)
)+1/6*g*(c*x^4+b*x^2+a)^(3/2)/c-1/8*g*b/c*(c*x^4+b*x^2+a)^(1/2)*x
^2-1/16*g*b^2/c^2*(c*x^4+b*x^2+a)^(1/2)-1/8*g*b/c^(3/2)*ln((1/2*b
+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))*a+1/32*g*b^3/c^(5/2)*ln((1
/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^4 + bx^2 + a}\left(gx^3 + fx^2 + ex + d\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(c*x^4 + b*x^2 + a)*(g*x^3 + f*x^2 + e*x + d),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2 + a)*(g*x^3 + f*x^2 + e*x + d), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{cx^4 + bx^2 + a}\left(gx^3 + fx^2 + ex + d\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(c*x^4 + b*x^2 + a)*(g*x^3 + f*x^2 + e*x + d),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2 + a)*(g*x^3 + f*x^2 + e*x + d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a + bx^2 + cx^4} \left(d + ex + fx^2 + gx^3\right) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**3+f*x**2+e*x+d)*(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*x**2 + c*x**4)*(d + e*x + f*x**2 + g*x**3), x
)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^4 + bx^2 + a}\left(gx^3 + fx^2 + ex + d\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(c*x^4 + b*x^2 + a)*(g*x^3 + f*x^2 + e*x + d),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2 + a)*(g*x^3 + f*x^2 + e*x + d), x)
```

3.105 $\int \frac{d+ex+fx^2+gx^3}{\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=359

$$\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\frac{\sqrt{c}d}{\sqrt{a}}+f\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2c^{3/4}\sqrt{a+bx^2+cx^4}}+\frac{(2ce-bg)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4c^{3/2}}-\frac{\sqrt[4]{a}f\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{c^{3/4}\sqrt{a+bx^2+cx^4}}+\frac{fx\sqrt{a+bx^2+cx^4}}{\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)}+\frac{g\sqrt{a+bx^2+cx^4}}{2c}$$

```
[Out] (g*Sqrt[a + b*x^2 + c*x^4])/(2*c) + (f*x*Sqrt[a + b*x^2 + c*x^4])
/(Sqrt[c]*(Sqrt[a] + Sqrt[c]*x^2)) + ((2*c*e - b*g)*ArcTanh[(b +
2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(4*c^(3/2)) - (a^(
1/4)*f*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a]
+ Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b
/(Sqrt[a]*Sqrt[c]))/4])/(c^(3/4)*Sqrt[a + b*x^2 + c*x^4]) + (a^(1
/4)*((Sqrt[c]*d)/Sqrt[a] + f)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b
*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1
/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*c^(3/4)*Sqrt[a
+ b*x^2 + c*x^4])
```

Rubi [A] time = 0.47256, antiderivative size = 359, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\frac{\sqrt{c}d}{\sqrt{a}}+f\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2c^{3/4}\sqrt{a+bx^2+cx^4}}+\frac{(2ce-bg)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4c^{3/2}}-\frac{\sqrt[4]{a}f\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{c^{3/4}\sqrt{a+bx^2+cx^4}}+\frac{fx\sqrt{a+bx^2+cx^4}}{\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)}+\frac{g\sqrt{a+bx^2+cx^4}}{2c}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x + f*x^2 + g*x^3)/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (g*Sqrt[a + b*x^2 + c*x^4])/(2*c) + (f*x*Sqrt[a + b*x^2 + c*x^4])
/(Sqrt[c]*(Sqrt[a] + Sqrt[c]*x^2)) + ((2*c*e - b*g)*ArcTanh[(b +
2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(4*c^(3/2)) - (a^(
1/4)*f*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a]
+ Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b
/(Sqrt[a]*Sqrt[c]))/4])/(c^(3/4)*Sqrt[a + b*x^2 + c*x^4]) + (a^(1
/4)*((Sqrt[c]*d)/Sqrt[a] + f)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b
*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1
/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*c^(3/4)*Sqrt[a
+ b*x^2 + c*x^4])
```

Rubi in Sympy [A] time = 61.9773, size = 325, normalized size = 0.91

$$- \frac{\sqrt[4]{a} f \sqrt{\frac{a + b x^{2} + c x^{4}}{\left(\sqrt{a} + \sqrt{c} x^{2}\right)^{2}}} \left(\sqrt{a} + \sqrt{c} x^{2}\right) E\left(2 \operatorname{atan}{\left(\frac{\sqrt[4]{c} x}{\sqrt[4]{a}} \right)}\middle| \frac{1}{2} - \frac{b}{4 \sqrt{a} \sqrt{c}}\right)}{c^{\frac{3}{4}} \sqrt{a + b x^{2} + c x^{4}}} + \frac{g \sqrt{a + b x^{2} + c x^{4}}}{2 c} + \frac{f x \sqrt{a + b x^{2} + c x^{4}}}{\sqrt{c} \left(\sqrt{a} + \sqrt{c} x^{2}\right)} - \frac{\left(b g - 2 c e\right) \operatorname{atanh}{\left(\frac{b + 2 c x^{2}}{2 \sqrt{c} \sqrt{a + b x^{2} + c x^{4}}} \right)}}{4 c^{\frac{3}{2}}} + \frac{\sqrt{\frac{a + b x^{2} + c x^{4}}{\left(\sqrt{a} + \sqrt{c} x^{2}\right)^{2}}} \left(\sqrt{a} + \sqrt{c} x^{2}\right) \left(\sqrt{a} f + \sqrt{c} d\right) F\left(2 \operatorname{atan}{\left(\frac{\sqrt[4]{c} x}{\sqrt[4]{a}} \right)}\middle| \frac{1}{2} - \frac{b}{4 \sqrt{a} \sqrt{c}}\right)}{2 \sqrt[4]{a} c^{\frac{3}{4}} \sqrt{a + b x^{2} + c x^{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((g*x**3+f*x**2+e*x+d)/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] -a**(1/4)*f*sqrt((a + b*x**2 + c*x**4)/(sqrt(a) + sqrt(c)*x**2)**
2)*(sqrt(a) + sqrt(c)*x**2)*elliptic_e(2*atan(c**(1/4)*x/a**(1/4)
), 1/2 - b/(4*sqrt(a)*sqrt(c)))/(c**(3/4)*sqrt(a + b*x**2 + c*x**
4)) + g*sqrt(a + b*x**2 + c*x**4)/(2*c) + f*x*sqrt(a + b*x**2 + c
*x**4)/(sqrt(c)*(sqrt(a) + sqrt(c)*x**2)) - (b*g - 2*c*e)*atanh((
b + 2*c*x**2)/(2*sqrt(c)*sqrt(a + b*x**2 + c*x**4)))/(4*c**(3/2))
 + sqrt((a + b*x**2 + c*x**4)/(sqrt(a) + sqrt(c)*x**2)**2)*(sqrt(
a) + sqrt(c)*x**2)*(sqrt(a)*f + sqrt(c)*d)*elliptic_f(2*atan(c**(
1/4)*x/a**(1/4)), 1/2 - b/(4*sqrt(a)*sqrt(c)))/(2*a**(1/4)*c**(3/
4)*sqrt(a + b*x**2 + c*x**4))
```

Mathematica [C] time = 2.60187, size = 526, normalized size = 1.47

$$-i\sqrt{2}\sqrt{c}\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}\left(f\left(\sqrt{b^2-4ac}-b\right)+2cd\right)F\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x\right)|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)+\sqrt{\ldots}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x + f*x^2 + g*x^3)/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (I*Sqrt[2]*Sqrt[c]*(-b + Sqrt[b^2 - 4*a*c])*f*Sqrt[(b - Sqrt[b^2
- 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 -
 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*EllipticE[I*ArcSinh[S
qrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c]
)/(b - Sqrt[b^2 - 4*a*c])] - I*Sqrt[2]*Sqrt[c]*(2*c*d + (-b + Sqr
t[b^2 - 4*a*c])*f)*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sq
rt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqr
t[b^2 - 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2
 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])]
+ Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*(2*Sqrt[c]*g*(a + b*x^2 + c*x^4
) + (2*c*e - b*g)*Sqrt[a + b*x^2 + c*x^4]*Log[b + 2*c*x^2 + 2*Sqr
t[c]*Sqrt[a + b*x^2 + c*x^4]]))/(4*c^(3/2)*Sqrt[c/(b + Sqrt[b^2 -
 4*a*c])]*Sqrt[a + b*x^2 + c*x^4])
```

Maple [A] time = 0.008, size = 453, normalized size = 1.3

$$\frac{d\sqrt{2}}{4}\sqrt{4-2\frac{\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\sqrt{4+2\frac{\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}EllipticF\left(\frac{\sqrt{2}x}{2}\sqrt{\frac{1}{a}\left(-b+\sqrt{-4ac+b^2}\right)},\frac{1}{2}\sqrt{-4+2\frac{b\left(b\ldots\right.}{\ldots}}\right.$$

$$+\frac{e}{2}\ln\left(1\left(\frac{b}{2}+cx^2\right)\frac{1}{\sqrt{c}}+\sqrt{cx^4+bx^2+a}\right)\frac{1}{\sqrt{c}}$$

$$-\frac{fa\sqrt{2}}{2}\sqrt{4-2\frac{\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\sqrt{4+2\frac{\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\left(EllipticF\left(\frac{\sqrt{2}x}{2}\sqrt{\frac{1}{a}\left(-b+\sqrt{-4ac+b^2}\right)},\frac{1}{2}\sqrt{-4+2\ldots}\right.\right.$$

$$+\frac{g}{2c}\sqrt{cx^4+bx^2+a}-\frac{bg}{4}\ln\left(1\left(\frac{b}{2}+cx^2\right)\frac{1}{\sqrt{c}}+\sqrt{cx^4+bx^2+a}\right)c^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] 1/4*d*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+
b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)
/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^
(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))+1/
2*e*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))/c^(1/2)-1/2*f
*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2
)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c
*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/
2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^
(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2
))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2)))+1/2*g
*(c*x^4+b*x^2+a)^(1/2)/c-1/4*g*b/c^(3/2)*ln((1/2*b+c*x^2)/c^(1/2)
+(c*x^4+b*x^2+a)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{gx^3 + fx^2 + ex + d}{\sqrt{cx^4 + bx^2 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3 + f*x^2 + e*x + d)/sqrt(c*x^4 + b*x^2 + a),x, algorithm="maxima")
```

```
[Out] integrate((g*x^3 + f*x^2 + e*x + d)/sqrt(c*x^4 + b*x^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{gx^3 + fx^2 + ex + d}{\sqrt{cx^4 + bx^2 + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3 + f*x^2 + e*x + d)/sqrt(c*x^4 + b*x^2 + a),x, algorithm="fricas")
```

```
[Out] integral((g*x^3 + f*x^2 + e*x + d)/sqrt(c*x^4 + b*x^2 + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{d + ex + fx^2 + gx^3}{\sqrt{a + bx^2 + cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((g*x**3+f*x**2+e*x+d)/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral((d + e*x + f*x**2 + g*x**3)/sqrt(a + b*x**2 + c*x**4), x
)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{gx^3 + fx^2 + ex + d}{\sqrt{cx^4 + bx^2 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((g*x^3 + f*x^2 + e*x + d)/sqrt(c*x^4 + b*x^2 + a),x, algorithm="giac")
```

```
[Out] integrate((g*x^3 + f*x^2 + e*x + d)/sqrt(c*x^4 + b*x^2 + a), x)
```

3.106 $\int \frac{d+ex+fx^2+gx^3}{\left(a+bx^2+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=447

$$\frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}(bd-2af)E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{a^{3/4}\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}-\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{c}d-\sqrt{a}f\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2a^{3/4}\sqrt[4]{c}\left(b-2\sqrt{a}\sqrt{c}\right)\sqrt{a+bx^2+cx^4}}-\frac{\sqrt{c}x\sqrt{a+bx^2+cx^4}(bd-2af)}{a\left(b^2-4ac\right)\left(\sqrt{a}+\sqrt{c}x^2\right)}+\frac{x\left(cx^2(bd-2af)-abf-2acd+b^2d\right)}{a\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}-\frac{-2ag+x^2(2ce-bg)+be}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}$$

```
[Out] (x*(b^2*d - 2*a*c*d - a*b*f + c*(b*d - 2*a*f)*x^2))/(a*(b^2 - 4*a
*c)*Sqrt[a + b*x^2 + c*x^4]) - (b*e - 2*a*g + (2*c*e - b*g)*x^2)/
((b^2 - 4*a*c)*Sqrt[a + b*x^2 + c*x^4]) - (Sqrt[c]*(b*d - 2*a*f)*
x*Sqrt[a + b*x^2 + c*x^4])/(a*(b^2 - 4*a*c)*(Sqrt[a] + Sqrt[c]*x^
2)) + (c^(1/4)*(b*d - 2*a*f)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*
x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/
4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(a^(3/4)*(b^2 - 4*a
*c)*Sqrt[a + b*x^2 + c*x^4]) - ((Sqrt[c]*d - Sqrt[a]*f)*(Sqrt[a]
+ Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2
]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c
]))/4])/(2*a^(3/4)*(b - 2*Sqrt[a]*Sqrt[c])*c^(1/4)*Sqrt[a + b*x^2
 + c*x^4])
```

Rubi [A] time = 0.755931, antiderivative size = 447, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.219$

$$\frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}(bd-2af)E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{a^{3/4}\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}-\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{c}d-\sqrt{a}f\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2a^{3/4}\sqrt[4]{c}\left(b-2\sqrt{a}\sqrt{c}\right)\sqrt{a+bx^2+cx^4}}-\frac{\sqrt{c}x\sqrt{a+bx^2+cx^4}(bd-2af)}{a\left(b^2-4ac\right)\left(\sqrt{a}+\sqrt{c}x^2\right)}+\frac{x\left(cx^2(bd-2af)-abf-2acd+b^2d\right)}{a\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}-\frac{-2ag+x^2(2ce-bg)+be}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x + f*x^2 + g*x^3)/(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (x*(b^2*d - 2*a*c*d - a*b*f + c*(b*d - 2*a*f)*x^2))/(a*(b^2 - 4*a
*c)*Sqrt[a + b*x^2 + c*x^4]) - (b*e - 2*a*g + (2*c*e - b*g)*x^2)/
((b^2 - 4*a*c)*Sqrt[a + b*x^2 + c*x^4]) - (Sqrt[c]*(b*d - 2*a*f)*
x*Sqrt[a + b*x^2 + c*x^4])/(a*(b^2 - 4*a*c)*(Sqrt[a] + Sqrt[c]*x^
2)) + (c^(1/4)*(b*d - 2*a*f)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*
x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/
4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(a^(3/4)*(b^2 - 4*a
*c)*Sqrt[a + b*x^2 + c*x^4]) - ((Sqrt[c]*d - Sqrt[a]*f)*(Sqrt[a]
+ Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2
]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c
]))/4])/(2*a^(3/4)*(b - 2*Sqrt[a]*Sqrt[c])*c^(1/4)*Sqrt[a + b*x^2
 + c*x^4])
```

Rubi in Sympy [A] time = 124.925, size = 435, normalized size = 0.97

$$
\frac{\sqrt{c}x\left(2af-bd\right)\sqrt{a+bx^2+cx^4}}{a\left(\sqrt{a}+\sqrt{c}x^2\right)\left(-4ac+b^2\right)}
+\frac{x\left(-abf-2acd+b^2d-cx^3\left(2ag-be\right)-cx^2\left(2af-bd\right)+x\left(-abg-2ace+b^2e\right)\right)}{a\left(-4ac+b^2\right)\sqrt{a+bx^2+cx^4}}
+\frac{\left(2ag-be\right)\sqrt{a+bx^2+cx^4}}{a\left(-4ac+b^2\right)}
-\frac{\sqrt[4]{c}\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}+\sqrt{c}x^2\right)\left(2af-bd\right)E\left(2\operatorname{atan}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}-\frac{b}{4\sqrt{a}\sqrt{c}}\right)}{a^{\frac{3}{4}}\left(-4ac+b^2\right)\sqrt{a+bx^2+cx^4}}
+\frac{\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}+\sqrt{c}x^2\right)\left(\sqrt{a}\left(bf-2cd\right)+\sqrt{c}\left(2af-bd\right)\right)F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}-\frac{b}{4\sqrt{a}\sqrt{c}}\right)}{2a^{\frac{3}{4}}\sqrt[4]{c}\left(-4ac+b^2\right)\sqrt{a+bx^2+cx^4}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((g*x**3+f*x**2+e*x+d)/(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] sqrt(c)*x*(2*a*f - b*d)*sqrt(a + b*x**2 + c*x**4)/(a*(sqrt(a) + s
qrt(c)*x**2)*(-4*a*c + b**2)) + x*(-a*b*f - 2*a*c*d + b**2*d - c*
x**3*(2*a*g - b*e) - c*x**2*(2*a*f - b*d) + x*(-a*b*g - 2*a*c*e +
 b**2*e))/(a*(-4*a*c + b**2)*sqrt(a + b*x**2 + c*x**4)) + (2*a*g
- b*e)*sqrt(a + b*x**2 + c*x**4)/(a*(-4*a*c + b**2)) - c**(1/4)*s
qrt((a + b*x**2 + c*x**4)/(sqrt(a) + sqrt(c)*x**2)**2)*(sqrt(a) +
 sqrt(c)*x**2)*(2*a*f - b*d)*elliptic_e(2*atan(c**(1/4)*x/a**(1/4
)), 1/2 - b/(4*sqrt(a)*sqrt(c)))/(a**(3/4)*(-4*a*c + b**2)*sqrt(a
 + b*x**2 + c*x**4)) + sqrt((a + b*x**2 + c*x**4)/(sqrt(a) + sqrt
```

```
(c)*x**2)**2)*(sqrt(a) + sqrt(c)*x**2)*(sqrt(a)*(b*f - 2*c*d) + s
qrt(c)*(2*a*f - b*d))*elliptic_f(2*atan(c**(1/4)*x/a**(1/4)), 1/2
 - b/(4*sqrt(a)*sqrt(c)))/(2*a**(3/4)*c**(1/4)*(-4*a*c + b**2)*sq
rt(a + b*x**2 + c*x**4))
```

Mathematica [C] time = 2.77069, size = 513, normalized size = 1.15

$$-\frac{4\sqrt{\frac{c}{\sqrt{b^2-4ac}+b}}\left(-2a^2g+ab(e+x(f-gx))+2acx(d+x(e+fx))-bdx\left(b+cx^2\right)\right)-i\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}\sqrt{\frac{-2\sqrt{b^2-4ac}+2b+4cx^2}{b-\sqrt{b^2-4ac}}}}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x + f*x^2 + g*x^3)/(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] -(4*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*(-2*a^2*g - b*d*x*(b + c*x^2)
 + 2*a*c*x*(d + x*(e + f*x)) + a*b*(e + x*(f - g*x))) + I*(-b + S
qrt[b^2 - 4*a*c])*(b*d - 2*a*f)*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c
*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] +
4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqr
t[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqr
t[b^2 - 4*a*c])] - I*(-(b^2*d) + 4*a*c*d + b*Sqrt[b^2 - 4*a*c]*d
- 2*a*Sqrt[b^2 - 4*a*c]*f)*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)
/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x
^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(
b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2
 - 4*a*c])])/(4*a*(b^2 - 4*a*c)*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*S
qrt[a + b*x^2 + c*x^4])
```

Maple [B] time = 0.009, size = 1005, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)^(3/2),x)
```

```
[Out] d*(-2*c*(1/2*b/a/(4*a*c-b^2)*x^3-1/2*(2*a*c-b^2)/a/(4*a*c-b^2)/c*
x)/((x^4+b/c*x^2+1/c*a)*c)^(1/2)+1/4*(1/a-(2*a*c-b^2)/a/(4*a*c-b^
2))*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^
2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(
c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1
/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-1/2*
```

```
b/(4*a*c-b^2)*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-
b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*
x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(Elliptic
F(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+
(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*((-b+(-4*
a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^
(1/2))))+e*(2*c*x^2+b)/(4*a*c-b^2)/(c*x^4+b*x^2+a)^(1/2)+f*(-2*c*
(-1/(4*a*c-b^2)*x^3-1/2*b/(4*a*c-b^2)/c*x)/((x^4+b/c*x^2+1/c*a)*c
)^(1/2)-1/4*b/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/
2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)
^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2
)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(
1/2))/a/c)^(1/2))+c/(4*a*c-b^2)*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2)
)/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*
a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2
)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/
2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x
*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c
+b^2)^(1/2))/a/c)^(1/2))))-g/(c*x^4+b*x^2+a)^(1/2)*(b*x^2+2*a)/(4
*a*c-b^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{gx^3 + fx^2 + ex + d}{(cx^4 + bx^2 + a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3 + f*x^2 + e*x + d)/(c*x^4 + b*x^2 + a)^(3/2),x, algorithm="maxima
```

```
[Out] integrate((g*x^3 + f*x^2 + e*x + d)/(c*x^4 + b*x^2 + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{gx^3 + fx^2 + ex + d}{(cx^4 + bx^2 + a)^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3 + f*x^2 + e*x + d)/(c*x^4 + b*x^2 + a)^(3/2),x, algorithm="fricas
```

```
[Out] integral((g*x^3 + f*x^2 + e*x + d)/(c*x^4 + b*x^2 + a)^(3/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{d + ex + fx^2 + gx^3}{(a + bx^2 + cx^4)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((g*x**3+f*x**2+e*x+d)/(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Integral((d + e*x + f*x**2 + g*x**3)/(a + b*x**2 + c*x**4)**(3/2)
, x)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3 + f*x^2 + e*x + d)/(c*x^4 + b*x^2 + a)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError
```

3.107 $\int \frac{d+ex+fx^2+gx^3}{(a+bx^2+cx^4)^{5/2}}\,dx$

Optimal. Leaf size=680

$$-\frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(6a^{3/2}\sqrt{c}f-3\sqrt{a}b\sqrt{c}d+abf-10acd+2b^2d\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{6a^{7/4}\left(b-2\sqrt{a}\sqrt{c}\right)\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}-\frac{\sqrt{c}x\sqrt{a+bx^2+cx^4}\left(12a^2cf+ab^2f-16abcd+2b^3d\right)}{3a^2\left(b^2-4ac\right)^2\left(\sqrt{a}+\sqrt{c}x^2\right)}+\frac{x\left(cx^2\left(12a^2cf+ab^2f-16abcd+2b^3d\right)+4a^2bcf+20a^2c^2d+ab^3f-17ab^2cd+2b^4d\right)}{3a^2\left(b^2-4ac\right)^2\sqrt{a+bx^2+cx^4}}+\frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(12a^2cf+ab^2f-16abcd+2b^3d\right)E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{3a^{7/4}\left(b^2-4ac\right)^2\sqrt{a+bx^2+cx^4}}+\frac{x\left(cx^2(bd-2af)-abf-2acd+b^2d\right)}{3a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^{3/2}}+\frac{4\left(b+2cx^2\right)(2ce-bg)}{3\left(b^2-4ac\right)^2\sqrt{a+bx^2+cx^4}}-\frac{-2ag+x^2(2ce-bg)+be}{3\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^{3/2}}$$

```
[Out] (x*(b^2*d - 2*a*c*d - a*b*f + c*(b*d - 2*a*f)*x^2))/(3*a*(b^2 - 4
*a*c)*(a + b*x^2 + c*x^4)^(3/2)) - (b*e - 2*a*g + (2*c*e - b*g)*x
^2)/(3*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^(3/2)) + (4*(2*c*e - b*g
)*(b + 2*c*x^2))/(3*(b^2 - 4*a*c)^2*Sqrt[a + b*x^2 + c*x^4]) + (x
*(2*b^4*d - 17*a*b^2*c*d + 20*a^2*c^2*d + a*b^3*f + 4*a^2*b*c*f +
 c*(2*b^3*d - 16*a*b*c*d + a*b^2*f + 12*a^2*c*f)*x^2))/(3*a^2*(b^
2 - 4*a*c)^2*Sqrt[a + b*x^2 + c*x^4]) - (Sqrt[c]*(2*b^3*d - 16*a*
b*c*d + a*b^2*f + 12*a^2*c*f)*x*Sqrt[a + b*x^2 + c*x^4])/(3*a^2*(
b^2 - 4*a*c)^2*(Sqrt[a] + Sqrt[c]*x^2)) + (c^(1/4)*(2*b^3*d - 16*
a*b*c*d + a*b^2*f + 12*a^2*c*f)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a +
 b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^
(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(3*a^(7/4)*(b^2
- 4*a*c)^2*Sqrt[a + b*x^2 + c*x^4]) - (c^(1/4)*(2*b^2*d - 3*Sqrt[
a]*b*Sqrt[c]*d - 10*a*c*d + a*b*f + 6*a^(3/2)*Sqrt[c]*f)*(Sqrt[a]
 + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^
2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[
c]))/4])/(6*a^(7/4)*(b - 2*Sqrt[a]*Sqrt[c])*(b^2 - 4*a*c)*Sqrt[a
+ b*x^2 + c*x^4])
```

Rubi [A] time = 1.23534, antiderivative size = 680, normalized size of antiderivative = 1., number of

steps used = 9, number of rules used = 8, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$-\frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(6a^{3/2}\sqrt{c}f-3\sqrt{a}b\sqrt{c}d+abf-10acd+2b^2d\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{6a^{7/4}\left(b-2\sqrt{a}\sqrt{c}\right)(b^2-4ac)\sqrt{a+bx^2+cx^4}}$$
$$-\frac{\sqrt{c}x\sqrt{a+bx^2+cx^4}\left(12a^2cf+ab^2f-16abcd+2b^3d\right)}{3a^2(b^2-4ac)^2\left(\sqrt{a}+\sqrt{c}x^2\right)}$$
$$+\frac{x\left(cx^2\left(12a^2cf+ab^2f-16abcd+2b^3d\right)+4a^2bcf+20a^2c^2d+ab^3f-17ab^2cd+2b^4d\right)}{3a^2(b^2-4ac)^2\sqrt{a+bx^2+cx^4}}$$
$$+\frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(12a^2cf+ab^2f-16abcd+2b^3d\right)E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{3a^{7/4}(b^2-4ac)^2\sqrt{a+bx^2+cx^4}}$$
$$+\frac{x\left(cx^2(bd-2af)-abf-2acd+b^2d\right)}{3a(b^2-4ac)(a+bx^2+cx^4)^{3/2}}+\frac{4\left(b+2cx^2\right)(2ce-bg)}{3(b^2-4ac)^2\sqrt{a+bx^2+cx^4}}-\frac{-2ag+x^2(2ce-bg)+be}{3(b^2-4ac)(a+bx^2+cx^4)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In]  Int[(d + e*x + f*x^2 + g*x^3)/(a + b*x^2 + c*x^4)^(5/2),x]
```

```
[Out] (x*(b^2*d - 2*a*c*d - a*b*f + c*(b*d - 2*a*f)*x^2))/(3*a*(b^2 - 4
*a*c)*(a + b*x^2 + c*x^4)^(3/2)) - (b*e - 2*a*g + (2*c*e - b*g)*x
^2)/(3*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^(3/2)) + (4*(2*c*e - b*g
)*(b + 2*c*x^2))/(3*(b^2 - 4*a*c)^2*Sqrt[a + b*x^2 + c*x^4]) + (x
*(2*b^4*d - 17*a*b^2*c*d + 20*a^2*c^2*d + a*b^3*f + 4*a^2*b*c*f +
 c*(2*b^3*d - 16*a*b*c*d + a*b^2*f + 12*a^2*c*f)*x^2))/(3*a^2*(b^
2 - 4*a*c)^2*Sqrt[a + b*x^2 + c*x^4]) - (Sqrt[c]*(2*b^3*d - 16*a*
b*c*d + a*b^2*f + 12*a^2*c*f)*x*Sqrt[a + b*x^2 + c*x^4])/(3*a^2*(
b^2 - 4*a*c)^2*(Sqrt[a] + Sqrt[c]*x^2)) + (c^(1/4)*(2*b^3*d - 16*
a*b*c*d + a*b^2*f + 12*a^2*c*f)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a +
 b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^
(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(3*a^(7/4)*(b^2
- 4*a*c)^2*Sqrt[a + b*x^2 + c*x^4]) - (c^(1/4)*(2*b^2*d - 3*Sqrt[
a]*b*Sqrt[c]*d - 10*a*c*d + a*b*f + 6*a^(3/2)*Sqrt[c]*f)*(Sqrt[a]
 + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^
2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[
c]))/4])/(6*a^(7/4)*(b - 2*Sqrt[a]*Sqrt[c])*(b^2 - 4*a*c)*Sqrt[a
+ b*x^2 + c*x^4])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((g*x**3+f*x**2+e*x+d)/(c*x**4+b*x**2+a)**(5/2),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 4.79479, size = 598, normalized size = 0.88

$$-4a\left(b^2-4ac\right)\left(-2a^2g+ab(e+x(f-gx))+2acx(d+x(e+fx))-bdx\left(b+cx^2\right)\right)+4\left(a+bx^2+cx^4\right)\left(4a^2\left(b^2(-g)+bc(2\cdots\right.\right.$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x + f*x^2 + g*x^3)/(a + b*x^2 + c*x^4)^(5/2),x]
```

```
[Out] (-4*a*(b^2 - 4*a*c)*(-2*a^2*g - b*d*x*(b + c*x^2) + 2*a*c*x*(d +
x*(e + f*x)) + a*b*(e + x*(f - g*x))) + 4*(a + b*x^2 + c*x^4)*(2*
b^3*d*x*(b + c*x^2) + a*b*x*(-17*b*c*d + b^2*f - 16*c^2*d*x^2 + b
*c*f*x^2) + 4*a^2*(-(b^2*g) + c^2*x*(5*d + x*(4*e + 3*f*x)) + b*c
*(2*e + x*(f - 2*g*x)))) + (I*Sqrt[2]*Sqrt[(b + Sqrt[b^2 - 4*a*c]
 + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b - Sqrt
[b^2 - 4*a*c])]*(a + b*x^2 + c*x^4)*(-((-b + Sqrt[b^2 - 4*a*c])*(
2*b^3*d - 16*a*b*c*d + a*b^2*f + 12*a^2*c*f)*EllipticE[I*ArcSinh[
Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c
])/(b - Sqrt[b^2 - 4*a*c])]) + (-2*b^4*d + b^3*(2*Sqrt[b^2 - 4*a*
c]*d - a*f) + 4*a*b*c*(-4*Sqrt[b^2 - 4*a*c]*d + a*f) + a*b^2*(18*
c*d + Sqrt[b^2 - 4*a*c]*f) + 4*a^2*c*(-10*c*d + 3*Sqrt[b^2 - 4*a*
c]*f))*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])
]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])]))/Sqrt[c/(
b + Sqrt[b^2 - 4*a*c])])/(12*a^2*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x
^4)^(3/2))
```

Maple [B] time = 0.082, size = 1395, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)^(5/2),x)
```

```
[Out] d*((-1/3*b/a/(4*a*c-b^2)/c*x^3+1/3*(2*a*c-b^2)/a/(4*a*c-b^2)/c^2*
x)*(c*x^4+b*x^2+a)^(1/2)/(x^4+b/c*x^2+1/c*a)^2-2*c*(1/3*b*(8*a*c-
b^2)/(4*a*c-b^2)^2/a^2*x^3-1/6*(20*a^2*c^2-17*a*b^2*c+2*b^4)/a^2/
```

```
(4*a*c-b^2)^2/c*x)/((x^4+b/c*x^2+1/c*a)*c)^(1/2)+1/4*(2/3*(5*a*c-
b^2)/(4*a*c-b^2)/a^2-1/3*(20*a^2*c^2-17*a*b^2*c+2*b^4)/a^2/(4*a*c
-b^2)^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a
*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1
/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^
2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))
-1/3*b*c*(8*a*c-b^2)/(4*a*c-b^2)^2/a*2^(1/2)/((-b+(-4*a*c+b^2)^(1
/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(
-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+
b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^
(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/
2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*
a*c+b^2)^(1/2))/a/c)^(1/2))))+1/3*e*(16*c^3*x^6+24*b*c^2*x^4+24*a
*c^2*x^2+6*b^2*c*x^2+12*a*b*c-b^3)/(c*x^4+b*x^2+a)^(3/2)/(16*a^2*
c^2-8*a*b^2*c+b^4)+f*((2/3/c/(4*a*c-b^2)*x^3+1/3*b/(4*a*c-b^2)/c^
2*x)*(c*x^4+b*x^2+a)^(1/2)/(x^4+b/c*x^2+1/c*a)^2-2*c*(-1/6*(12*a*
c+b^2)/(4*a*c-b^2)^2/a*x^3-1/6*b*(4*a*c+b^2)/c/(4*a*c-b^2)^2/a*x)
/((x^4+b/c*x^2+1/c*a)*c)^(1/2)+1/4*(-1/3*b/(4*a*c-b^2)/a-1/3*b*(4
*a*c+b^2)/(4*a*c-b^2)^2/a)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1
/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2
)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/
2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^
(1/2))/a/c)^(1/2))+1/6*c*(12*a*c+b^2)/(4*a*c-b^2)^2*2^(1/2)/((-b+
(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^
(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1
/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+
b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2
))-EllipticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*
(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))))-1/3*g*(8*b*c^2*x^6+1
2*b^2*c*x^4+12*a*b*c*x^2+3*b^3*x^2+8*a^2*c+2*a*b^2)/(c*x^4+b*x^2+
a)^(3/2)/(16*a^2*c^2-8*a*b^2*c+b^4)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{gx^3 + fx^2 + ex + d}{(cx^4 + bx^2 + a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3 + f*x^2 + e*x + d)/(c*x^4 + b*x^2 + a)^(5/2),x, algorithm="maxima
```

```
[Out] integrate((g*x^3 + f*x^2 + e*x + d)/(c*x^4 + b*x^2 + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{gx^3 + fx^2 + ex + d}{(c^2x^8 + 2\,bcx^6 + (b^2 + 2\,ac)x^4 + 2\,abx^2 + a^2)\sqrt{cx^4 + bx^2 + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3 + f*x^2 + e*x + d)/(c*x^4 + b*x^2 + a)^(5/2),x, algorithm="fricas
```

```
[Out] integral((g*x^3 + f*x^2 + e*x + d)/((c^2*x^8 + 2*b*c*x^6 + (b^2 +
 2*a*c)*x^4 + 2*a*b*x^2 + a^2)*sqrt(c*x^4 + b*x^2 + a)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**3+f*x**2+e*x+d)/(c*x**4+b*x**2+a)**(5/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3 + f*x^2 + e*x + d)/(c*x^4 + b*x^2 + a)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.108 $\int \frac{ag-cgx^4}{(a+bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=19

$$\frac{gx}{\sqrt{a+bx^2+cx^4}}$$

```
[Out] (g*x)/Sqrt[a + b*x^2 + c*x^4]
```

Rubi [A] time = 0.0152488, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036

$$\frac{gx}{\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*g - c*g*x^4)/(a + b*x^2 + c*x^4)^(3/2),x]

[Out] (g*x)/Sqrt[a + b*x^2 + c*x^4]
```

Rubi in Sympy [A] time = 13.0074, size = 17, normalized size = 0.89

$$\frac{gx}{\sqrt{a+bx^2+cx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((-c*g*x**4+a*g)/(c*x**4+b*x**2+a)**(3/2),x)

[Out] g*x/sqrt(a + b*x**2 + c*x**4)
```

Mathematica [A] time = 0.0451867, size = 19, normalized size = 1.

$$\frac{gx}{\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a*g - c*g*x^4)/(a + b*x^2 + c*x^4)^(3/2),x]

[Out] (g*x)/Sqrt[a + b*x^2 + c*x^4]
```

Maple [A] time = 0.007, size = 18, normalized size = 1.

$$gx\frac{1}{\sqrt{cx^4+bx^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-c*g*x^4+a*g)/(c*x^4+b*x^2+a)^(3/2),x)

[Out] g*x/(c*x^4+b*x^2+a)^(1/2)
```

Maxima [A] time = 0.768361, size = 23, normalized size = 1.21

$$\frac{gx}{\sqrt{cx^4+bx^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(c*g*x^4 - a*g)/(c*x^4 + b*x^2 + a)^(3/2),x, algorithm="maxima")

[Out] g*x/sqrt(c*x^4 + b*x^2 + a)
```

Fricas [A] time = 0.259511, size = 23, normalized size = 1.21

$$\frac{gx}{\sqrt{cx^4+bx^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(c*g*x^4 - a*g)/(c*x^4 + b*x^2 + a)^(3/2),x, algorithm="fricas")

[Out] g*x/sqrt(c*x^4 + b*x^2 + a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-g\left(\int\left(-\frac{a}{a\sqrt{a+bx^2+cx^4}+bx^2\sqrt{a+bx^2+cx^4}+cx^4\sqrt{a+bx^2+cx^4}}\right)dx+\int\frac{cx^4}{a\sqrt{a+bx^2+cx^4}+bx^2\sqrt{a+bx^2+cx^4}+cx^4\sqrt{a+bx^2+cx^4}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-c*g*x**4+a*g)/(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] -g*(Integral(-a/(a*sqrt(a + b*x**2 + c*x**4) + b*x**2*sqrt(a + b*
x**2 + c*x**4) + c*x**4*sqrt(a + b*x**2 + c*x**4)), x) + Integral
(c*x**4/(a*sqrt(a + b*x**2 + c*x**4) + b*x**2*sqrt(a + b*x**2 + c
*x**4) + c*x**4*sqrt(a + b*x**2 + c*x**4)), x))
```

GIAC/XCAS [A] time = 0.329233, size = 95, normalized size = 5.

$$\frac{\left(b^4g-8\,ab^2cg+16\,a^2c^2g\right)x}{32\left(ab^4c^2-8\,a^2b^2c^3+16\,a^3c^4\right)\sqrt{cx^4+bx^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(c*g*x^4 - a*g)/(c*x^4 + b*x^2 + a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/32*(b^4*g - 8*a*b^2*c*g + 16*a^2*c^2*g)*x/((a*b^4*c^2 - 8*a^2*b
^2*c^3 + 16*a^3*c^4)*sqrt(c*x^4 + b*x^2 + a))
```

3.109 $\int \frac{ag+ex-cgx^4}{(a+bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=57

$$\frac{gx}{\sqrt{a+bx^2+cx^4}}-\frac{e\left(b+2cx^2\right)}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}$$

```
[Out] (g*x)/Sqrt[a + b*x^2 + c*x^4] - (e*(b + 2*c*x^2))/((b^2 - 4*a*c)*
Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] time = 0.108386, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161

$$\frac{gx}{\sqrt{a+bx^2+cx^4}}-\frac{e\left(b+2cx^2\right)}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a*g + e*x - c*g*x^4)/(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (g*x)/Sqrt[a + b*x^2 + c*x^4] - (e*(b + 2*c*x^2))/((b^2 - 4*a*c)*
Sqrt[a + b*x^2 + c*x^4])
```

Rubi in Sympy [A] time = 17.8847, size = 48, normalized size = 0.84

$$-\frac{be+2cex^2-gx\left(-4ac+b^2\right)}{\left(-4ac+b^2\right)\sqrt{a+bx^2+cx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-c*g*x**4+a*g+e*x)/(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] -(b*e + 2*c*e*x**2 - g*x*(-4*a*c + b**2))/((-4*a*c + b**2)*sqrt(a
 + b*x**2 + c*x**4))
```

Mathematica [A] time = 0.0726, size = 51, normalized size = 0.89

$$\frac{-4acgx+b^2gx-be-2cex^2}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a*g + e*x - c*g*x^4)/(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (-(b*e) + b^2*g*x - 4*a*c*g*x - 2*c*e*x^2)/((b^2 - 4*a*c)*Sqrt[a
+ b*x^2 + c*x^4])
```

Maple [A] time = 0.007, size = 52, normalized size = 0.9

$$\frac{4\,acgx-b^2gx+2\,cex^2+be}{4\,ac-b^2}\frac{1}{\sqrt{cx^4+bx^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-c*g*x^4+a*g+e*x)/(c*x^4+b*x^2+a)^(3/2),x)
```

```
[Out] (4*a*c*g*x-b^2*g*x+2*c*e*x^2+b*e)/(c*x^4+b*x^2+a)^(1/2)/(4*a*c-b^
2)
```

Maxima [A] time = 0.762189, size = 69, normalized size = 1.21

$$-\frac{2\,cex^2+be-\left(b^2g-4\,acg\right)x}{\sqrt{cx^4+bx^2+a}(b^2-4\,ac)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(c*g*x^4 - a*g - e*x)/(c*x^4 + b*x^2 + a)^(3/2),x, algorithm="maxima")
```

```
[Out] -(2*c*e*x^2 + b*e - (b^2*g - 4*a*c*g)*x)/(sqrt(c*x^4 + b*x^2 + a)
*(b^2 - 4*a*c))
```

Fricas [A] time = 0.260076, size = 111, normalized size = 1.95

$$-\frac{\sqrt{cx^4+bx^2+a}\left(2\,cex^2-\left(b^2-4\,ac\right)gx+be\right)}{(b^2c-4\,ac^2)x^4+ab^2-4\,a^2c+(b^3-4\,abc)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(c*g*x^4 - a*g - e*x)/(c*x^4 + b*x^2 + a)^(3/2),x, algorithm="fricas")
```

```
[Out] -sqrt(c*x^4 + b*x^2 + a)*(2*c*e*x^2 - (b^2 - 4*a*c)*g*x + b*e)/((
b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int\left(-\frac{ag}{a\sqrt{a+bx^2+cx^4}+bx^2\sqrt{a+bx^2+cx^4}+cx^4\sqrt{a+bx^2+cx^4}}\right)dx-\int\left(-\frac{ex}{a\sqrt{a+bx^2+cx^4}+bx^2\sqrt{a+bx^2+cx^4}+cx^4\sqrt{a+bx^2+cx^4}}\right)dx-\int\frac{cgx^4}{a\sqrt{a+bx^2+cx^4}+bx^2\sqrt{a+bx^2+cx^4}+cx^4\sqrt{a+bx^2+cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-c*g*x**4+a*g+e*x)/(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] -Integral(-a*g/(a*sqrt(a + b*x**2 + c*x**4) + b*x**2*sqrt(a + b*x
**2 + c*x**4) + c*x**4*sqrt(a + b*x**2 + c*x**4)), x) - Integral(
-e*x/(a*sqrt(a + b*x**2 + c*x**4) + b*x**2*sqrt(a + b*x**2 + c*x*
*4) + c*x**4*sqrt(a + b*x**2 + c*x**4)), x) - Integral(c*g*x**4/(
a*sqrt(a + b*x**2 + c*x**4) + b*x**2*sqrt(a + b*x**2 + c*x**4) +
c*x**4*sqrt(a + b*x**2 + c*x**4)), x)
```

GIAC/XCAS [A] time = 0.315821, size = 228, normalized size = 4.

$$-\frac{\left(\frac{2\,(b^2ce-4\,ac^2e)\,x}{ab^4c^2-8\,a^2b^2c^3+16\,a^3c^4}-\frac{b^4g-8\,ab^2cg+16\,a^2c^2g}{ab^4c^2-8\,a^2b^2c^3+16\,a^3c^4}\right)x+\frac{b^3e-4\,abce}{ab^4c^2-8\,a^2b^2c^3+16\,a^3c^4}}{16\,\sqrt{cx^4+bx^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(c*g*x^4 - a*g - e*x)/(c*x^4 + b*x^2 + a)^(3/2),x, algorithm="giac")
```

```
[Out] -1/16*((2*(b^2*c*e - 4*a*c^2*e)*x/(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16
*a^3*c^4) - (b^4*g - 8*a*b^2*c*g + 16*a^2*c^2*g)/(a*b^4*c^2 - 8*a
^2*b^2*c^3 + 16*a^3*c^4))*x + (b^3*e - 4*a*b*c*e)/(a*b^4*c^2 - 8*
a^2*b^2*c^3 + 16*a^3*c^4))/sqrt(c*x^4 + b*x^2 + a)
```

3.110 $\int \frac{ag+fx^3-cgx^4}{(a+bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=57

$$\frac{f(2a+bx^2)}{(b^2-4ac)\sqrt{a+bx^2+cx^4}}+\frac{gx}{\sqrt{a+bx^2+cx^4}}$$

```
[Out] (g*x)/Sqrt[a + b*x^2 + c*x^4] + (f*(2*a + b*x^2))/((b^2 - 4*a*c)*
Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] time = 0.132408, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152

$$\frac{f(2a+bx^2)}{(b^2-4ac)\sqrt{a+bx^2+cx^4}}+\frac{gx}{\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a*g + f*x^3 - c*g*x^4)/(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (g*x)/Sqrt[a + b*x^2 + c*x^4] + (f*(2*a + b*x^2))/((b^2 - 4*a*c)*
Sqrt[a + b*x^2 + c*x^4])
```

Rubi in Sympy [A] time = 18.3178, size = 46, normalized size = 0.81

$$\frac{2af+bfx^2+gx\left(-4ac+b^2\right)}{\left(-4ac+b^2\right)\sqrt{a+bx^2+cx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-c*g*x**4+f*x**3+a*g)/(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] (2*a*f + b*f*x**2 + g*x*(-4*a*c + b**2))/((-4*a*c + b**2)*sqrt(a
+ b*x**2 + c*x**4))
```

Mathematica [A] time = 0.0751125, size = 48, normalized size = 0.84

$$\frac{2a(f-2cgx)+bx(bg+fx)}{(b^2-4ac)\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a*g + f*x^3 - c*g*x^4)/(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (b*x*(b*g + f*x) + 2*a*(f - 2*c*g*x))/((b^2 - 4*a*c)*Sqrt[a + b*x
^2 + c*x^4])
```

Maple [A]   time = 0.008, size = 53, normalized size = 0.9

$$\frac{4\,acgx - b^2gx - bf x^2 - 2\,f a}{4\,ac - b^2}\frac{1}{\sqrt{cx^4 + bx^2 + a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-c*g*x^4+f*x^3+a*g)/(c*x^4+b*x^2+a)^(3/2),x)
```

```
[Out] (4*a*c*g*x-b^2*g*x-b*f*x^2-2*a*f)/(c*x^4+b*x^2+a)^(1/2)/(4*a*c-b^
2)
```

Maxima [A]   time = 0.771128, size = 66, normalized size = 1.16

$$\frac{bf x^2 + 2\,af + \left(b^2g - 4\,acg\right)x}{\sqrt{cx^4 + bx^2 + a}(b^2 - 4\,ac)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(c*g*x^4 - f*x^3 - a*g)/(c*x^4 + b*x^2 + a)^(3/2),x, algorithm="maxima
```

```
[Out] (b*f*x^2 + 2*a*f + (b^2*g - 4*a*c*g)*x)/(sqrt(c*x^4 + b*x^2 + a)*
(b^2 - 4*a*c))
```

Fricas [A]   time = 0.281872, size = 108, normalized size = 1.89

$$\frac{\sqrt{cx^4 + bx^2 + a}\left(bf x^2 + \left(b^2 - 4\,ac\right)gx + 2\,af\right)}{(b^2c - 4\,ac^2)x^4 + ab^2 - 4\,a^2c + (b^3 - 4\,abc)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(c*g*x^4 - f*x^3 - a*g)/(c*x^4 + b*x^2 + a)^(3/2),x, algorithm="fricas
```

```
[Out] sqrt(c*x^4 + b*x^2 + a)*(b*f*x^2 + (b^2 - 4*a*c)*g*x + 2*a*f)/((b
^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int\left(-\frac{ag}{a\sqrt{a+bx^2+cx^4}+bx^2\sqrt{a+bx^2+cx^4}+cx^4\sqrt{a+bx^2+cx^4}}\right)dx-\int\left(-\frac{fx^3}{a\sqrt{a+bx^2+cx^4}+bx^2\sqrt{a+bx^2+cx^4}+cx^4\sqrt{a+bx^2+cx^4}}\right)dx-\int\frac{cgx^4}{a\sqrt{a+bx^2+cx^4}+bx^2\sqrt{a+bx^2+cx^4}+cx^4\sqrt{a+bx^2+cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-c*g*x**4+f*x**3+a*g)/(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] -Integral(-a*g/(a*sqrt(a + b*x**2 + c*x**4) + b*x**2*sqrt(a + b*x
**2 + c*x**4) + c*x**4*sqrt(a + b*x**2 + c*x**4)), x) - Integral(
-f*x**3/(a*sqrt(a + b*x**2 + c*x**4) + b*x**2*sqrt(a + b*x**2 + c
*x**4) + c*x**4*sqrt(a + b*x**2 + c*x**4)), x) - Integral(c*g*x**
4/(a*sqrt(a + b*x**2 + c*x**4) + b*x**2*sqrt(a + b*x**2 + c*x**4)
 + c*x**4*sqrt(a + b*x**2 + c*x**4)), x)
```

GIAC/XCAS [A] time = 0.314071, size = 221, normalized size = 3.88

$$\frac{\left(\frac{(b^3f-4\,abcf)x}{ab^4c^2-8\,a^2b^2c^3+16\,a^3c^4}+\frac{b^4g-8\,ab^2cg+16\,a^2c^2g}{ab^4c^2-8\,a^2b^2c^3+16\,a^3c^4}\right)x+\frac{2\left(ab^2f-4\,a^2cf\right)}{ab^4c^2-8\,a^2b^2c^3+16\,a^3c^4}}{16\sqrt{cx^4+bx^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(c*g*x^4 - f*x^3 - a*g)/(c*x^4 + b*x^2 + a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/16*(((b^3*f - 4*a*b*c*f)*x/(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*
c^4) + (b^4*g - 8*a*b^2*c*g + 16*a^2*c^2*g)/(a*b^4*c^2 - 8*a^2*b^
2*c^3 + 16*a^3*c^4))*x + 2*(a*b^2*f - 4*a^2*c*f)/(a*b^4*c^2 - 8*a
^2*b^2*c^3 + 16*a^3*c^4))/sqrt(c*x^4 + b*x^2 + a)
```

3.111 $\int \frac{ag+ex+fx^3-cgx^4}{(a+bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=69

$$\frac{gx}{\sqrt{a+bx^2+cx^4}}-\frac{-2af+x^2(2ce-bf)+be}{(b^2-4ac)\sqrt{a+bx^2+cx^4}}$$

```
[Out] (g*x)/Sqrt[a + b*x^2 + c*x^4] - (b*e - 2*a*f + (2*c*e - b*f)*x^2)
/((b^2 - 4*a*c)*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] time = 0.201119, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111

$$\frac{gx}{\sqrt{a+bx^2+cx^4}}-\frac{-2af+x^2(2ce-bf)+be}{(b^2-4ac)\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a*g + e*x + f*x^3 - c*g*x^4)/(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (g*x)/Sqrt[a + b*x^2 + c*x^4] - (b*e - 2*a*f + (2*c*e - b*f)*x^2)
/((b^2 - 4*a*c)*Sqrt[a + b*x^2 + c*x^4])
```

Rubi in Sympy [A] time = 22.647, size = 54, normalized size = 0.78

$$\frac{2af-be+gx\left(-4ac+b^2\right)+x^2\left(bf-2ce\right)}{\left(-4ac+b^2\right)\sqrt{a+bx^2+cx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((-c*g*x**4+f*x**3+a*g+e*x)/(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] (2*a*f - b*e + g*x*(-4*a*c + b**2) + x**2*(b*f - 2*c*e))/((-4*a*c
 + b**2)*sqrt(a + b*x**2 + c*x**4))
```

Mathematica [A] time = 0.0915398, size = 61, normalized size = 0.88

$$\frac{-4acgx+2af+b^2gx-be+bfx^2-2cex^2}{(b^2-4ac)\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a*g + e*x + f*x^3 - c*g*x^4)/(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (-(b*e) + 2*a*f + b^2*g*x - 4*a*c*g*x - 2*c*e*x^2 + b*f*x^2)/((b^
2 - 4*a*c)*Sqrt[a + b*x^2 + c*x^4])
```

Maple [A] time = 0.008, size = 63, normalized size = 0.9

$$\frac{4\,acgx-b^{2}gx-bfx^{2}+2\,cex^{2}-2\,fa+be}{4\,ac-b^{2}}\frac{1}{\sqrt{cx^{4}+bx^{2}+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-c*g*x^4+f*x^3+a*g+e*x)/(c*x^4+b*x^2+a)^(3/2),x)
```

```
[Out] (4*a*c*g*x-b^2*g*x-b*f*x^2+2*c*e*x^2-2*a*f+b*e)/(c*x^4+b*x^2+a)^(
1/2)/(4*a*c-b^2)
```

Maxima [A] time = 0.780754, size = 127, normalized size = 1.84

$$-\frac{\sqrt{cx^{4}+bx^{2}+a}\left((2\,ce-bf)x^{2}+be-2\,af-\left(b^{2}g-4\,acg\right)x\right)}{(b^{2}c-4\,ac^{2})x^{4}+ab^{2}-4\,a^{2}c+(b^{3}-4\,abc)x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(c*g*x^4 - f*x^3 - a*g - e*x)/(c*x^4 + b*x^2 + a)^(3/2),x, algorithm="m
```

```
[Out] -sqrt(c*x^4 + b*x^2 + a)*((2*c*e - b*f)*x^2 + b*e - 2*a*f - (b^2*
g - 4*a*c*g)*x)/((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 -
 4*a*b*c)*x^2)
```

Fricas [A] time = 0.266905, size = 124, normalized size = 1.8

$$\frac{\sqrt{cx^{4}+bx^{2}+a}\left(\left(b^{2}-4\,ac\right)gx-(2\,ce-bf)x^{2}-be+2\,af\right)}{(b^{2}c-4\,ac^{2})x^{4}+ab^{2}-4\,a^{2}c+(b^{3}-4\,abc)x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(c*g*x^4 - f*x^3 - a*g - e*x)/(c*x^4 + b*x^2 + a)^(3/2),x, algorithm="f
```

```
[Out] sqrt(c*x^4 + b*x^2 + a)*((b^2 - 4*a*c)*g*x - (2*c*e - b*f)*x^2 -
b*e + 2*a*f)/((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*
a*b*c)*x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int\left(-\frac{ag}{a\sqrt{a+bx^2+cx^4}+bx^2\sqrt{a+bx^2+cx^4}+cx^4\sqrt{a+bx^2+cx^4}}\right)dx$$
$$-\int\left(-\frac{ex}{a\sqrt{a+bx^2+cx^4}+bx^2\sqrt{a+bx^2+cx^4}+cx^4\sqrt{a+bx^2+cx^4}}\right)dx$$
$$-\int\left(-\frac{fx^3}{a\sqrt{a+bx^2+cx^4}+bx^2\sqrt{a+bx^2+cx^4}+cx^4\sqrt{a+bx^2+cx^4}}\right)dx$$
$$-\int\frac{cgx^4}{a\sqrt{a+bx^2+cx^4}+bx^2\sqrt{a+bx^2+cx^4}+cx^4\sqrt{a+bx^2+cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-c*g*x**4+f*x**3+a*g+e*x)/(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] -Integral(-a*g/(a*sqrt(a + b*x**2 + c*x**4) + b*x**2*sqrt(a + b*x
**2 + c*x**4) + c*x**4*sqrt(a + b*x**2 + c*x**4)), x) - Integral(
-e*x/(a*sqrt(a + b*x**2 + c*x**4) + b*x**2*sqrt(a + b*x**2 + c*x*
*4) + c*x**4*sqrt(a + b*x**2 + c*x**4)), x) - Integral(-f*x**3/(a
*sqrt(a + b*x**2 + c*x**4) + b*x**2*sqrt(a + b*x**2 + c*x**4) + c
*x**4*sqrt(a + b*x**2 + c*x**4)), x) - Integral(c*g*x**4/(a*sqrt(
a + b*x**2 + c*x**4) + b*x**2*sqrt(a + b*x**2 + c*x**4) + c*x**4*
sqrt(a + b*x**2 + c*x**4)), x)
```

GIAC/XCAS [A] time = 0.313919, size = 262, normalized size = 3.8

$$\frac{\left(\frac{\left(b^3f-4\,abcf-2\,b^2ce+8\,ac^2e\right)x}{ab^4c^2-8\,a^2b^2c^3+16\,a^3c^4}+\frac{b^4g-8\,ab^2cg+16\,a^2c^2g}{ab^4c^2-8\,a^2b^2c^3+16\,a^3c^4}\right)x+\frac{2\,ab^2f-8\,a^2cf-b^3e+4\,abce}{ab^4c^2-8\,a^2b^2c^3+16\,a^3c^4}}{8\,\sqrt{cx^4+bx^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(c*g*x^4 - f*x^3 - a*g - e*x)/(c*x^4 + b*x^2 + a)^(3/2),x, algorithm="g
```

```
[Out] 1/8*(((b^3*f - 4*a*b*c*f - 2*b^2*c*e + 8*a*c^2*e)*x/(a*b^4*c^2 -
8*a^2*b^2*c^3 + 16*a^3*c^4) + (b^4*g - 8*a*b^2*c*g + 16*a^2*c^2*g
```

```
)/(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4))*x + (2*a*b^2*f - 8*a^
2*c*f - b^3*e + 4*a*b*c*e)/(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^
4))/sqrt(c*x^4 + b*x^2 + a)
```

4 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Mathematica/Rubi followed by one for Maple. The following are links to the source code.

The following are the listing of the above functions.

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)
(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)

(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)

GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)
```

```
ExpnType[expn_] :=
  If[AtomQ[expn], 1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]]
```

```
ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]

SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]
HypergeometricFunctionQ[func_] := MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]
AppellFunctionQ[func_] := MemberQ[{AppellF1},func]
```

```
# File: GradeAntiderivative.mpl Original version thanks to Albert Rich emailed on 03/21/2017
#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);
  #This check below actually is not needed, since I only call this grading only for
  #passed integrals. i.e. I check for "F" before calling this.

  if not type(result,freeof('int')) then
     return "F";
  end if;

  if ExpnType_result<=ExpnType_optimal then
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
         if leaf_count_result<=2*leaf_count_optimal then
            return "A";
         else
            return "B";
         end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      return "C";
  end if
end proc:
```

```
# is_contains_complex(result) takes expressions and returns true if it contains "I"
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
      1 else
    max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
        1 else
      max(2,ExpnType(op(1,expn))) end if else
    max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' or op(0,expn)='integrate' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:
```

```
ElementaryFunctionQ := proc(func)
  member(func,[exp,log,ln, sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[erf,erfc,erfi,FresnelS,FresnelC,Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:
AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
    op(2,u) else
  apply(op(0,u),op(2..nops(u),u))
  end if
end proc:
#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```